The Ships Containers to storage areas Assignment Problems: Complexity and a Case study

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***Abstract*- In order to reduce the time the ship stays in the port, we consider the Ships containers to Storage areas Assignment Problem (SSAP) which finds an allocation of ship containers to storage area that minimizes the travelling time of the Reach Stacker and containers dispersion. In a first step, a Mixed Integer Linear Program model is presented to address the SSAP with a view to reduce both travelling time and containers dispersion while satisfying storage capacities when the containers of the same ship can be affected into one or two different storage areas. In a second step, the complexity of the SSAP is established by reduction from the Numerical Matching with Target Sum Problems where the containers of one ship can be partitioned into one or two different storage areas. Our mathematical model is illustrated with a real case study in the Tunisian port of Rades.**

Keywords: **Storage Area Assignment, Containers Dispersion, Complexity.**

1. Introduction

Increased by globalization and the intensification of competition, maritime transport has become a ubiquitous force in the supply chain over the last few decades as a result of the increase in world trade volume, its low cost of energy consumption and its adaptation to the volume of freight exchanged. The importance of maritime transport continues to grow, making it a credible and attractive alternative to air and ground transportation. It is with the introduction of containers since the 1950s that the containerization of goods has made it possible to standardize this mode of transport, thus accelerating the transfer of the goods from one mode of transport to another, to lighten the various operations in container terminals and thus increase the volume of goods transported.

This containerization requires the proper operation of a container port terminal considered as an important link in the global logistics chain and constituting intermodal interfaces for the global transport network. Its competitiveness is mainly influenced by factors such as total time spent by ships at port, loading and unloading time, container transfer time, etc.

Several problems may arise during core activities at a port terminal, namely: planning problems that arise either in the dock area (such as berth allocation problems, container stowage and dock crane planning) or in the courtyard area (scheduling of yard cranes, storage of containers and scheduling of container vehicles).

Problems of allocating berths, transferring and storing containers are considered to be the major problems encountered in the various container port terminals. Otherwise, a good allocation of the containers to the storage areas will also minimize the time required for receiving and transferring operations using the Reach Stackers or others. However, in order to be competitive, port authorities must optimize their logistical resources and resolve not only problems in the dock area but also those that appear in the courtyard area, in order to increase their port productivity, reduce turnaround times and increase their incomes, while keeping customer satisfaction at a desired level. So the performance of a container terminal is not limited to dock planning problems, but it also encompasses planning problems in the courtyard.

This paper examines the complexity of the SSAP by reduction from the numerical Matching with Target Sum Problems and with the aim of minimizing the traveling time of containers unloaded between the berth and the storage area as well as decreasing the dispersion of each ship containers in the storage areas. To address these problems, we present a mathematical model solved using CPLEX.

The outline of the paper including a review of the literature is provided in section 2. In section 3 a description of the SSAP and its mathematical model are presented in case of one or two consecutive storage areas. A proof of complexity for this problem is established. Section 4 presents the real example from the port of Radès, in Tunisia. Finally, concluding remarks and directions for future research are suggested in Section 5.

2. Literature review

Shipping lines are mainly concerned with the waiting time and berthing time of the ship at the port [1]. Consequently, many constraints must be taken into consideration before the assignment of containers of storage areas. For example, there are some characteristics of the storage area which need to be taken into account such as the travelling time between the berth and the storage area, the capacity of storage area, the velocity and the number of the yard crane, dispersion containers, etc.

However, in practice, the SSAP consists in assigning all the containers to the storage area. Many researchers have investigated several methods in an attempt to find a best solution to the SSAP. For example, Kim and Kim [2] devised a method for determining the optimal amount of storage space and the optimal number of transfer cranes for handling import containers. Lee et al. [3] proposed an integrated model for yard truck scheduling and storage allocation for import containers. This problem was formulated as a MIP with the objective of minimizing the make-span of operations and to solve this problem a Genetic Algorithm was developed.

Preston and Kozan [1] formulated a mathematical optimization model to minimize the time spent during the transfer of containers from the storage area to the ship and vice versa. A Genetic Algorithm was used to address this problem. Kim and Kim [4] determined the optimal routes for a single Straddle Carries SC to retrieve containers which need to be loaded on a ship from the stock more efficiently. Their objective was to reduce the total travelling time of the Straddle Carries. As for Zhang et al.[5],they studied the Storage Space Allocation Problem in the storage yards of a terminal so that the total transportation distance for moving containers between blocks and ship berthing locations is reduced.

Lee et al. [6] devised an approach that integrates the problems of yard truck scheduling and storage allocation, their work sought to minimize the heavy sum of the total delay of requests and the total travel time of yard trucks, and to solve this problem a hybrid algorithm was developed. Moussi et al. [7] examined the Container Stacking Problem and suggested a model that would determine the optimal storage strategy for various container-handling schedules.

Our work seeks to decrease the traveling time of containers unloaded between the berth and the storage area and to reduce also the dispersion of containers in the storage area. In light of the seminal work of Zeinebou and Abdellatif [8], we propose two mathematical models in this paper. Our models have distinct additional characteristics compared to that adopted by Zeinebou and Abdellatif [8]. In fact, our models find an independent optimal solution for the SSA problems. A new formulation of the SSAP problem where the travelling time is modelled as a variable is presented. Besides, the minimisation of the dispersion of containers in the storage areas is guaranteed in the SSAP model which rendered evident the complexity of the problems.

