Support Functions and Combination Rules in Hyper Power Set and Super Power Set

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***Abstract-* The probability theory and belief function theory are two important tools dealing with uncertainty. The concepts of probability theory and belief function theory are exchangeable with proper modifications. Rarely, authors are using approach from probability theory to belief function theory. The branch of belief function theory viz. evidence theory is useful in many fields such as behavioral sciences, investigation bureaus, data mining etc. The simple support functions are particular type of belief functions. The combination rules of simple support functions are introduced by Dempster [8] in two dimensions for heterogeneous evidences and conflicting evidences. Super power set includes hyper power set and hyper power set includes power set. In this paper, we generalize these combination rules of simple support functions in three dimensions for heterogeneous evidences and conflicting evidences in hyper power set and apply these combination rules of simple support functions for two dimensions for heterogeneous evidences and conflicting evidences in super power set. This approach is supported by an illustrative example. Also, we discuss about combination rules of simple support functions for higher dimensions in hyper power set and super power set.**

Index Terms -- **Simple support function, belief function, probability density function, hyper power set, super power set**.

INTRODUCTION

In the world of uncertainty, each and every incidence occurring in our day to day life, always follows some known or unknown probability distribution. Therefore choice of appropriate probability distribution plays an important role in decision making. Hence it becomes necessary that we should know common characteristics of all probability distributions. Introduction of simple support functions is useful in categorizing cases of belief functions, infact it helps us in withdrawing final conclusion. Super power set consists of all combinations of subsets under study in sense of set operations viz. union, intersection and complementation.

In this paper, firstly, we apply combination rules of simple support functions in three dimensions for heterogeneous evidences and conflicting evidences with cases, in hyper power set. Secondly, we apply combination rules of simple support functions for two dimensions for heterogeneous evidences and conflicting evidences in super power set with illustrative example. Finally, we discuss on combination rules of simple support functions for higher dimensions in hyper power set and super power set. Now we summarize preliminaries of discrete belief functions, evidence theory and power set, hyper power set and super power set.

2 PRELIMERIES

2.1 Discrete Belief Function Theory

From Shafer's book [8] and Gawn & Bell book [4], frame of discernment $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$. Every element of Θ is a proposition. The propositions of interest, are in one -to -one correspondence with the subsets of Θ. If Θ is *frame of discernment*, then a function $m : 2^\Theta \to [0,1]$ is called **basic probability assignment** whenever $m(\emptyset) = 0$ and $\sum_{A \subset \Theta} m(A) = 1$. The quantity $m(A)$ is called A's **basic probability number** and it is a measure of the belief committed exactly to A.The *total belief* committed to A is sum of $m(B)$, for all proper subsets B of A. A function $Bel : 2^\Theta \to [0,1]$ is called **belief function** over Θ if it satisfies $Bel(A) = \sum_{B \subset A} m(B)$.

Theorem 2.1 *If* Θ *is a frame of discernment, then a function* $Bel : 2^\Theta \to [0,1]$ *is belief function if and only if it satisfies following conditions*

1. $Bel(\emptyset) = 0$.
2. $Bel(\Theta) = 1$.
3. For every positive integer n and every collection $A_1, A_2, \ldots, A_n$ of subsets of Θ

$$Bel(A_1 \cup A_2 \cup \ldots \cup A_n) \geq \sum_{I \subset \{1,2,\cdots,n\}} (-1)^{|I|+1} Bel(\bigcap_{i \in I} A_i). \qquad (1)$$

Degree of doubt :

$$\mathrm{Dou(A)} = \mathrm{Bel}(\overline{\mathrm{A}}) \text{ or } \mathrm{Bel(A)} = 1 - \mathrm{Dou}(\overline{\mathrm{A}}) \text{ and } \mathrm{pl(A)} = 1 - \mathrm{Dou(A)} = \sum_{\mathrm{A} \cap \mathrm{B} \neq \emptyset} \mathrm{m(B)} \qquad (2)$$

which expresses the extent to which one finds A credible or plausible [8]. In [1, 2, 7], a function $P:\Theta \to [0,1]$ is called probability function if

1. $\forall A \in \Theta, \quad 0 \le P(A) \le 1$.
2. $P(\Theta) = 1$.

2.2 Simple Support Functions and Combination Rules

In [3, 8], an evidence supports many propositions discerned by frame of discernment Θ but in different degrees.

Theorem 2.2 *Suppose an evidence points precisely and ambiguously to a single non-empty subset* A *of* Θ. *The effect of evidence is to provide certain degree of support* s *for* A *with* $0 \le s \le 1$. *Let* s *be a degree of support for* A *with* $0 \le s \le 1$. *The* ***simple support function*** $S:2^{\Theta} \to [0,1]$ *is*

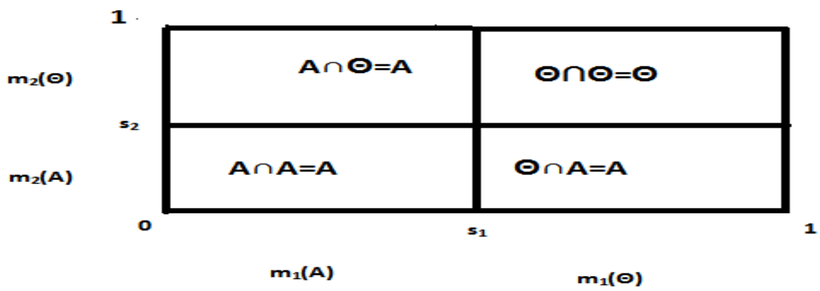
$$S(B) = \begin{cases} 0 & \text{if B does not contain A} \\ s & \text{if B contains A but B} \neq \Theta. \\ 1 & \text{if B} = \Theta \end{cases} \tag{3}$$

Here $S(B)$ is degree of support for $B \subseteq \Theta$ and function S is a simple support function focused on A. We convert S into belief function by assigning basic Probability numbers

$$m(C) = \begin{cases} S(A) & \text{if C} = \text{A} \\ 1 - S(A) & \text{if C} = \Theta \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Bernoulli's Rule of Combination [4, 8]:

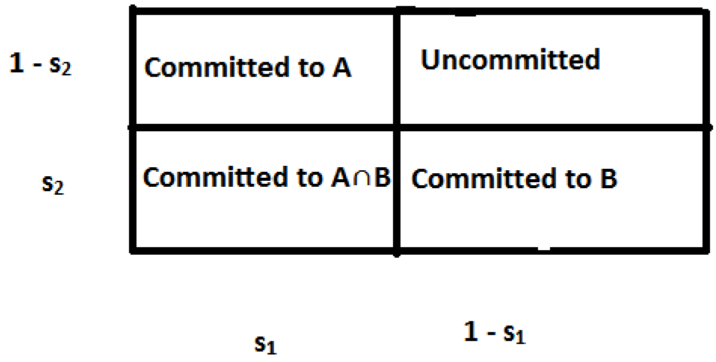
Figure 1: Bernoulli rule of Combination.



