

# Phi Music On Fibonacci Sequence And Golden Ratio

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**Abstract:** The Music perception based on Fibonacci sequence is Phi Music. Fibonacci sequence has every new number is the result of adding the two previous ones: like 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89.... etc. In mathematical notation we can describe the Fibonacci Sequence as  $X_n = X_{n-1} + X_{n-2}$ . The ratio of two adjacent Fibonacci numbers is the golden number or Phi ( $\Phi$ ). Musical frequencies are based on Fibonacci ratios, in Tempered Scale, i.e. an octave of 13 notes consists of 8 white keys and 5 black keys. In a scale, the middle dominant note is the 5th note (Pa), and 8th note (Sa') of all 13 notes that make up the octave of tempered scale. The Fibonacci Sequence, in the Music composition and in Tabala Talls, is described in this paper.

**Keywords:** Phi Music, Music theory, Fibonacci sequence, Golden ratio, Musical composition. Modulations in Music.

**INTRODUCTION:** Music is melodious harmonic sound, and sound is the mechanical energy of the vibrating medium developed from the mechanical energy of the vibrating source. Gioseffo Zarlino (1560) an Italian music theorist and composer of the Renaissance. He made a large contribution to the theory of counterpoint as well as to musical tuning. Zarlino was the first to theorize the primacy of triad over interval as a means of structuring harmony. His exposition of just intonation based on proportions within the "Senario" (1, 2, 3, 4, 5, 6) and 8 is a departure from the previously established Pythagorean diatonic system as passed on by Boethius. [1]

Wolfgang Amadeus Mozart was a prolific and influential composer of the Classical period. Despite his short life, his rapid pace of composition resulted in more than 800 works representing virtually every Western classical genre of his time. Many of these compositions are acknowledged as pinnacles of the music. Mozart is widely regarded as one of the greatest composers in the history of Western music. [2]

Elliott Sharp an in New York City released over eighty-five recordings ranging from contemporary classical, avant-garde, free improvisation, jazz, experimental, and orchestral music to noise, no wave, and electronic music. He pioneered the use of personal computers in live performance with his Virtual Stance project of the 1980s. He has used algorithms and fibonacci numbers in experimental composition since the 1970s, and has cited literature as an inspiration for his music and often favors improvisation. He is an inveterate performer, playing mainly guitar, saxophone and bass clarinet. Sharp has led many ensembles over the years, including the blues-oriented Terraplane, Orchestra Carbon, and SysOrk, a group dedicated to the realization of algorithmic and graphic scores. [3]

Gresham College London UK, Now provides free Lectures on Mathematics of music. Exempli gratia: Musical Consonance and Dissonance: The Good, Bad and Beautifully Ugly by Musician Milton Mermikides and Carla J. Pinkney studied Statistics in Music on Great Music and the Fibonacci Sequence. [4]

Rochelle Gutierrez (2023) made a connection with tabala and mathematics in both the long and short combinations. [5] Tall or beat is a musical meter of composition. There is a group of tall characterized by Matra, i.e. number of beats in a defined time cycle. The starting beat of each cycle is called Sama or slow, the double speed of beats is called Madhya or Theka (middle), and four times the speed of beats is called Droot or fast. [6]

Objective of this paper is to explore the phi music by literature search and apply Fibonacci sequence in many popular bhartiya tabala Talls European mathematicians in Gresham College, Madam Kaija Saariaho, Pr Milton Mermikides- guitarist, Pr. Sarah Hart Geometry. Milton Babbitt, an American musician who set a set theory among notes of an octave. Pierre Boulez (2011) played double Piano and set group theory in tone rows describing slow notes and sharp notes.

**MUSICAL SCALE:** The vibration must be within a certain limit of audibility, i.e. 20 vibrations per second to 20,000 vibrations per second (frequency). Less than 20 vibrations per second are ultrasonic and upper than 20000 vibrations per second is supersonic sound. [7]. The science of sound is called Acoustics. The music is a melodious (interesting) acoustics.