Many effort has been made in order to solve many optimization problems using Enumerative methods or Heuristics and Meta-Heuristics methods. Yet, heuristic and meta-heuristic methods did not guarantee optimality; they only provided an approximate solution searching through the set of feasible ones. They were usually time-consuming, especially for big problems. More recently, optimization problems have been classified according to their computational complexity and the difficulty to find the optimal solution.

Indeed, several works addressed the problem of complexity. For example; Kamoun and Sriskandarajah [9] highlighted the complexity of scheduling jobs in the repetitive manufacturing system while Lawler et al. [10] presented the complexity of sequencing and scheduling. Matsuo [11] described the complexity of cyclical sequencing problems in the two permutation- machines flow shop. Similarly, Munier [12] pinpointed the complexity of a cyclic scheduling problem with identical machine; Hall et al. [13] dealt with scheduling in robotic cell and Elloumi et al. [14], examined the classroom assignment problem. Kamoun and Sriskandarajah [9], Hall et al. [13] and Elloumi et al. [14] used the Numerical Matching with Target Sums problem in their proofs of NP-hardness by reduction.

Based on the work of Elloumi et al. [14], we proved that the SSA problem is strongly NP-hard by the reduction from the Numerical with Target Sums problem.

3. Finding an Optimal Assignment of Ships Containers to Storage Areas: The complexity of assigning Ships to the Storage Area

In this section, we seek to find an optimal assignment of ships containers in terminals to storage areas so that the travelling time and containers dispersion are minimized. In this respect, we propose a mathematical model and, then, study the complexity of assigning ships containers in terminals to storage areas.

3.1. The Mathematical Model

Our mathematical model has already been presented in one of our works kallel et al. [15] and it aims to minimize the transfer time of containers from the berth to the storage area. This will affect the time taken by the whole process i.e. while the containers are unloaded from the berth and then allocated to the storage areas. Simultaneously, our model would minimize the dispersion of all the containers in the ship to the storage areas.

As we have already indicated our mathematical model was presented in one of our research works and in this part we will just recall it in order to be able to follow the study of its complexity

a) Parameters

i : The index of all the berths, i (= 1…I) ∈ B
j : The index of all the ships, j (=1…T) ∈ V
z : The index of all the storage areas, z (=1…A) ∈ D
c : The index of moving all containers, c (=1…C) ∈ F
n : The index of all the Reach Stacker, n (= 1…..N) ∈ E
B: The set of the berths.
V: The sets of all the ships.
D: The set of storage areas with A=|D|.

D1: The set of storage areas with $\mathrm{D1} = \{1,3,5 \ldots, \mathrm{A}-2\} \in \mathrm{D}$, if A is odd.
D1: The set of storage areas with $\mathrm{D1} = \{1,3,5 \ldots, \mathrm{A}-1\} \in \mathrm{D}$, if A is even.
D2: The set of storage areas with $\mathrm{D2} = \{\mathrm{D}\backslash\mathrm{D1}\backslash\{\mathrm{A}\}\} = \{2,4,6, \ldots, \mathrm{A}-1\} \in \mathrm{D}$, if A is odd.
D2: The set of storage areas with $\mathrm{D2} = \{\mathrm{D}\backslash\mathrm{D1}\} = \{2,4,6, \ldots, \mathrm{A}\} \in \mathrm{D}$, if A is even.

F: The set of all containers.
$\boldsymbol{V^n}$: The velocity of the Reach stacker..
***lock₁*:** This is unload time for the Reach stacker to hold on to a container before taking up.
***lock2*:** This is the load time required for the Reach Stacker to hold on to a container before taking up.
$\boldsymbol{d_{iz}}$: The distance between the berth *i* and the storage area *z*.
$\boldsymbol{C_j}$**:** The number of containers from the ship *j*.
$\boldsymbol{T_{iz}}$**:** The traveling time of a container from the berth *i* until the storage area *z*, and is given by the formula: $T_{iz} = lock_1 + \frac{d_{iz}}{V^n} + lock_2$

$\boldsymbol{X^*_{ij}}$: *this is the result of assigning ships to berths, is equal* 1 if the ship j from the import set is assigned to berth i, 0 *otherwise*
$\boldsymbol{Q_z}$**:** The capacity of storage area *z*.
***M*:** A very large number.
$\boldsymbol{C'_j = M\, C_j}$

b) Decision Variables

$Y_{jz} = 1$ if the containers of ship *j* are allocated in storage area *z*.0 otherwise.
$YY_{jz} = 1$ if the containers of ship *j* that will be affected in the storage area *z* and (*z +1*). *0* otherwise, for z ∈ D1.
Note that we have added ZZ_{jz} for the linearization of a non linear product ($CC_{jz} * YY_{jz}$).
ZZ_{jz} = The containers with the ship *j* that will be affected in the storage area *z* if $YY_{jz} = 1$.
$\boldsymbol{C_{jz}}$: The containers with the ship *j* that will be affected in the storage area *z*, if $Y_{jz} = 1$.
$\boldsymbol{CC_{jz}}$: The containers with the ship *j* that will be affected in the storage area *z*, if $YY_{jz} = 1$.
This problem can be presented as follows :

$$\mathbf{F} = \mathbf{Min} \sum_{i \in B} \sum_{j \in V} \sum_{z \in D} T_{iz} * C_j * X^*_{ij} * Y_{jz} + \sum_{i \in B} \sum_{j \in V} \sum_{z \in D1} (T_{iz} + T_{i(z+1)})/2 * C'_j * X^*_{ij} * YY_{jz} \quad (1)$$