Theorem 2.3 *Suppose one body of evidence have precisely supporting effect for set* $A \subset \Theta$ *with degree* s_1 *and the another completely separate body of evidence have precisely supporting effect for set* $A \subset \Theta$ *with degree* s_2. *Then degree of supporting* A *by these two bodies of evidences together is* $m(A) = 1-(1-s_1)(1-s_2)$ *with* $m(\Theta) = (1-s_1)(1-s_2)$.

Heterogeneous Evidences [4, 8]:

Figure 2: Heterogeneous Evidences



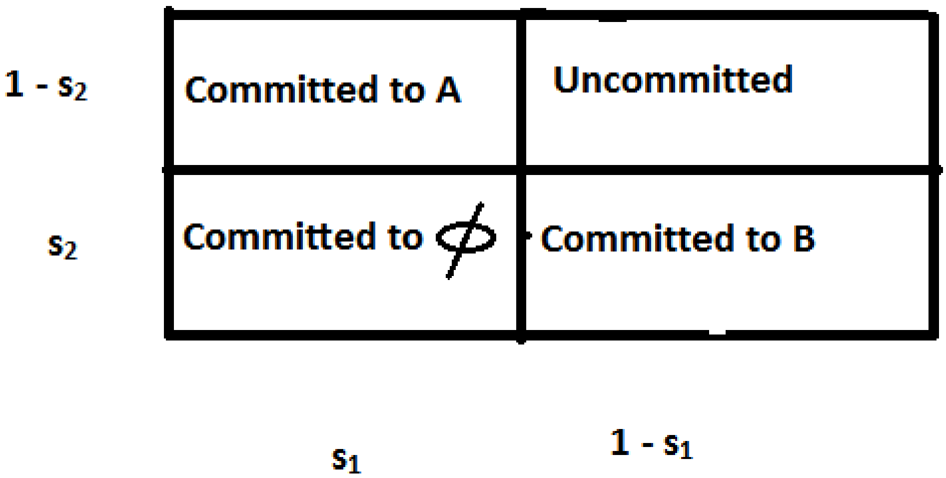
Theorem 2.4 *Suppose* $A \cap B \neq \varnothing$. *Let* S_1 *be a simple support function focused on* A *with* $S_1(A) = s_1$ *and* $m(\Theta) = 1 - s_1$. *Let* S_2 *be another simple support function focused on* B *with* $S_2(B) = s_2$ *and* $m(\Theta) = 1 - s_2$. *Then the orthogonal sum* $S = S_1 \oplus S_2$ *focused on* $A \cup B$, *have basic probability numbers*
$m(A \cap B) = s_1 s_2, m(A) = s_1(1 - s_2), m(B) = s_2(1 - s_1)$ *and* $m(\Theta) = (1 - s_1)(1 - s_2)$ *and*

$$S(C) = \begin{cases} 0 & \text{if C does not contain } A \cap B \\ s_1 s_2 & \text{if C contains } A \cap B \text{ but neither A nor B} \\ s_1 & \text{if C contains A but not B} \\ s_2 & \text{if C contains B but not A} \\ 1-(1-s_1)(1-s_2) & \text{if C contains both A and B and } C \neq \Theta \\ 1 & \text{if } C = \Theta \end{cases} \quad (5)$$

for all $C \subset \Theta$.
Note that $1-(1-s_1)(1-s_2) = s_1 + s_2 - s_1 s_2$ [4, 8].

2.2.1 Conflicting Evidences

Figure 3: Conflicting Evidences



Theorem 2.5 *If* $A \cap B = \varnothing$ *then the combination of evidences pointing to* A *and evidences pointing to* B *is simple as compared to* $A \cap B = \neq \varnothing$. *The effect of one body of evidence decreases the effect of other body of evidence. While applying Dempster's rule of combination, we eliminate lower-left rectangle (see the figure of conflicting evidences) and increase the*

measures of remaining rectangles by multiplying factor $\frac{1}{1-s_1 s_2}$. *Let* S_1 *be a simple support function focused on* A *with* $S_1(A) = s_1$. *Let* S_2 *be a simple support function focused on* B *with* $S_2(B) = s_2$. *Then the* ***orthogonal sum*** $S = S_1 \oplus S_2$ *focused on* $A \cup B$ *have basic probability numbers:*

$$m(C) = \begin{cases} 0 & \text{if } C = \varnothing \\ \frac{s_1(1-s_2)}{1-s_1 s_2} & \text{if } C = A \\ \frac{s_2(1-s_1)}{1-s_1 s_2} & \text{if } C = B \\ \frac{(1-s_1)(1-s_2)}{1-s_1 s_2} & \text{if } C = \Theta. \end{cases} \quad (6)$$

Also, the orthogonal sum S of simple support functions S_1 and S_2 focused on $A \cup B$, is given by

$$S(C) = \begin{cases} 0 & \text{if } C \text{ contains neither } A \text{ nor } B \\ \frac{s_1(1-s_2)}{1-s_1 s_2} & \text{if } C \text{ contains } A \text{ but not } B \\ \frac{s_2(1-s_1)}{1-s_1 s_2} & \text{if } C \text{ contains } B \text{ but not } A \\ \frac{s_1(1-s_2)+s_2(1-s_1)}{1-s_1 s_2} & \text{if } C \text{ contains } A \text{ and } B \text{ but } C \neq \Theta \\ 1 & \text{if } C = \Theta \end{cases} \quad (7)$$

for all $C \subset \Theta$.

A belief function is called **separable support function** if it is a simple support function or equal to the orthogonal sum of two or more simple support functions.

$$S = S_1 \oplus S_2 \oplus, \cdots, \oplus S_n.$$

where $n \geq 1$ and each S_i is a simple support function citeGuanBell1991, Shafer1976. This representation is unique since Dempster‘s rule of combination is commutative and associative.

In [5], degrees of support are substituted by probabilities obtained by probability distributions and in [6], a new basic probability assignment is introduced : For any $A \subseteq \Theta, \quad m(A) = \frac{p(A)}{2^{n-1}}$.

2.3 Super Power Set

A set 2^{Θ} is called power set of Θ if it closed for union of subsets of Θ. A set D^{Θ} is called hyper power set of Θ if it closed for union and intersection of subsets of Θ. A set S^{Θ} is called super power set of Θ if it closed for union, intersection and complementation of subsets of Θ [4, 3].