Frequency  $n = 1 / T$  Or  $T = 1 / n$  or  $n T = 1$ .  
 Sound energy moves in the waveform in the medium with a definite wavelength  $\lambda$   
 Speed of Sound  $C = n \lambda$ . Or, Length of sound wave  $\lambda = C / n = C.T$

A musical scale is a series of notes (keys or reads) selected out of hundreds of notes available. Such that they produce a harmonic effect, when, they are played together. It is a combination of seven notes called Sargam, Saptaka = Octave. In the Bharteeya music system, they are: Sa, Re, Ga, Ma, Pa, Dha, Ni, Sa, Generally, There are three octaves in the harmonium. Slow octave (Mand), medium octave (Madhya), and tempered octave. (Teebra Saptaka) The slowest normal vowel should be lower Sa of harmonium note = read = Key, next notes of Sargam increasing frequency (intensity) reaching Eighth Sa Reaches at just double frequency  $n$ . [8]  
 A musical scale is a series of notes (keys) selected out of infinite variety of notes available. They produce harmonious effect when they are played together. The octave is taken as a group of notes. The construction of a musical scale depends on how an octave is divided in to number of notes. In this way musical Diatonic scale are 08 key octave, such that, Sa, Re, Ga, Ma, Pa, Dha, Ni, and again Sa. They are used as pure form, and Gioseffo Zarlino (1565) named it Diatonic Scale and when Sa(s), Re (s), Ma (s), Pa(s) Dha(s), 5 notes are more used of their adjacent notes. Thus it becomes 13 notes scale is called Tempered scale, Thus the Octave in Tempered scales have notes Sa, Sa(s), Re, Re(s), Ga, Ma, Ma(s), Pa, Pa(s), Dha, Dha(s), Ni, and Sa. [10]

The Fibonacci series appears in the foundation of aspects of art, beauty and life. Even music has a foundation in the series, such that, There are 13 notes in the span of any note through its octave. A scale is composed of 8 notes, of which the 5th and 3rd notes create the basic foundation of all chords, and are based on a tone which are combination of 2 steps and 1 step from the root tone that is the 1st note of the scale [2]. It is less well known that these numbers also underlie certain musical intervals and compositions. This paper considers the presence of the Fibonacci sequence in the structure of the octave of tempered scale and also notes the use of the golden ratio in instrument design and in certain musical works of composer Howat R. Debussy. [11]

## MUSICAL INTERVALS:

The pitch of a musical note is expressed in frequency or number of vibrations per second. The absolute frequency of the notes is immaterial because the ear can recognize only the ratio in which their frequency alters but not their numerical differences. The musical interval between two notes is expressed by the ratio of their frequencies. If the two notes frequencies  $m$  and  $n$ , the musical interval between them is equal to  $m/n$ . If the two notes change in such a way that, their ratio remains the same to be harmonic (lyrics) Music.

Table 1 represents a relation of the ratio between the notes (keys) of an octave, (Sa, Re, Ga, Ma, Pa, Dha, Ni, Sa) and their Savart Units with their intervals. Savart divided Octave into Zero to 300 Savart units (s). Where the First notes Sa has 0s. And, the last notes of octave Sa has 300S. Savart 0S to 300S is in between six notes that have their Savart unit in the ratio of their wave frequencies. If three successive notes have their frequencies  $m$ ,  $n$ , and  $p$ . the interval between the successive notes are  $m/n$  and  $n/p$ . The interval between the first and the third,  $m/p$  is obtained by multiplication of two intervals  $m/n$  and  $n/p$ . Therefore, any two intervals are said to be added logarithms of ratios, when their frequency ratio are multiplied.

Thus,  $\log M/p = \log m/n + \log n/p$ .