Subject to :

$$C_{jz} \leq C_j + M\,(1 - Y_{jz}) \quad ; \forall j \in V, z \in D \quad (2)$$

$$C_{jz} + M\,(1 - Y_{jz}) \geq C_j \quad ; \forall j \in V, z \in D \quad (3)$$

$$CC_{jz} + CC_{j(z+1)} \leq C_j + M\,(1 - YY_{jz}) \quad ; \forall j \in V, z \in D1 \quad (4)$$

$$CC_{jz} + CC_{j(z+1)} + M\,(1 - YY_{jz}) \geq C_j \quad ; \forall j \in V,\ z \in D1 \quad (5)$$

$$\sum_{z \in D} Y_{jz} + \sum_{z \in D1} YY_{jz} = 1 \quad ; \forall\ j \in V \quad (6)$$

$$\sum_{j \in V} C_j * Y_{jz} + \sum_{j \in V} ZZ_{jz} \leq Q_z \quad ; \forall\ z \in D1 \quad (7)$$

$$ZZ_{jz} \leq CC_{jz} + M\left(1 - YY_{jz}\right) \quad ;\forall j \in V, z \in D1 \quad (8)$$

$$ZZ_{jz} + M\left(1 - YY_{jz}\right) \geq CC_{jz} \quad ;\forall j \in V,\ z \in D1 \quad (9)$$

$$ZZ_{jz} \leq M\ YY_{jz} \quad ;\forall j \in V, z \in D1 \quad (10)$$

$$\sum_{j \in V} C_j * Y_{jz} + \sum_{j \in V} ZZ_{jz} \leq Q_z \quad ;\forall\ z \in D2 \quad (11)$$

$$ZZ_{jz} \leq CC_{jz} + M\left(1 - YY_{j(z-1)}\right) \quad ;\forall j \in V, z \in D2 \quad (12)$$

$$ZZ_{jz} + M\left(1 - YY_{j(z-1)}\right) \geq CC_{jz} \quad ;\forall j \in V,\ z \in D2 \quad (13)$$

$$ZZ_{jz} \leq M\ YY_{j(z-1)} \quad ;\forall j \in V,\quad z \in D2 \quad (14)$$

$$\sum_{j \in V} C_j * Y_{jz} \leq Q_z \quad ; z = A;\ if\ A\ is\ odd \quad (15)$$

$$Y_{jz}, YY_{jz} \in \{0, 1\} \quad ;\forall j \in V,\quad z \in D \quad (16)$$

$$C_{jz}, CC_{jz}, ZZ_{jz} \in \mathbb{R} \quad ;\forall j \in V,\quad z \in D \quad (17)$$

- The objective function (1) aims to minimize the transfer time of the containers as well as their dispersion in the storage areas.
- Constraints (2) and (3) ensure that all containers C_j are affected to one storage area *z* if $\boldsymbol{Y_{jz}}$ is equal to 1.
- Constraints (4) and (5) guarantee that all containers C_j are affected to two storage areas *z* and *(z+1)*, if YY_{jz} is equal 1.
- Constraint (6) guarantee that each ship *j* is affected to one storage area, or to two storage areas.
- Constraint (7) ensures that all the containers of the ship *j* which are assigned to one storage area z or to two storage areas z and (z + 1*)* for all odd number of z.
- Constraints (8), (9) and (10) are the linearization constraints for the storage areas D1.
- Constraint (11) guarantee that all the containers of the ship *j* which are assigned to one storage area z, or to two storage areas(z-*1) and z* must not exceed the total capacity of the storage area *z*, (for all even number of z).
- Constraints (12), (13) and (14) are the linearization constraints for the storage areas D2.
- Constraint (15) ensures that all the containers of the ship *j* are assigned to one storage area z. They must not exceed the total capacity of the storage area *z*, and *z*=*A* if A is odd.
- Constraint (16) and (17) defines the decision variables.

3.2. The Complexity of Assigning Ships to the Storage Areas :

We consider the Bin Packing Problem (BPP) where ***n*** bins with different capacities are to be filled with ***m*** items having different sizes. The problem involves packing the items into the bins in such a way that the capacity of each bin does not exceed a given size. The BPP has been extensively studied in the literature taking into account all its variants ([16] and [17]).

With regard to our real case, the bins play the role of storage areas with different capacities $\boldsymbol{Q_z}$, **z (=1… *A*) ∈*D*** , while the items are considered as ships having different sizes (number of containers) C_j, ***j (=1…T)*** ∈ ***V***.

Our objective is to assign the ships (items) to the storage areas (bins) in such a way that the total size of ships assigned to a storage area does not exceed its capacity while minimizing the traveling time from berths to the storage area.

In the following instance of our problem (F), we have an optimal solution if each ship is assigned to only one storage area.

The following are the values for different parameters:

- T_{iz}=1, for all i and for all z,

- C_j=1, for all j,
- $L = \sum_{i=1}^{n}(X_i + Y_i)$ where, X_i and Y_i are parameters from the Numerical Matching with Target Sums (NMTS) problem,
- $C_j' = M * Cj$, where M = 4L * n,
- $\boldsymbol{X_{jj}^*}$ **=1** and j =1,2,…n (we take the case where the number of berth *i* is equal to the number of ships *j* then *i* = *j*),
- $\boldsymbol{Y_{jz}}$ =1 if the ships *j* is assigned to the storage area z and 0 otherwise (z ∈ D),
- $\boldsymbol{YY_{jz}}$ =1 if the ships *j* is assigned to the storage area z and (z+1), 0 otherwise (z ∈ D1).