3 Heterogeneous Evidences for 3D in Hyper Power Set

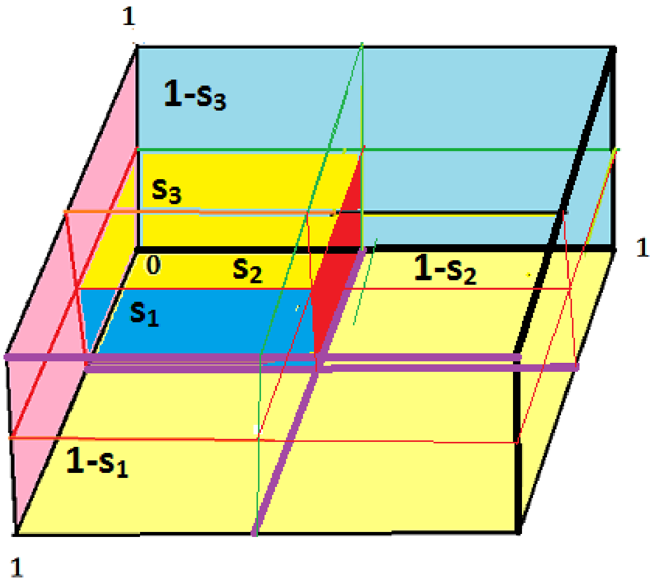


Figure 4: Heterogeneous Evidences for 3D

Now, we generalize result of heterogeneous evidences in two dimensions (2D) to three dimensions (3D). We combine evidence which points to a proposition A_1 with evidences which does not point to a proposition A_1 but to different propositions which are compatible with proposition A_1 viz. propositions A_2, A_3. Such combination provides a support not only for A_1, A_2 and A_3 separately but also for the conjunction $A_1 \cap A_2 \cap A_3$.

Theorem 3.1 *Suppose* $A_1 \cap A_2 \cap A_3 \neq \varnothing$. *Let* S_j, $j = 1,2,3$ *be a simple support function focused on* A_j *with* $S_j(A) = s_j(A)$. *We combine* S_1, S_2 *and* S_3. *By Dempster's rule,* $A_1 \cap A_2 \cap A_3$ *is supported by the quantity* $s_1(A_1)s_2(A_2)s_3(A_3)$. *Then by figure of heterogeneous evidences, the orthogonal sum* $S = S_1 \oplus S_2 \oplus S_3$ *has basic probability numbers as:*

$$
\begin{aligned}
m(A_1 \cap A_2 \cap A_3) &= s_1(A_1)s_2(A_2)s_3(A_3),\\
m(A_1 \cap A_2) &= s_1(A_1)s_2(A_2)(1-s_3(A_3)),\\
m(A_1 \cap A_3) &= s_1(A_1)s_3(A_3)(1-s_2(A_2)),\\
m(A_2 \cap A_3) &= s_2(A_2)s_3(A_3)(1-s_1(A_1)),\\
m(A_1) &= s_1(A_1)(1-s_2(A_2))(1-s_3(A_3)),\\
m(A_2) &= s_2(A_2)(1-s_1(A_1))(1-s_3(A_3)),\\
m(A_3) &= s_3(A_3)(1-s_1(A_1))(1-s_2(A_2)),
\end{aligned}
$$

$$m(\Theta) = (1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))$$

$$\text{and } m(B) = 0 \text{ if } B \text{ is none of the above mentioneds ubsets of } \Theta.$$

Also, orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is

$$S(C)=\begin{cases} 0 & \text{if C does not contain } A_1\cap A_2\cap A_3 \\ s_1(A_1)s_2(A_2)s_3(A_3) & \text{if C contains } A_1\cap A_2\cap A_3 \text{but neither} \\ & A_1\cap A_2 \text{ nor } A_1\cap A_3 \text{ nor } A_2\cap A_3 \\ s_1(A_1)s_2(A_2) & \text{if C contains } A_1\cap A_2 \text{ but neither} \\ & A_1\cap A_3 \text{ nor } A_2\cap A_3 \\ s_1(A_1)s_3(A_3) & \text{if C contains } A_1\cap A_3 \text{ but neither} \\ & A_1\cap A_2 \text{ nor } A_2\cap A_3 \\ s_2(A_2)s_3(A_3) & \text{if C contains } A_2\cap A_3 \text{ but neither} \\ & A_1\cap A_2 \text{ nor } A_1\cap A_3 \\ s_1(A_1)s_2(A_2)+s_1(A_1)s_3(A_3) & \text{if C contains } A_1\cap A_2 \text{ and} \\ -s_1(A_1)s_2(A_2)s_3(A_3) & A_1\cap A_3 \text{ but not } A_2\cap A_3 \\ s_1(A_1)s_2(A_2)+s_2(A_2)s_3(A_3) & \text{if C contains } A_1\cap A_2 \text{ and} \\ -s_1(A_1)s_2(A_2)s_3(A_3) & A_2\cap A_3 \text{but not } A_1\cap A_3 \\ s_1(A_1)s_2(A_3)+s_2(A_2)s_3(A_3) & \text{if C contains } A_1\cap A_3 \text{ and} \\ -s_1(A_1)s_2(A_2)s_3(A_3) & A_2\cap A_3 \text{ but not } A_1\cap A_2 \\ s_1(A_1)s_2(A_2)+s_1(A_1)s_3(A_3) & \text{if C contains } A_1\cap A_2, A_1\cap A_3 \text{ and} \\ +s_2(A_2)s_3(A_3)-2s_1(A_1)s_2(A_2)s_3(A_3) & A_2\cap A_3 \text{but not their proper superset in } \Theta \\ s_1(A_1) & \text{if C contains } A_1 \text{ but neither } A_2 \text{ nor } A_3 \\ s_2(A_2) & \text{if C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3 \\ s_3(A_3) & \text{if C contains } A_3 \text{ but neither } A_2 \text{ nor } A_3 \\ s_3(A_3)+s_1(A_1)s_2(A_2)(1-s_3(A_3)) & \text{if C contains } (A_1\cap A_2)\cup A_3 \\ & \text{but not its propersuperset in } \Theta \\ s_2(A_2)+s_1(A_1)s_3(A_3)(1-s_2(A_2)) & \text{if C contains } (A_1\cap A_3)\cup A_2 \\ & \text{but not its propersuperset in } \Theta \\ s_1(A_1)+s_2(A_2)s_3(A_3)(1-s_1(A_1)) & \text{if C contains } (A_2\cap A_3)\cup A_1 \\ & \text{but no tits proper superset in } \Theta \\ s_1(A_1)+s_2(A_2)-s_1(A_1)s_2(A_2) & \text{if C contains } A_1 \text{ and } A_2 \text{ but not } A_3 \\ s_1(A_1)+s_3(A_3)-s_1(A_1)s_3(A_3) & \text{if C contains } A_1 \text{and } A_3 \text{ but not } A_2 \\ s_2(A_2)+s_3(A_3)-s_2(A_2)s_3(A_3) & \text{if C contains } A_2 \text{ and } A_3 \text{but not } A_1 \\ 1-(1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3)) & \text{if C contains } A_1, A_2 \text{ and } A_3 \text{ but } C\neq\Theta \\ 1 & \text{if } C=\Theta \end{cases} \quad (8)$$

for all $C\subseteq\Theta$.