## SAVARTS LAW

String Length is constant.  
 Frequency =  $\sqrt{T/m}$ .  
 Here,  $T$  = tension and,  $m$  = mass.  
 Therefore thin wire and adjusted tension at calibration. [10]  
 Savart described the frequency of the string;  $N_1^2 = N^2 + N_0^2$ .  
 Where  $N_1$  = Actual frequency,  $N$  = ideal frequency,  
 and  $N_0$  = frequency due to stiffness of string.  
 The equation of motion of the vibrating string is represented as  
 $\frac{d^2y}{dt^2} = T \cdot \frac{d^2y}{m dx^2} = v^2 \frac{d^2y}{dx^2}$ .

**Table1. Musical frequencies and their intervals in temporal scale (13 notes octave):**

Srl.	Name of Indian Octave	Indian Octave	Western Octave	Frequency	Savart Frequency	Frequency Interval from First Note	Value of Frequency Interval
1	Shhadaj	Sa	C	264	00	1/1	1.00
2	Shhadaj sharp	Sa#	C#	280	25.0	16/15	1.06666...
3	Rishabh	Re	D	297	51.0	9/8	1.125
4	Rishabh sharp	Re#	D#	314	75.0	6/5	1.2
5	Gandhar	Ga,	E	330	96.6	5/4	1.25
6	Madhyam	Ma	F	352	124.5	4/3	1.33333...
7	Madhyam sharp	Ma#	F#	374	160	64/45	1.4222...
8	Pancham	Pa	G	396	175.5	3/2	1.5
9	Pancham sharp	Pa#	G#	418	200	8/5	1.6
10	Dhaiwat	Dha	A	440	221.1	5/3	<b>1.666..</b> Fibonacci ratio
11	Dhaiwat sharp	Dha#	A#	484	250	16/9	1.7777...
12	Nishad	Ni	B	495	272.1	15/8	1.875
13	Shhadaj''	Sa''	C''	528	300	2/1	2.0

Here,  $X$  and  $y$  are two points of the string.  
 $T =$  stiffness, and  $m$  is the mass of the vibrating string.  
 Where  $v = \sqrt{T/m}$

The interval between two notes is expressed in the logarithm of any number of component intervals. Hence, a scale can be set up by dividing octave logarithmically. The Logarithm to the Base 10 of any two successive notes apart differs by 0.03103. Any two notes are said to be a centi-octave apart if frequency differs by  $n \cdot 0.03103 / 100$ . Here, two notes are said to be Sa-centi. If the logarithm of their frequencies differs by  $0.03103 / 1200$ , then it is called Sa Cent. Sawart S is the logarithm of their frequencies that differs by  $S/1000$ . Thus a whole Octave is equal to 100 centi octaves=1200 cent octaves=301.03 S.

Savart is a convenient size of unit, but it is very inconvenient to have friction for the octave. Therefore a slight modification of Savart is 300S in an Octave.[9]

**GOLDEN RATIO ( $\Phi$ ):** This paper also presents an original composition based on Fibonacci numbers; to explore the inherent aesthetic appeal of this mathematical phenomenon 8 divided by 13 equals **0.61538...** the approximate **Golden Ratio**. The golden ratio, denoted by ( $\Phi$ ), in decimal form is approximately 1.618033988 ...1.618 033988 ... When dividing each term by its previous term in the Fibonacci Sequence, the golden ratio can be seen in the answer. Furthermore, this value becomes more accurate as we get farther into the sequence. For example:  $13/8=1.625$ ;  $13/8=1.625$ ;  $89/55=1.61818$ ;  $89/55=1.61818$ ;  $610/377=1.61803$ ;  $610/377=1.61803$ ; and so on, converging to 1.618033988 ...1.618033988 .... This golden ratio is what is commonly known as "The Divine Proportion" due to its frequency in the natural world.[ 12, 13, 14, 15]

Table 1 represents the musical frequencies and their intervals of 13 notes scale or Tempered scale. Here, in the tenth row of (of 'A' Note) is based on Fibonacci ratio. And all Notes are about of Fibonacci ratio. Notes in the scale of western music are based on natural harmonics that are created by ratios of frequencies. Ratios found in the first seven numbers of the Fibonacci series ( 0, 1, 1, 2, 3, 5, 8 ) are related to key frequencies of musical notes.