The following NP hard in the strong sense of the problem is used to show the complexity of the ships assignment.

Numerical Matching with Target Sums (NMTS)
Let $\boldsymbol{X}=\{X_1, X_2, \dots, X_n\}$, $\boldsymbol{Y}=\{Y_1, Y_2, \dots, Y_n\}$, $\boldsymbol{Q}=\{Q_1, Q_2, \dots, Q_n\}$ be sets of positive numbers. Can $X \cup Y$ be partitioned into $\boldsymbol{n}$ disjoint subsets $\Gamma_1, \Gamma_2, \dots, \Gamma_n$ with $\Gamma_k = \{X_{i(k)}, Y_{i(k)}\}$ such that $Q_k = X_{i(k)} + Y_{i(k)}$, $k=1,2,\dots,n$?.
Clearly, the condition

$$\sum_{i=1}^{n}(X_i + Y_i - \boldsymbol{Q}_i) = 0$$

Is necessary for a 'yes' answer. Since it is readily proved, we may assume that it always holds [14].
The construction of the presented instance from the Numerical Matching with Target Sums (NMTS) problem is done in polynomial time. Therefore, we need to show that there is a solution to (F) if and only if there is a solution to the NMTS problem.
Let's consider the following instance of the problem (F) constructed from the NMTS problem.
There are two sets of ships. Each contains *n* ships with the following number of containers:

- Ships of type ***I*** with size :

$\boldsymbol{C_m^I = L + X_m}$; $m = 1 \dots . n$

- Ships of type ***Π*** with size :

$\boldsymbol{C_m^{\Pi} = 2L + Y_m}$; $m = 1 \dots . n$

The set of storage areas has the following capacities:
$\boldsymbol{QQ_z = 3L + Q_z}$; $z = 1, \dots, n$

<u>If part</u>:
If there is a solution to NMTS, there exist a solution to our problem (F), with one ship of type ***I*** and one ship of type ***Π*** which are stored in the same storage area, such that :

$$\boldsymbol{L + X_m + 2L + Y_m = 3L + Q_z}$$

And the objective function is equal to $3n\,L + \sum_{i=1}^{n}(X_i + Y_i)$.

<u>Only if part</u>:
We need to prove that there is a solution to our problem (F) with the objective function less or equal to $3n\,L + \sum_{i=1}^{n}(X_i + Y_i)$, only if there exists a solution to NMTS.
***Claim 1*:**
In an optimal solution, all the ships are not assigned to two storage areas.
***Proof*:**
If all $\boldsymbol{YY_{jz} = 0}$, in this case all the ships are assigned in one storage area and the objective function can be equal to $3n\,L + \sum_{i=1}^{n}(X_i + Y_i)$.
.
If $\boldsymbol{YY_{jz}}$ =1 for some *j* and z. In this case the ship is assigned in two storage areas and the objective function will be greater or equal to $4\,L * n > 3n\,L + \sum_{i=1}^{n}(X_i + Y_i)$.

***Claim 2*:**
In a feasible solution, all storage areas are full.

***Proof*:**
The total size of the ships is equal to:

$$nL + \sum_{m=1}^{n} \boldsymbol{X_m} + 2\,nL + \sum_{m=1}^{n} \boldsymbol{Y_m} = 3\,nL + \sum_{m=1}^{n} \boldsymbol{X_m} + \sum_{m=1}^{n} \boldsymbol{Y_m}$$

The total capacity of the storage areas is equal to:

$$3\,nL + \sum_{z=1}^{n} \boldsymbol{Q_z} \ since \sum_{m=1}^{n} \boldsymbol{X_m} + \sum_{m=1}^{n} \boldsymbol{Y_m} = \sum_{Z=1}^{n} \boldsymbol{Q_Z}$$

Is assumed as given in the NMTS problem. This result completes the proof of Claim 2.

***Claim 3*:**
In a feasible solution, exactly one ship of type ***I*** and one ship of type ***II*** are assigned to one storage area, respectively.
***Proof*:**
Assigning two or more ships of type ***II*** to a storage area is not feasible as the size of the ships exceeds the capacity by:

$$\boldsymbol{4L} + \boldsymbol{X_{i(m)}} + \boldsymbol{Y_{i(m)}} - \boldsymbol{3L} - \boldsymbol{Q_z} > 0$$

Or equivalently

$$\boldsymbol{L} + \boldsymbol{X_{i(m)}} + \boldsymbol{Y_{i(m)}} - \boldsymbol{Q_z} > 0$$

Since the number of the storage areas is equal to the number of ships of type ***II***, we cannot have a storage area which is not assigned a ship of type ***II***.
Therefore, one portion of a storage area holds exactly one ship of type ***II*** .
If we do not assign any ship of type ***I*** to a storage area, we have no feasible solution. One ship of type ***II*** is not enough.
If we assign two or more ships of type ***I*** to a storage area, this assignment is not feasible. The capacity is exceeded by at least:

$$\boldsymbol{L} + \boldsymbol{X_{i(m)}} + \boldsymbol{Y_{i(m)}} - \boldsymbol{Q_z} > 0$$

Therefore, one ship of type ***I*** and one ship of type ***II*** are assigned to each storage area.