Proof: Here, for basic probability numbers, we consider sets which are intersection of any number of sets among $A_1, A_2, \text{and} A_3$. For $i=1,2,3$; if set A_i present in set under consideration then multiply by s_i and if set A_i absent in set under consideration then multiply by $(1-s_i)$. The sum of probability numbers of all subsets of Θ is one. For value of $S(C)$, we consider immediate proper superset C of subset of Θ, which does not contain properly any super set of subset of Θ under consideration. For calculation of $S(C)$, we consider subsets of Θ which are contained in C. For $i=1,2,3$; if set A_i present in set under consideration then multiply by $s_i(A_i)$ and if set A_i absent in set under consideration then multiply by $(1-s_i(A_i))$. Same rule is applied if C contains more subsets of Θ.

Here
$$\begin{aligned}&s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))+s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))\\&+s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))+s_1(A_1)s_2(A_2)(1-s_3(A_3))\\&+s_1(A_1)s_3(A_3)(1-s_2(A_2))+s_2(A_2)s_3(A_3)(1-s_1(A_1))+s_1(A_1)s_2(A_2)s_3(A_3)\\&=s_1(A_1)+s_2(A_2)+s_3(A_3)-s_1(A_1)s_2(A_2)-s_1(A_1)s_3(A_3)-s_2(A_2)s_3(A_3)+s_1(A_1)s_2(A_2)s_3(A_3)\\&=1-(1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3)).\end{aligned}$$

3.1 Conflicting Evidences in Hyper Power Set

Now, we generalize result of conflicting evidences in two dimensions (2D) to three dimensions (3D). For three dimensions, conflicting evidences have following cases:

1. Evidences are pairwise disjoint.
2. $A_1 \cap A_2 \cap A_3 = \varnothing$.
3. Mixing of above two cases.

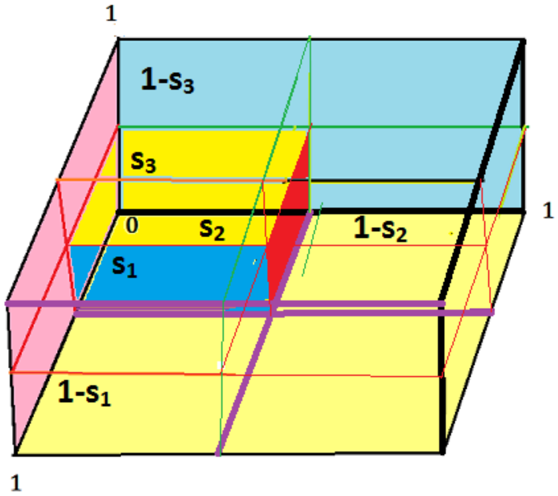


Figure 5: Conflicting Evidences for 3D

Case I: Evidences are pairwise disjoint

Theorem 3.2 *Suppose that for* $i=1,2,3$ *; simple support function* S_i *is focused on* $A_1 i$ *with degree of support* $s_i(A_i)$. *Then the orthogonal sum* S *of* S_1, S_2 *and* S_3 *have basic probability numbers*

$$\begin{aligned}&\mathrm{m(A_1) = s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))/K},\\&\mathrm{m(A_2) = s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))/K},\\&\mathrm{m(A_3) = s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))/K},\\&\mathrm{m(\Theta(= (1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))/K},\end{aligned}$$

and $\quad \mathrm{m(B) = 0}$ if B is none of the above mentioned subsets of Θ,

where

$$K = 1 - s_1(A_1)s_2(A_2)s_3(A_3) - s_1(A_1)s_2(A_2)(1-s_3(A_3)) - s_1(A_1)s_3(A_3)(1-s_2(A_2)) - s_2(A_2)s_3(A_3)(1-s_1(A_1)) \ .$$

Also, sum of basic probability numbers corresponding to all parallelepipeds is one and orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is a separable support function and is given by

$$S(C)=\begin{cases} 0 & \text{If C contains neither } A_1 \text{ nor } A_2 \text{ nor } A_3 \\ \dfrac{s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))}{K} & \text{If C contains } A_1 \text{ but neither } A_2 \text{ nor } A_3. \\ \dfrac{s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))}{K} & \text{If C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3. \\ \dfrac{s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))}{K} & \text{If C contains } A_3 \text{ but neither } A_1 \text{ nor } A_2. \\ \dfrac{s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))}{K} + \dfrac{s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))}{K} & \text{If C contains } A_1 \text{ and } A_2 \text{ but not } A_3. \\ \dfrac{s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))}{K} + \dfrac{s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))}{K} & \text{If C contains } A_1 \text{ and } A_3 \text{ but not } A_2. \\ \dfrac{s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))}{K} + \dfrac{s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))}{K} & \text{If C contains } A_2 \text{ and } A_3 \text{ but not } A_1. \\ \dfrac{s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))}{K} + \dfrac{s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))}{K} + \dfrac{s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))}{K} & \text{If C contains } A_1, A_2 \text{ and } A_3 \text{but } C \neq \Theta. \\ 1 & \text{If } C = \Theta. \end{cases} \tag{9}$$

for all $C \subseteq \Theta$.

Proof: Here we consider all subsets of Θ which are considered in above theorem. By figure of conflicting evidences for three dimensions, the parallelopipeds corresponding to $A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3$ and $A_1 \cap A_2 \cap A_3$ are related to $\varnothing$. For application of Dempster‘s rule, we must eliminate parallelopipeds which are related to $\varnothing$ viz. parallelopipeds $s_1(A_1) \cap s_2(A_2), s_1(A_1) \cap s_3(A_3), s_2(A_2) \cap s_3(A_3)$ and $s_1(A_1) \cap s_2(A_2) \cap s_3(A_3)$. Therefore decrease the measures of the remaining parallelopipeds by quantity which is reciprocal of $1-s_1(A_1)s_2(A_2)s_3(A_3)-s_1(A_1)s_2(A_2)(1-s_3(A_3))-s_1(A_1)s_3(A_3)(1-s_2(A_2))-s_2(A_2)s_3(A_3)(1-s_1(A_1))$.

Here, for basic probability numbers, we consider sets $A_1, A_2, \text{and} A_3$. For $i=1,2,3$; if set A_i present in set under consideration then multiply by $\dfrac{s_i(A_i)\prod_{j=1;i\neq j}^{3}(1-s_j(A_j))}{1-s_1(A_1)s_2(A_2)s_3(A_3)-s_1(A_1)s_2(A_2)(1-s_3(A_3))-s_1(A_1)s_3(A_3)(1-s_2(A_2))-s_2(A_2)s_3(A_3)(1-s_1(A_1))}$. The sum of basic probability numbers of all subsets of Θ under consideration is one.