**FIBONACCI SEQUENCE IN MUSICAL SCALE:** Fibonacci sequence of numbers and the associated "golden ratio" are manifested in nature and in certain works of Music An octave is the interval between a note and the next instance of that same note name on the Harmonium . In Figure1 an octave interval is from the C on the left to the C on the right of the keyboard. An octave spans 13 notes. For example, an octave starting on C would include C, C#, D, D#, E, F, F#, G, G#, A, A#, B, C. This is called a "chromatic" scale. The interval between two consecutive notes in a chromatic scale is a "semitone" interval. A "whole-tone" interval is twice a semitone interval. The interval between F and G[16] Composers have been using the Fibonacci Sequence and the Golden Ratio for hundreds of years to compose amazing music. A sonata is traditionally made up of 3 parts: exposition, development and recapitulation, in which the musical subject matter is stated, explored or expanded, and repeated. It transpires that Mozart arranged his piano sonatas so that the number of bars (a bar is a small segment of music that holds a certain number of beats) in the development and recapitulation divided by the number of bars in the exposition would equal approximately **1.618**, the Golden Ratio.

**FIBONACCI SEQUENCE IN TABALA TALLS:** Assume, There is a running music. A dancer minds of the emotion of that music, tabAla players listen the tall, string player think of melody of running octave. Linguistic men think of song paraphrase. And common audition enjoys on observation of music function. A mathematician observes mathematical relation among octaves and tall like Fibonacci sequence, logarithm law, set theory etc. Even some musicians construct music following pi ratio of octaves (3.1). Or An artist make a Xronomorphy for light and sound programme. Therefore, the perspective of music perception is different for different types of auditions . Table 2 represents some Tall of tabala, their matra ( points of beats) it's parts of playing boll ( speaking of tabala) but in view of Fibonacci sequence , the parts of matra- boll of the tall is reconstructed as shown in table2 just before Fibonacci sequence of tabala tall. A Phi Tall of tabala could be played on Fibonacci sequence: I e. 1,2,3,5 =11 beats ( matra ) it is also called Rudra Tall. The boll of fibonacci tall would be such that " tom, tatom, takita, tagitagetom" . And "Dom dadom, dhadhiga, Dhagi dhage dham ".[20-27]

**Table2**                      **Fibonacci,**                      **sequence**                      **in**                      **Tabala**                      **talls:**

Srl NO	Tall	Boll (Speaking )	Parts	Matra (Points)	Fibonacci, Parts	Fibonacci, sequence
1	Dadra	Dha Dhi Na , Dha Ti Na	2	6	3	1,2,3,
2	Kaharawa	Dha Ge Na , Tu Na Ka, Dhi Na.	3	8	2	3,5
3	Jhap tall	Dhi Na , Dhi Dhee, Na , Ti Na , Dhi Dhee Na	4	10	3	2,3,5

4	Bhajan Theka	Dhin Na Dhin Dhin Na. Tin Na Tin Tin Na	2	10	3	2,3,5
5	Drupad	Ka Ta, Ta Ta, Ki Ta , Ga Di Gan Dha	3	10	3	2,3,5
6	Ek Tall	Dhin Dhin Dha Ge , Tir Kit , Too Na. Tin Tin Ta Ge , Dhir Kit , Dhoo Na	6	12	5	1,1,2,3,5
7	Chaotall (4 Talls)	Dha Dha , Di Tta , Kit Dha , Tin Ta, Ga Di, Gan Dha	6	12	5	1,1,2,3,5
8	Tritall	Dha Dhin Dhin Dha, Dha Tin Tin Ta. Ta, Tin Tin Ta, Dha Dhin Dhin Dha	4	16	5	1,2,3,5,5

**FIBONACCI SEQUENCE IN MUSIC:** The Fibonacci Sequence is an integral part of Western harmony and music scales. Some examples are given below:

- 1 An octave on the piano consists of 13 notes: 8 white keys and 5 black keys.
- 2 A scale consists of 8 notes, of which the 3rd and 5th notes make up a basic chord
- 3 In a scale, the dominant note is the 5th note, which is also the 8th note of all 13 notes that make up the octave
- 4 in an octave, 8 divided by 13 equals, such that Semitone scale / temperate scale = **0.61538...** the approximate **Golden Ratio**

There are all the numbers which feature in the Fibonacci Sequence: 0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610... composers have been using the Fibonacci Sequence and the Golden Ratio for hundreds of years to compose amazing music. For example, many of Mozart's piano sonatas are based on the Golden Ratio. A sonata is traditionally made up of 3 parts: exposition, development and recapitulation, in which the musical subject matter is stated, explored or expanded, and repeated. It transpires that Mozart arranged his piano sonatas so that the number of bars (a bar is a small segment of music that holds a certain number of beats) in the development and recapitulation divided by the number of bars in the exposition would equal approximately **1.618**, the Golden Ratio. This can be seen in many of Mozart's works, but as an illustrative example we take the first movement of his Piano Sonata No. 1 in C Major. C is the sonata's first movement as a whole, B is the development and recapitulation, and A is the exposition. The exposition has 38 bars and the development and recapitulation has 62. The first movement as a whole contains 100 bars. 62 divided by 38 equals 1.63 (approximately the **Golden Ratio**). The Golden Ratio has also been used in other areas of music including being used to craft violins, saxophones mouthpieces, and in speaker wires. [29-34]

**FIBONACCI MUSIC BOX:** Samurai guitarist and Peter Bence Pianist called it python music. They made a quartz like a graph paper, in the intersecting point is fit and illumination of colored light. They made a digit transition of big numbers into a small using first digit of the mode 10. Mode is here just being divided by ten and remaining balance digit is taken for phi music sequence. In The pigeon hole model in Fibonacci music box if mode ten is taken, the Fibonacci sequence will be such that, 1,1,2,3,5,8,3,1,4,5,9,4,3,7,0. Apart from this, if a music box is of mode 06, big number transition in to small number is made by formula :  $B / A = C / B = D / C = E / D \dots = \text{Phi}$ . Or we can divide to the big Fibonacci series by the mode number and balance is took as transited Fibonacci digits.

For example : in Fibonacci sequence = 13, 21, 44, ^ 65  
 If mode = 06, dividing by 06, = 13/6, 21/6, 44/6 ^ .65/ 6.  
 Resultant number for application in music box 1, 3, 2, 5.



Phi value is about  $\sim 1.61$ ....such that  $21/13 = 44/21 = 65/44 = \dots = 1.615 \sim 2.095 \sim 1.477 = \text{phi}$ .

Figure 1 represents an Octave in Middle scale of my Casio (model: SA-75). It's clear that in Fibonacci sequence, Any Fibonacci number  $F_n = (F_{n-1}) + (F_{n-2})$ . If we plot a graph assuming  $F_{n-1}$  value on x axis and  $F_n$  in y axis. Then Fibonacci graph line goes in exponential theory as it represented in figure 2. Therefore the transition into small number by mode system make applicable in the graph paper simulated equally on music box screens having light bulb at intersecting points. When phi music is played concerning bulbs glow light and make beautiful line on the screen of music box of an orchestra music program. Figure 3 represents its just starting glows. Phi is the ratio among two adjacent fibonacci numbers. But it is proved to be natural curvature of living system of nature. like curvature of Conch (Aliger gigas ; Molusca), archetecture of hives of Apis mellifera (honey bee home ) and mini parts of Annona squamosa fruits micro flowers of Anthocephalus kadamba. phi curvature is represented in fig 4.

Fabinacci music is a digital music ,always used base 10. But Octaves have notes 7 or 12, in .... Scale and temperate smultaniously, Thus, the fabonacci music is a more elegant solution than a number as arbitrary as 10. Devidmann views of echo effectis specifically given the feeling of discovering a mystic cave, with every different cycle being more mysterious and magical than the last, especially with that echo effect. [21] Fi music box is modeling of Fi music by lamping on a square board following the Fibonacci relation In the frame work of fi music box their on screens, X axis shows the value of  $F_{n-1}$  ( before number and in y axis is value of  $F_n$ . There are infinite pigeon holes point ( P ) on the crossing points on various values of x and y axis. When phi music is played the multicolored bulbs on the pigeon hole Lights. And line form is also found to make its more attractive in the light and sound programs of orchestra industries.