Theorem 1: The problem (F) is NP-hard in the strong sense.
Proof:
It is easy to show that our problem (F) belongs to the NP class. Given an assignment of ship containers to storage areas, we can check in polynomial time whether this assignment is feasible.
If we have a solution to the NMTS problem, then it is clear that we have a solution to (F). There is a solution where each ship of size $\boldsymbol{L+X_m}$, is joined with another ship of size $\boldsymbol{2L+Y_m}$.
They are both assigned to a storage area of capacity $3\boldsymbol{L}+\boldsymbol{Q_z}$; z= 1…n **,** where

$$\mathbf{L} = \sum_{z=1}^{n} \boldsymbol{Q_z}$$

Furthermore, using Claim 3,we can conclude that we have a feasible solution of (F) where a ship of size $\boldsymbol{L+X_{i(m)}}$ and a ship of size $\boldsymbol{2L+Y_{i(m)}}$, are assigned to a storage area with a capacity of $3\boldsymbol{L}+\boldsymbol{Q_z}$ if $3\boldsymbol{L}+\boldsymbol{Q_z} > 3\boldsymbol{L}+\boldsymbol{X_{i(m)}}+\boldsymbol{Y_{i(m)}}$. This causes an empty space in the storage area, which is prohibited by Claim 2. Besides, let's suppose that the storage area of capacity $3\boldsymbol{L}+\boldsymbol{Q_z}$, is assigned to a ship of size $\boldsymbol{L+X_{i(m)}}$ and another ship of size $\boldsymbol{2L+Y_{i(m)}}$ such that:
$3\boldsymbol{L}+\boldsymbol{Q_z} < \boldsymbol{L}+\boldsymbol{X_{i(m)}} + \boldsymbol{2L}+\boldsymbol{Y_{i(m)}}$ .This case is not feasible as the capacity of the storage area is exceeded.
Therefore, there is a feasible solution to (F) only if we have $\boldsymbol{X_{i(m)}} + \boldsymbol{Y_{i(m)}} = Q_z$. In other words, there is a solution to the NMTS problem. This completes the proof of the theorem.

4. Container transfer and storage problem: A real case on the port

Before presenting our results of the problem of transfer and storage of containers, we will first present some information on the port of Radès, we have learned that the various operations of unloading, loading and handling within this port in export or import are handled by the STAM dealer. We were also able to detect that there is no principle used for the assignment of containers to storage areas.

In order to use the mathematical model of container transfer and storage, we relied on real data recorded during the first 10 days of November 2021 at the port of Radès. But before providing these data, we will briefly present the mechanism for unloading and transferring containers from the ship to the storage area adopted by STAM. Note that in this part we will only focus on container unloading and transfer operations in the case of importing containers.

The operation of unloading the container from the ship starts from the container ship already docked on the berth using a quay crane, then it is placed on the berth. The average time required for a crane to hook onto the container (20 feet) and place it on the quay is ten minutes.
A Reach Stacker will then take care of moving this container and putting it on a tray (it takes an average of five minutes), which will then be tracked by a tractor (RO RO Truck) to the storage area (whose speed does not exceed 40 km/h). Then, the container will be unloaded by another Reach Stacker in the pick-up area that is part of the storage area (this operation takes an average of five minutes).
At the port of Rades there are seven storage areas reserved for containers. The capacity of each storage area being respectively, zone 1: 500 TEU, Zone 2: 1400 TEU, Zone 3: 800 TEU, Zone 4: 2200 TEU , Zone 5: 800 TEU, Zone 6: 2400 TEU and Zone 7: 2300 TEU, (note that a 20-feet container is worth 1 TEU and a 40-feet container is worth 2 TEU)
The distance in meters separating the three berths dedicated to container ships and the seven container storage areas are presented in the following table.

	Berth 1	**Berth 2**	**Berth 3**
Storage Area 1	520m	250m	42m
Storage Area 2	605m	344m	108m
Storage Area 3	320m	63m	48m
Storage Area 4	625m	375m	204m
Storage Area 5	42m	172m	375m
Storage Area 6	306m	203m	344m
Storage Area	465m	406m	562m

Table 1 : Distance between berth and storage area

Now based on these different data we can calculate the transfer time of a container based on the formula already presented:

$\boldsymbol{T_{iz}= lock_1 + (d_{iz}/V^n) + lock_2}$.

The values of the traveling time of a container of 1 TEU (20') are presented in minutes in the following table.

$\boldsymbol{T_{iz}}$	**Berth 1**	**Berth 2**	**Berth 3**
Storage Area 1	20.780mn	20.375mn	20.063mn
Storage Area 2	20.907mn	20.516mn	20.162mn
Storage Area 3	20.480mn	20.094mn	20.072mn
Storage Area 4	20.937mn	20.562mn	20.306mn
Storage Area 5	20.063mn	20.258mn	20.562mn
Storage Area 6	20.459mn	20.304mn	20.516mn
Storage Area 7	20.697mn	20.609mn	20.843mn

Table 2: The values in minutes of the transfer time from the berth to the storage area

Note that according to STAM the value of the traveling time for a 40' container (2 TEU) is considered almost double that of a 20' container (1 TEU).
From the data collected from the port of Radès, we will first start our assignment of containers to storage areas by taking the date of 01/11/2021 as a start date of assignment while taking into account the number of TEUs available in each storage area and the allocation of ships to berths which is already given by the port authorities.