For value of $S(C)$, we consider immediate proper super set C of subset of Θ, which does not contain properly any super set of subset of Θ under consideration. For calculation of $S(C)$, we consider subsets of Θ which are contained in C. For $i=1,2,3$; if set A_i present in set under consideration then add quantity $\dfrac{s_i(A_i)\prod_{j=1;i\neq j}^{3}(1-s_j(A_j))}{1-s_1(A_1)s_2(A_2)s_3(A_3)-s_1(A_1)s_2(A_2)(1-s_3(A_3))-s_1(A_1)s_3(A_3)(1-s_2(A_2))-s_2(A_2)s_3(A_3)(1-s_1(A_1))}$. Same rule is applied if C contains more subsets of Θ.

Case II: $A_1 \cap A_2 \cap A_3 = \varnothing$:

Theorem 3.3 *Suppose that for* $i=1,2,3$; *simple support function* S_i *is focused on* $A_1 i$ *with degree of support* $s_i(A_i)$. *Then* *the orthogonal sum* $S = S_1 \oplus S_2 \oplus S_3$ *have basic probability numbers*

$$\begin{aligned}
&\mathrm{m}(\mathrm{A}_1 \cap \mathrm{A}_2) = \mathrm{s}_1(\mathrm{A}_1)\mathrm{s}_2(\mathrm{A}_2)(1-\mathrm{s}_3(\mathrm{A}_3))/\mathrm{K},\\
&\mathrm{m}(\mathrm{A}_1 \cap \mathrm{A}_3) = \mathrm{s}_1(\mathrm{A}_1)\mathrm{s}_3(\mathrm{A}_3)(1-\mathrm{s}_2(\mathrm{A}_2))/\mathrm{K},\\
&\mathrm{m}(\mathrm{A}_2 \cap \mathrm{A}_3) = \mathrm{s}_2(\mathrm{A}_2)\mathrm{s}_3(\mathrm{A}_3)(1-\mathrm{s}_1(\mathrm{A}_1))/\mathrm{K},\\
&\mathrm{m}(\mathrm{A}_1) = \mathrm{s}_1(\mathrm{A}_1)(1-\mathrm{s}_2(\mathrm{A}_2))(1-\mathrm{s}_3(\mathrm{A}_3))/\mathrm{K},\\
&\mathrm{m}(\mathrm{A}_2) = \mathrm{s}_2(\mathrm{A}_2)(1-\mathrm{s}_1(\mathrm{A}_1))(1-\mathrm{s}_3(\mathrm{A}_3))/\mathrm{K},\\
&\mathrm{m}(\mathrm{A}_3) = \mathrm{s}_3(\mathrm{A}_3)(1-\mathrm{s}_1(\mathrm{A}_1))(1-\mathrm{s}_2(\mathrm{A}_2))/\mathrm{K},\\
&\mathrm{m}(\Theta(= (1-\mathrm{s}_1(\mathrm{A}_1))(1-\mathrm{s}_2(\mathrm{A}_2))(1-\mathrm{s}_3(\mathrm{A}_3))/\mathrm{K},\\
\text{and}\quad &\mathrm{m}(\mathrm{B}) = 0 \text{ if } \mathrm{B} \text{ is none of the above mentioned subsets of } \Theta,
\end{aligned}$$

where $K = 1 - s_1(A_1)s_2(A_2)s_3(A_3)$.

Also, sum of basic probability numbers corresponding to all parallelepipeds is one and the orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is a separable support function and is given by

$$S(C)=\begin{cases} 0 & \text{If C contains neither } A_1\cap A_2 \text{ nor } A_1\cap A_3 \text{ nor } A_2\cap A_3 \\ \dfrac{s_1(A_1)s_2(A_2)(1-s_3(A_3))}{K} & \text{if C contains } A_1\cap A_2 \text{ but neither } A_1\cap A_3 \text{ nor } A_2\cap A_3 \\ \dfrac{s_1(A_1)s_3(A_3)(1-s_2(A_2))}{K} & \text{if C contains } A_1\cap A_3 \text{ but neither } A_1\cap A_2 \text{ nor } A_2\cap A_3 \\ \dfrac{s_2(A_2)s_3(A_3)(1-s_1(A_1))}{K} & \text{if C contains } A_2\cap A_3 \text{ but neither } A_1\cap A_2 \text{ nor } A_1\cap A_3 \\ \dfrac{s_1(A_1)s_2(A_2)(1-s_3(A_3))}{K}+\dfrac{s_1(A_1)s_3(A_3)(1-s_2(A_2))}{K} & \text{if C contains } A_1\cap A_2 \text{ and } A_1\cap A_3 \text{ but no t} A_2\cap A_3 \\ \dfrac{s_1(A_1)s_2(A_2)(1-s_3(A_3))}{K}+\dfrac{s_2(A_2)s_3(A_3)(1-s_1(A_1))}{K} & \text{if C contains } A_1\cap A_2 \text{ and } A_2\cap A_3 \text{ but not } A_1\cap A_3 \\ \dfrac{s_1(A_1)s_2(A_3)(1-s_3(A_3))}{K}+\dfrac{s_2(A_2)s_3(A_3)(1-s-1(A_1))}{K} & \text{if C contains } A_1\cap A_3 \text{ and } A_2\cap A_3 \text{ but not } A_1\cap A_2 \\ \dfrac{s_1(A_1)s_2(A_2)(1-s_3(A_3))}{K}+\dfrac{s_1(A_1)s_3(A_3)(1-s_2(A_2))}{K}+\dfrac{s_2(A_2)s_3(A_3)(1-s_1(A_1))}{K} & \text{if C contains } A_1\cap A_2, A_1\cap A_3 \text{ and } A_2\cap A_3 \text{ but not their proper superset in } \Theta \\ \dfrac{s_1(A_1)-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1 \text{ but neither } A_2 \text{ nor } A_3 \\ \dfrac{s_2(A_2)-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3 \\ \dfrac{s_3(A_3)-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_3 \text{ but neither } A_2 \text{ nor } A_3 \\ \dfrac{s_3(A_3)+s_1(A_1)s_2(A_2)-2s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains} (A_1\cap A_2)\cup A_3 \text{ but not its proper superset in } \Theta \\ \dfrac{s_2(A_2)+s_1(A_1)s_3(A_3)-2s_1(A_1)s_2(A_2)s_3(A_3))}{K} & \text{if C contains } (A_1\cap A_3)\cup A_2 \text{ but not its proper superset in } \Theta \\ \dfrac{s_1(A_1)+s_2(A_2)s_3(A_3)-2s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } (A_2\cap A_3)\cup A_1 \text{ but not its proper superset in } \Theta \\ \dfrac{s_1(A_1)+s_2(A_2)-s_1(A_1)s_2(A_2)-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1 \text{ and } A_2 \text{ but not } A_3 \\ \dfrac{s_1(A_1)+s_3(A_3)-s_1(A_1)s_3(A_3)-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1 \text{ and } A_3 \text{ but not } A_2 \\ \dfrac{s_2(A_2)+s_3(A_3)-s_2(A_2)s_3(A_3)-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_2 \text{ and } A_3 \text{but not } A_1 \\ \dfrac{1-(1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))-s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1, A_2 \text{ and } A_3 \text{ but } C\neq\Theta \\ 1 & \text{if } C=\Theta \end{cases} \tag{10}$$

for all $C\subseteq\Theta$.