**CONCLUSION:** Conclusion It is clear that the Fibonacci sequence of numbers and the golden ratio are manifested in music. The numbers are present in the octave, the foundational unit of melody and harmony. Stradivarius used the golden ratio to make the greatest string instruments ever created. Roy Howat's research on Debussy's works shows that the composer used the golden ratio and Fibonacci numbers to structure his music. The Fibonacci Composition reveals the inherent aesthetic appeal of this mathematical phenomenon. Fibonacci numbers harmonise naturally and the exponential growth which the Fibonacci sequence typically defines in nature is made present in music by using Fibonacci 77 notes. Perhaps it is present in other categories of things, such as tastes or smells. It has already been discovered in quantum mechanics and in time.



Figure1- Octave in Middle scale

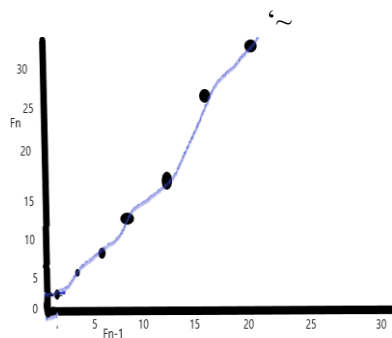


Figure2-FibonacciGraph

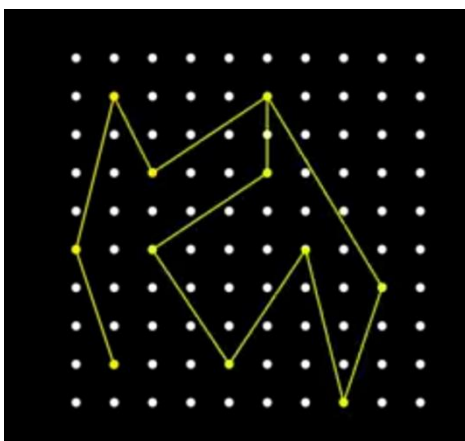


Figure3-Fibonacci Music Box (From Ref 30)

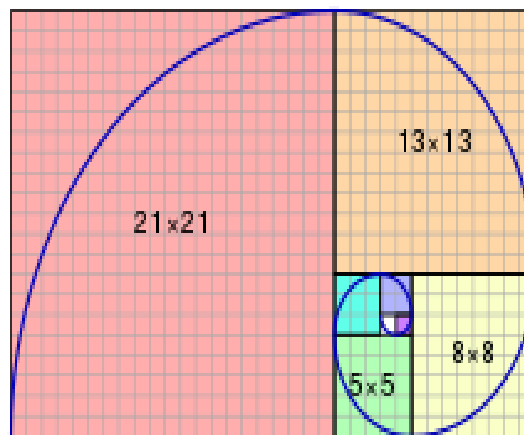


Figure4- Phi Curvature (From Ref 34)

**FURTHER SCOPE OF THE STUDY :** This is resonance = same frequency matching. Simba canis barks on my pipe sounding. Equus ass dechu dechu does on violin. Galope Dances on Indian Drumm...on Dha & Ni.  $22/7\pi$  {  $\pi$  } law is also have a place in Music. I was looking log law in music read Descartes in seventeenth century. But with those literature. is Pi music, So sound wave scatter in all angles ...making spherical ballooning.. Assume a Bird Cuckoo is cooking on a branch of Mengifera tree and its Sound source will be center of spherical moving of waves, if timings is frequency beTime the increasing diameter Or it's just minus- value i.e wavelength.

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**DECLARATION:** I firmly declare that the content used in this paper from external sources is properly cited in references. And for any liability regarding plagiarism would be my responsibility.

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