At the beginning of our first assignment plan, we should calculate the number of TEUs available in each storage area. For this we relied on the latest container assignment results to the storage areas for the month of October as well as on the actual remaining capacity in TEUs in each zone as of 01/11/2021

	Storage Area 1	**Storage Area 2**	**Storage Area 3**	**Storage Area 4**	**Storage Area 5**	**Storage Area 6**	**Storage Area 7**
Storage capacity	230	368	190	574	110	430	642

Table 3: Capacity of storage areas in TEUs as of 01/11/2021

Then we will present the number of containers to unload for each ship. The total containers to be unloaded for each ship is calculated in TEU (20').

Ship number	**Date assigned to the berth**	**Number of containers to be unloaded for each ship C_j**		
		20 TEUs	**40 TEUs**	**Total TEUs**
1	02/11/2021 16:30	150	0	150
2	03/11/2021 13:40	119	95	309
3	03/11/2021 11:00	215	20	255
4	05/11/2021 8:00	195	67	329
5	07/11/2021 10:25	98	34	166
6	08/11/2021 22:00	115	87	289
7	09/11/2021 07:00	256	86	428
		Total containers to be unloaded in TEUs		**1926**

Table 4: Number of containers to be unloaded in TEUs on 01/11/2021

Let us indicate that the result of assignment of ships to berths is given by the port authorities is as follows $\boldsymbol{X^*_{21} = 1, X^*_{32} = 1, X^*_{13} = 1, X^*_{24} = 1, X^*_{15} = 1, X^*_{36} = 1, X^*_{27} = 1}$

The result of assigning containers from the same ship to storage areas is obtained in a time of 0.094 seconds using CPLEX. It is given as follows:

$Y_{16} = 1,\ Y_{57} = 1,\ Y_{62} = 1,\ YY_{21} = 1,\ YY_{22} = 1,\ YY_{35} = 1,\ YY_{37} = 1,\ YY_{43} = 1$ *et* $YY_{46} = 1$

$C_{16} = 150,\ C_{57} = 166,\ C_{62} = 289,\ CC_{21} = 230,\ CC_{22} = 79,\ CC_{35} = 110,\ CC_{37} = 145,\ CC_{43} = 49$ *et* $CC_{46} = 280.$

The objective function in minutes is $F = 38447$.

This result of assigning containers to storage areas for each ship can be represented as follows.

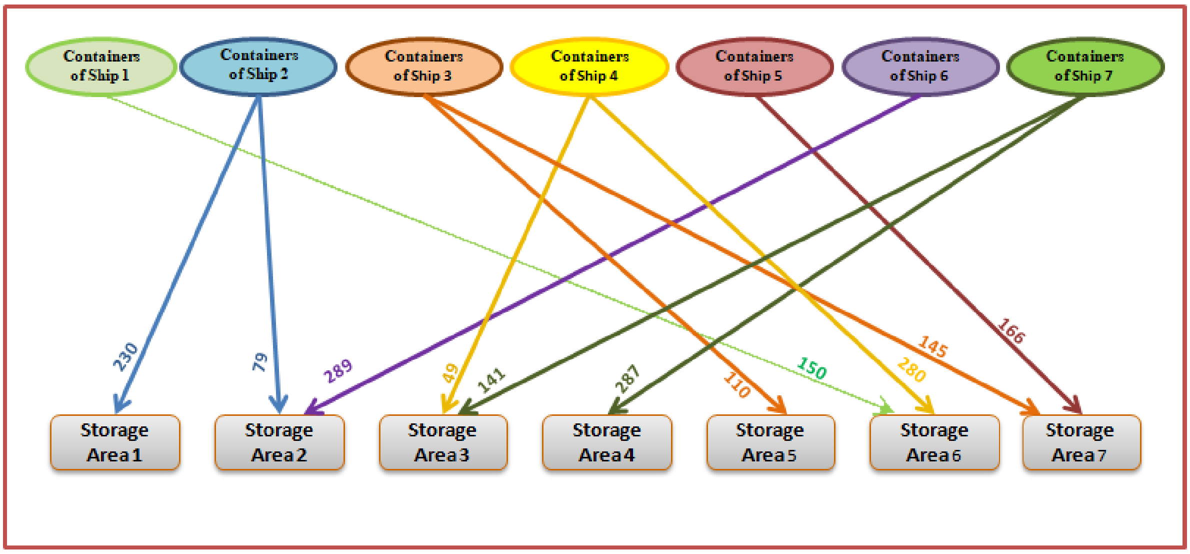

Figure 1: Result of assigned ship containers to storage areas (in 1 TEU)

We notice through this figure that the containers of the same ship are assigned to a single storage area. The containers of ship 1 and those of ship 5 and ship 6 are assigned to a single storage area while the containers of ships 2, 3, 4 and 7 are assigned to two storage areas, and this subsequently allows the minimization of the dispersion of containers to storage areas as well as the reduction of traveling time from docks to storage areas.
Thus the number of 20' and 40' size containers from the same ship which are assigned to the storage areas is presented in table 5.