Proof: Here we consider all subsets of Θ which are considered in heterogeneous evidences in 3D. By figure of conflicting evidence for three dimensions, the parallelopiped $A_1\cap A_2\cap A_3$ is related to $\varnothing$. For application of Dempster's rule, we eliminate corresponding parallelopipeds. Therefore decrease the measures of the remaining parallelopiped by quantity $\dfrac{1}{1-s_1(A_1)s_2(A_2)s_3(A_3)}$. let $K=1-s_1(A_1)s_2(A_2)s_3(A_3)$. The basic probability numbers are obtained from basic probability numbers of all subsets of Θ except $A_1\cap A_2\cap A_3$ by multiplying K. Here sum of basic probability numbers of all subsets of Θ under consideration is one. We apply remaining procedure of calculation of $S(C)$ similar to Case I.

Case III :- Mixing of Above Two Cases:

In this case, any two evidences may be heterogeneous or conflicting. If given two evidences are heterogeneous then their intersection is not related to $\varnothing$. If given two evidences are conflicting then their intersection is related to $\varnothing$. Such combinations of evidences should be done for all possible pairs of given evidences. The effect of conflicting evidences is taken into account while calculating the value of K as in above cases. Also eliminate cases in $S(C)$ whose intersection is related to $\varnothing$. As Dempster's rule of combination is associative and commutative hence Bernaulli's rule of combination, the order of combination of support functions is not important.

Theorem 3.4 *Suppose that for* $i=1,2,3$ *; simple support function* S_i *is focused on* A_i *with degree of support* $s_i(A_i)$ *and the parallelopiped* $A_2 \cap A_3$ *is related to* $\varnothing$*. Then the orthogonal sum* $S = S_1 \oplus S_2 \oplus S_3$ *have basic probability numbers*

$$
\begin{aligned}
&m(A_1 \cap A_2) = s_1(A_1)s_2(A_2)(1-s_3(A_3))/K,\\
&m(A_1 \cap A_3) = s_1(A_1)s_3(A_3)(1-s_2(A_2))/K,\\
&m(A_1) = s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))/K,\\
&m(A_2) = s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))/K,\\
&m(A_3) = s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))/K,\\
&m(\Theta(= (1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))/K,\\
\text{and}\quad &m(B) = 0 \text{ if } B \text{ is none of the above mentioned subsets of } \Theta.
\end{aligned}
$$

where $K = 1 - s_2(A_2)s_3(A_3) - s_1(A_1)s_2(A_2)s_3(A_3)$.

Also, sum of basic probability numbers corresponding to all parallelopipeds is one and the orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is a separable support function and is given by

$$
S(C)=\begin{cases}
0 & \text{If C contains neither } A_1 \cap A_2 \text{ nor } A_1 \cap A_3\\
\dfrac{s_1(A_1)s_2(A_2)(1-s_3(A_3))}{K} & \text{if C contains } A_1 \cap A_2 \text{ but not } A_1 \cap A_3\\
\dfrac{s_1(A_1)s_3(A_3)(1-s_2(A_2))}{K} & \text{if C contains } A_1 \cap A_3 \text{ but not } A_1 \cap A_2\\
\dfrac{s_1(A_1)s_2(A_2)(1-s_3(A_3))}{K} + \dfrac{s_1(A_1)s_3(A_3)(1-s_2(A_2))}{K} & \text{if C contains } A_1 \cap A_2 \text{ and } A_1 \cap A_3\\
\dfrac{s_1(A_1) - s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains} A_1 \text{ but neither } A_2 \text{ nor} A_3\\
\dfrac{s_2(A_2) - s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3\\
\dfrac{s_3(A_3) - s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_3 \text{ but neither } A_2 \text{ nor } A_3\\
\dfrac{s_3(A_3) + s_1(A_1)s_2(A_2) - s_2(A_2)s_3(A_3) - s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } (A_1 \cap A_2) \cup A_3 \text{ but not its proper superset in } \Theta\\
\dfrac{s_2(A_2) + s_1(A_1)s_3(A_3) - s_2(A_2)s_3(A_3) - s_1(A_1)s_2(A_2)s_3(A_3)}{K} & \text{if C contains } (A_1 \cap A_3) \cup A_2 \text{ but not its proper superset in } \Theta\\
\dfrac{s_1(A_1) + s_2(A_2) - s_1(A_1)s_2(A_2) - s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1 \text{ and } A_2 \text{ but not } A_3\\
\dfrac{s_1(A_1) + s_3(A_3) - s_1(A_1)s_3(A_3) - s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1 \text{ and } A_3 \text{ but not } A_2\\
\dfrac{s_2(A_2) + s_3(A_3) - 2s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_2 \text{ and } A_3 \text{ but not } A_1\\
\dfrac{(1-(1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))) - -s_2(A_2)s_3(A_3)}{K} & \text{if C contains } A_1, A_2 \text{ and } A_3 \text{ but } C \neq \Theta\\
1 & \text{if } C = \Theta
\end{cases}
$$

(11)

for all $C \subseteq \Theta$.

Proof: Here we consider all subsets of Θ which are considered in conflicting evidences in 3D in Case II. By figure of conflicting evidence for three dimensions, WOLOG, assume that the parallelopiped is related to $A_2 \cap A_3 = \varnothing$ hence $A_1 \cap A_2 \cap A_3 = \varnothing$. Here $A_1 \cap A_2 \neq \varnothing$ and $A_1 \cap A_3 \neq \varnothing$. For application of Dempster's rule, we eliminate corresponding parallelepipeds viz. $A_2 \cap A_3$ and $A_1 \cap A_2 \cap A_3$. Therefore we decrease the measures of the remaining parallelopipeds by quantity $\dfrac{1}{1 - s_2(A_2)s_3(A_3) - s_1(A_1)s_2(A_2)s_3(A_3)}$. Let $K = 1 - s_2(A_2)s_3(A_3) - s_1(A_1)s_2(A_2)s_3(A_3)$. The basic probability numbers are obtained from basic probability numbers of all subsets of Θ except $A_2 \cap A_3$ and $A_1 \cap A_2 \cap A_3$ by multiplying K. Here sum of basic probability numbers of all subsets of Θ under consideration is one. We apply remaining procedure of calculation of $S(C)$ similar to Case II.

Simillarly, we can apply other cases only one pair of evidences are conflicting and two pairs of evidences are conflicting.

4 Combination of Evidences in 2D in Super Power Set

Now we replace hyper power set by super power set. The complements of subsets in hyper power set are included.