Ship number	**assigned containers ships to storage areas**													
	Area 1		**Area 2**		**Area 3**		**Area 4**		**Area 5**		**Area 6**		**Area 7**	
	20'	**40'**	**20'**	**40'**	**20'**	**40'**	**20'**	**40'**	**20'**	**40'**	**20'**	**40'**	**20'**	**40'**
1	0	0	0	0	0	0	0	0	0	0	150	0	0	0
2	0	115	79	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	70	20	0	0	145	0
4	0	0	0	0	49	0	0	0	0	0	146	67	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	98	34
6	0	0	115	87	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	141	0	115	86	0	0	0	0	0	0
Totals	**0**	**115**	**194**	**87**	**190**	**0**	**115**	**86**	**70**	**20**	**296**	**67**	**243**	**34**

Table 5: Number of containers per ship assigned to storage areas

This result of assignment of the containers to the storage zones will be used later for the calculation of the transfer time of these containers.
In this part, we presented the result of allocation of containers to storage areas.We will compare our results obtained with those actually achieved at the port of Radès.

Table 6: Number of containers per ship assigned to storage areas according to STAM

Through this table we can notice according to the assignment plan carried out at the port there are containers of the same ship which are assigned to four storage areas such as the contents of ships 1, 2, 4 and 5, and the contents of ships 3 and 6 are assigned to five storage areas, on the other hand the containers of ship 7 are dispersed over six storage areas.

Ship number	Number of containers to be unloaded for each ship C_j			assigned of containers ships to storage areas													
	20TEUs	40TEUs	Total TEUs	Area 1		Area 2		Area 3		Area 4		Area 5		Area 6		Area 7	
				20'	40'	20'	40'	20'	40'	20'	40'	20'	40'	20'	40'	20'	40'
1	150	0	150	20	0	65	0	37	0	0	0	28	0	0	0	0	0
2	119	95	309	0	0	0	20	90	0	29	63	0	0	0	0	0	12
3	215	20	255	78	0	30	0	42	3	0	0	39	0	26	17	0	0
4	195	67	329	0	0	4	7	0	0	0	60	0	0	68	0	123	0
5	98	34	166	0	0	58	6	0	0	0	17	0	0	23	11	17	0
6	115	87	289	0	0	0	0	0	4	0	31	15	0	14	18	86	34
7	256	86	428	0	47	83	0	0	0	0	18	5	0	26	0	142	21
	Totals		**1926**	**98**	**47**	**240**	**33**	**169**	**7**	**29**	**189**	**87**	**0**	**157**	**46**	**368**	**67**

The assignment results produced by our mathematical model and that carried out by the port authorities at the port of Rades can be illustrated as follows:

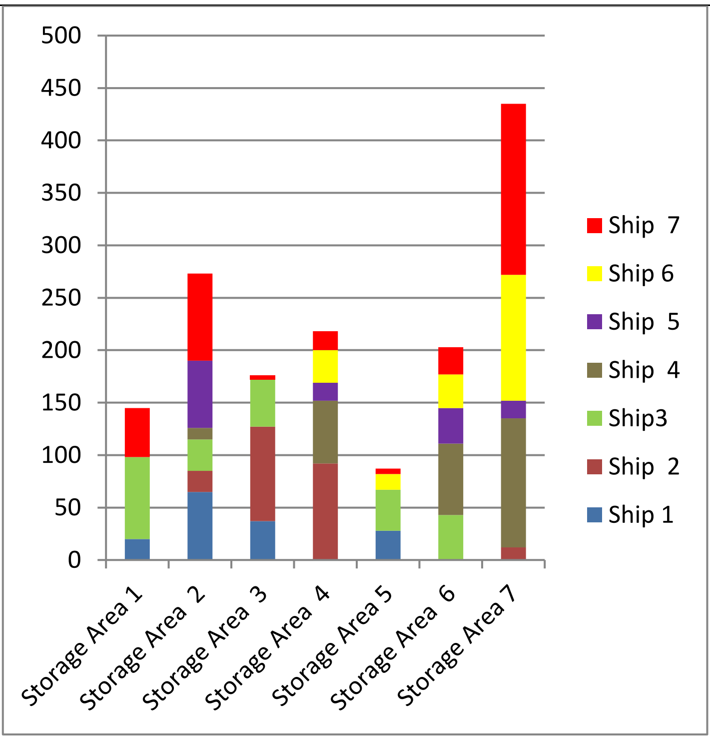

Figure 2 Result of assigned containers ships to storage areas carried out at the port

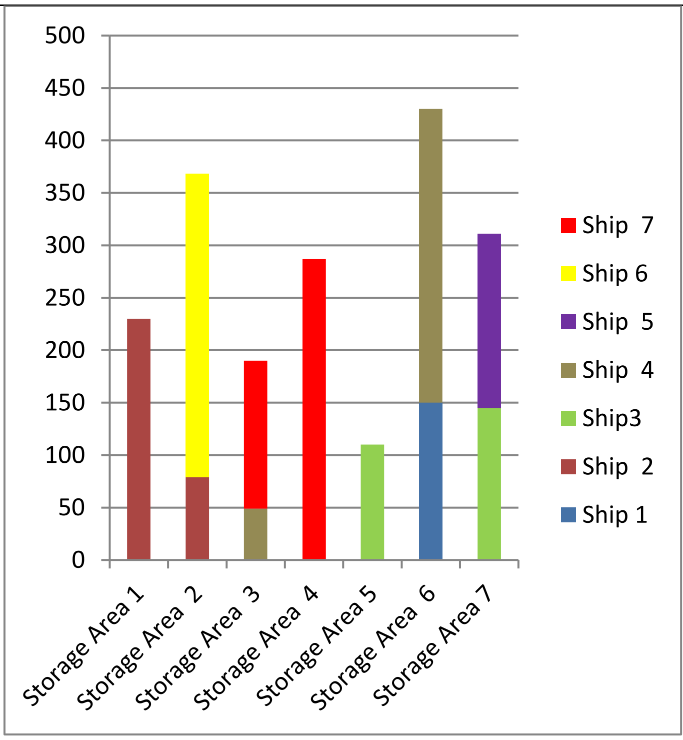

Figure 3: Result of assigned containers ships to storage areas according our mathematical model