4.1 Heterogeneous Evidences in 2D in Super Power Set

We have super power set for subsets A and B of Θ as
$\{\varnothing, A\cap B, A, B, A\cup B, c(A\cap B), c(A), c(B), c(A\cup B), c(\varnothing) = \Theta\}$,
where $c(X) =$ complement of ssubset X of Θ in Θ.

Theorem 4.1 *Suppose* $A \cap B \neq \varnothing$. *Let* S_1 *be a simple support function focused on* A *with* $S_1(A) = s_1$. *Let* S_2 *be another simple support function focused on* B *with* $S_2(B) = s_2$. *Then the orthogonal sum* $S = S_1 \oplus S_2$ *focused on* $A \cup B$, *have basic probability numbers* $m(A\cap B) = s_1s_2, m(A) = s_1(1-s_2), m(B) = s_2(1-s_1)$ *and* $m(\Theta) = (1-s_1)(1-s_2)$. *Also,*

$$S(C) = \begin{cases} 0 & \text{if C does not contain } A\cap B \\ s_1s_2 & \text{if C contains } A\cap B \text{ but neither A nor B} \\ s_1 & \text{if C contains A but not B} \\ s_2 & \text{if C contains B but not A} \\ 1-(1-s_1)(1-s_2) & \text{if C contains both A and B and } C \neq \Theta \\ (1-s_1)(1-s_2) & \text{if C contains } c(A\cup B) \text{ but not its proper superset in } \Theta \\ 1-s_1 & \text{if C contains } c(A) \text{ but not its proper superset in } \Theta \\ 1-s_2 & \text{if C contains } c(B) \text{ but not its proper superset in } \Theta \\ 1-s_1s_2 & \text{if C contains } c(A\cap B) \text{ but not its proper superset in } \Theta \\ 1 & \text{if } C = \Theta \end{cases} \quad (12)$$

for all $C \subset \Theta$.

Proof: Here we apply method of heterogeneous evidences in 2D with subsets from super power set of Θ . Also, $1-(1-s_1)(1-s_2) = s_1 + s_2 - s_1s_2$. While obtaining support functions for complement of subsets of Θ, we can use result : For any subset $A \subseteq \Theta$, $Bel(c(A)) = 1 - Pl(A)$, since support function is a belief function.

4.1.1 Conflicting Evidences in 2D in Super Power Set

Theorem 4.2 *If* S_1 *be a simple support function focused on* A *with* $S_1(A) = s_1$ *and* S_2 *be a simple support function focused on* B *with* $S_2(B) = s_2$ *and* $A \cap B = \varnothing$. *Then the* ***orthogonal sum*** $S = S_1 \oplus S_2$ *focused on* $A \cup B$ *have basic probability numbers:*

$$m(C)=\begin{cases} 0 & if \;\; C=\varnothing \\ \dfrac{s_1(1-s_2)}{1-s_1s_2} & if \;\; C=A \\ \dfrac{s_2(1-s_1)}{1-s_1s_2} & if \;\; C=B \\ \dfrac{(1-s_1)(1-s_2)}{1-s_1s_2} & if \;\; C=\Theta. \end{cases} \tag{13}$$

Also, the orthogonal sum S of simple support functions S_1 and S_2 focused on $A\cup B$, is given by

$$S(C)=\begin{cases} 0 & \text{if C contains neither A nor B} \\ \dfrac{s_1(1-s_2}{1-s_1s_2} & \text{if C contains A but not B} \\ \dfrac{s_2(1-s_1)}{1-s_1s_2} & \text{if C contains B but not A} \\ \dfrac{s_1(1-s_2)+s_2(1-s_1)}{1-s_1s_2} & \text{if C contains both A and B and } C\neq\Theta \\ 1-\dfrac{s_1(1-s_2)+s_2(1-s_1)}{1-s_1s_2} & \text{if C contains } c(A\cup B) \text{ but not its proper superset in } \Theta \\ 1-\dfrac{s_1(1-s_2)}{1-s_1s_2} & \text{if C contains c(A) but not its proper superset in } \Theta \\ 1-\dfrac{s_2(1-s_1)}{1-s_1s_2} & \text{if C contains c(B) but not its proper superset in } \Theta \\ 1 & \text{if } C=\Theta \end{cases} \tag{14}$$

for all $C\subset\Theta$.

Proof: Here, we consider sets from super power set of Θ. While applying Dempster‘s rule of combination in 2D, we eliminate lower-left rectangle (see the figure of conflicting evidences in 2D) and increase the measures of remaining rectangles by multiplying factor $\dfrac{1}{1-s_1s_2}$. For calculating basic probability numbers by multiplying basic probability numbers except for set $A\cap B$ by quantity $\dfrac{1}{1-s_1s_2}$. We apply procedure similar to conflicting evidences in 2D for calculating $S(C)$.

4.2 Generalizations of Above Results to k Dimensions

In hyper power set, the number of cases for heterogeneous evidences, follows the sequence of Dedekind numbers with addition of 1 viz. $2,6,20,168,7581,7828354,\ldots$. For $k\geq 4$, it becomes difficult to enumerate all cases. In super power set, the number of cases for heterogeneous evidences, follows the sequence $2^{2^{n}-1}+n$ viz. $1,3,10,131,32772,\ldots$. For $k\geq 3$, it becomes difficult to enumerate all cases.

In each case, subset of Θ whose support function to be calculated, is union and intersection of subsets of Θ in some prescribed order. Whenever operation union is performed, we use rule : For any subsets $A,B\subseteq\Theta,\;\; S(A\cup B)=S(A)+S(B)-S(A\cap B)$, since support function S is a belief function. Whenever operation intersection is performed, we use rule : For any subsets $A,B\subseteq\Theta,\;\; S(A\cap B)=S(A)S(B)$. Using these two rules in prescribed order in required subset of Θ, we obtain support function of every subset of Θ.

5 Simple Support Function Based on $s=\dfrac{p(A)}{2^{n-1}}$

By using basic belief assignment from [6], $m = \frac{p(A)}{2^{n-1}}$ as degree of support, we have a advantage as : For any subset A of Θ, $m(A) \le m(\Theta)$ and a disadvantage as : Values involved in calculations are very small as compared to degree of support s which is based on probability from probability distributions. The advantage is more appealing than disadvantage.

6 Illustrative Example

From [1, 7], suppose that one body of evidence focused on subset A_1 of $\Theta = \{0,1,2,3,4,5,6,7,8,9,10\}$ with degrees of support s_1 follows binomial distribution

$$p(x) = \binom{n}{x} p^x q^{1-x}, \quad x = 0,1,2,3,4,5,6,7,8,9,10$$

$$\text{with}\, p = 0.7, 1 = 1 - p = 0.3 \text{and} n = 10$$

Therefore,

$$\begin{aligned} &p(0) = 0.0000059049, && p(1) = 0.000137781, \\ &p(2) = 0.0014467005, && p(3) = 0.009001692, \\ &p(4) = 0.036756909, && p(5) = 0.1029193452, \\ &p(6) = 0.2001209449, && p(7) = 0.266827932, \\ &p(8) = 0.2334744405, && p(9) = 0.121060821, \\ &p(10) = 0.0282475249 \end{aligned} \tag{15}$$

Here $\sum_{x=0}^{10} p(x) = 1$.