Through figures 2 and 3 we notice a great dispersion of the containers of the same ship when they are assigned by the port authorities to the storage areas, sometimes reaching six areas. This could be due to a poor container allocation policy and the lack of consideration of the transfer time when allocating containers from the berth to the storage area, which subsequently leads to an increase assignment time and consequently the container unloading time.

However, our results of assigning containers to storage areas obtained by our model allowed us to minimize the dispersion of containers belonging to the same ship to storage areas and which will allow us subsequently to reduce the transfer time as well as the unloading time of the containers. These two assignment results will allow us to deduce the variation in the transfer time per ship and subsequently that of all the ships which are presented below

Ship number	Transfer time according to STAM assignment results (in minutes)	Transfer time according to the assignment results of our model (in minutes)	Transfer time variation (in minutes)
1	3051.742	3045.600	(-) 6.142
2	6260.622	6207.288	(-) 53.334
3	5241.087	5207.995	(-) 33.092
4	6752.307	6669.726	(-) 82.581
5	3447.852	3435.702	(-) 12.15
6	5963.600	5826.818	(-) 136.782
7	8820.432	8734.548	(-) 85.884
Totaux	39537.642	39127.677	(-) 409.965

Table 7: Variation in transfer time by ship

This variation in the transfer time from containers to storage areas can also be shown in the following figure.

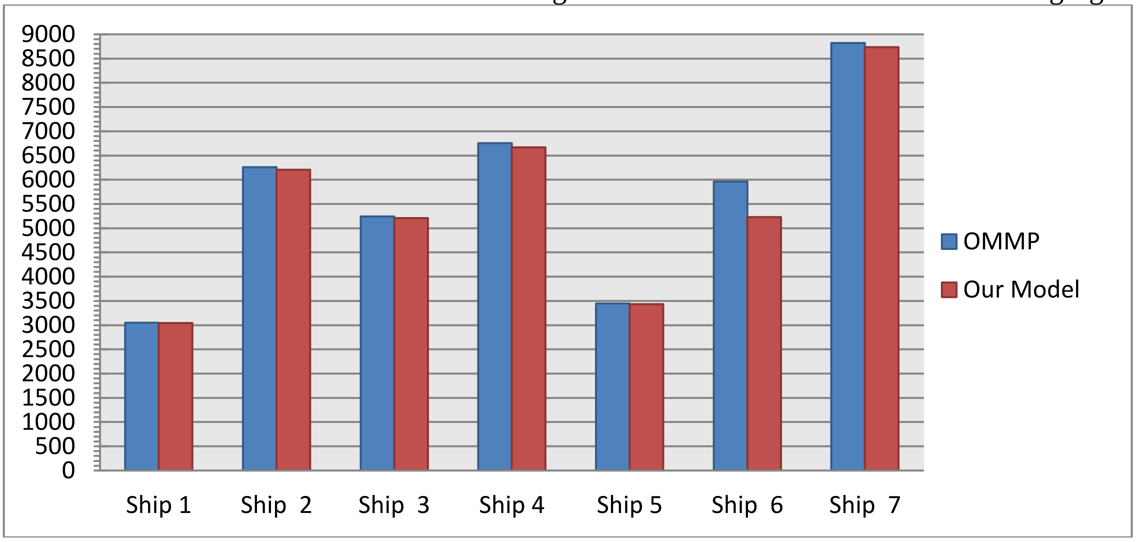

Figure 4: Variation in container transfer time from ships to storage areas

From table 7 and figure 4 we can notice a decrease in the transfer time of containers to the storage areas at the port of Radès. This drop is confirmed by a simple comparison between the results of our model and that of the transfer and storage of containers with the actual results carried out at the port of Radès by the port authorities. Thus, the total transfer time of containers from ships to storage areas at the port has decreased by 409 minutes, i.e. 6 hours and 50 minutes, which saves time during unloading and the assignment of containers to storage areas. . This variation in the transfer time seems minimal for this case treated over a period of ten days at the port of Rades, but this value will be enormous if the number of days is increased, and which will subsequently have a significant impact on the length of stay of the ships at the berths and therefore in the port. Admittedly, minimizing the dispersion of containers from the same ship in storage areas will allow port authorities to easily locate containers from the same ship. In addition, minimizing the dispersion of containers belonging to the same customer will facilitate their search during their delivery and therefore their traceability.

Conclusion

Given that the problems of yard planning and more precisely the allocation of containers to storage areas are considered today as one of the most complex optimization problems when solving them, we have addressed in this article the problem of complexity of ship assignment in two consecutive storage areas. We also applied the mathematical model to a real case at the port of Rades which allowed us to compare our results with the assignment result that was actually achieved at the port. We have shown its advantages allowing the reduction of the transfer time to the port as well as minimizing the dispersion of the containers belonging to the same ship in the storage areas. Solving larger instances of the ships to storage areas assignment requires the use of meta heuristics as solving tools due to the NP-hardeness of the problem.

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