Suppose that one body of evidence focused on subset A_2 of $\Theta = \{0,1,2,3,4,5,6,7,8,9,10\}$ with degrees of support s_2 folllows Poison distribution

$$p(x) = \frac{e^{\lambda} \lambda^x}{x!}, \quad x = 0,1,2,3,4,5,6,7,8,9,10$$

$$\text{with}\, \lambda = 2, \text{and}\ n = 10$$

Therefore,

$$\begin{aligned} &p(0) = 0.1353352832 36612, && p(1) = 0.2706705664 73225, \\ &p(2) = 0.2706705664 73225, && p(3) = 0.1804470443 15483, \\ &p(4) = 0.0902235221 57741, && p(5) = 0.0360894088 63096, \\ &p(6) = 0.1202980295 4365, && p(7) = 0.0034370865 58390, \\ &p(8) = 0.0008592716 39597, && p(9) = 0.0001909492 53243, \\ &p(10) = 0.0000381898 50648779 \end{aligned} \tag{16}$$

Here $\sum_{x=0}^{10} p(x) = 0.999991691775625 \approx 1$.

From [1, 7], suppose that one body of evidence focused on subset A_3 of $\Theta = \{0,1,2,3,4,5,6,7,8,9,10\}$ with degrees of support s_3 folllows discrete uniform distribution

$$p(x) = \frac{1}{n+1}, \quad x = 0,1,2,3,4,5,6,7,8,9,10. \qquad n = 10$$

$$p(x) = \frac{1}{11} = 0.09090909090909$$

Here $\sum_{x=0}^{10} p(x) = 0.99999999999999 \approx 1$.

6.1 Heterogeneous Evidences for 3D in Hyper Power Set

Suppose $A_1 \cap A_2 \cap A_3 \neq \emptyset$. Let S_j, $j = 1,2,3$ be a simple support function focused on A_j with $S_j(A) = s_j(A)$. We combine S_1, S_2 and S_3. By Dempster‘s rule, $A_1 \cap A_2 \cap A_3$ is supported by the quantity $s_1(A_1)s_2(A_2)s_3(A_3)$. From [1, ?], suppose that first body of evidence focused on A_1 with degree of support s_1 is probability obtained from binomial distribution $Binomial(10,0.7)$, second body of evidence focused on A_2 with degree of support s_2 is probability obtained from Poison distribution $Poison(2)$ and third body of evidence focused on A_3 with degree of support s_3 is probability obtained from discrete uniform distribution. Here $A_1 = \{1,2,3,5,6,8,9\}$, $A_2 = \{0,1,3,6,8,10\}, A_3 = \{0,2,3,5,7,8\}$.

Therefore $A_1 \cap A_2 = \{1,3,6,8\}$, $A_1 \cap A_3 = \{2,3,5,8\}$, $A_2 \cap A_3 = \{0,3,8\}$,

and $A_1 \cap A_2 \cap A_3 = \{3,8\}$.

Here $s_1 = p(A_1) = 0.6681617292$, $s_2 = p(A_2) = 0.59938015846993$ and $s_3 = p(A_3) = 0.54545454545454$

By figure of heterogeneous evidences, the orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ has basic probability numbers as:

$$
\begin{aligned}
m(A_1 \cap A_2 \cap A_3) &= s_1(A_1)s_2(A_2)s_3(A_3) = 0.218445208980782,\\
m(A_1 \cap A_2) &= s_1(A_1)s_2(A_2)(1-s_3(A_3)) = 0.182037674150655,\\
m(A_1 \cap A_3) &= s_1(A_1)s_3(A_3)(1-s_2(A_2)) = 0.146006664331012,\\
m(A_2 \cap A_3) &= s_2(A_2)s_3(A_3)(1-s_1(A_1)) = 0.108489422911903,\\
m(A_1) &= s_1(A_1)(1-s_2(A_2))(1-s_3(A_3)) = 0.1214271770566773,\\
m(A_2) &= s_2(A_2)(1-s_1(A_1))(1-s_3(A_3)) = 0.90407852426588,\\
m(A_3) &= s_3(A_3)(1-s_1(A_1))(1-s_2(A_2)) = 0.072513270251731,
\end{aligned}
$$

$$m(\Theta) = (1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3)) = 0.06042772520977$$

$$\text{and} \quad m(B) = 0 \text{ if B is none of the above mentioned subsets of } \Theta.$$

We consider sets which are intersection of any sets among $A_1, A_2, \text{and} A_3$. For value of $S(C)$, we consider proper super set C of subset of Θ, which does not contain its proper super set of subset of Θ under consideration.

Therefore, by equation (8), orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is

$$S(C)=\begin{cases} 0 & \text{if } C \text{ does not contain } A_1\cap A_2\cap A_3 \\ 0.218445208980782 & \text{if } C \text{ contains } A_1\cap A_2\cap A_3 \text{ but neither} \\ & A_1\cap A_2 \text{ nor } A_1\cap A_3 \text{ nor } A_2\cap A_3 \\ 0.400482883131438 & \text{if } C \text{ contains } A_1\cap A_2 \text{ but neither} \\ & A_1\cap A_3 \text{ nor } A_2\cap A_3 \\ 0.364451852290905 & \text{if } C \text{ contains } A_1\cap A_3 \text{ butneither} \\ & A_1\cap A_2 \text{ nor } A_2\cap A_3 \\ 0.326934631892685 & \text{if } C \text{ contains } A_2\cap A_3 \text{ but neither} \\ & A_1\cap A_2 \text{ nor } A_1\cap A_3 \\ 0.546489526441561 & \text{if } C \text{ contains } A_1\cap A_2 \text{ and} \\ & A_1\cap A_3 \text{butnot} A_2\cap A_3 \\ 0.508972306043341 & \text{if } C \text{ contains } A_1\cap A_2 \text{ and} \\ & A_2\cap A_3 \text{ but not } A_1\cap A_3 \\ 0.472941275202808 & \text{if } C \text{ contains } A_1\cap A_3 \text{ and} \\ & A_2\cap A_3 \text{ but not } A_1\cap A_2 \\ 0.654978949353464 & \text{if } C \text{ contains } A_1\cap A_2, A_1\cap A_3 \text{ and} \\ & A_2\cap A_3 \text{ but not their proper superset in } \Theta \\ 0.6681617292 & \text{if } C \text{ contains } A_1 \text{ but neither } A_2 \text{ nor } A_3 \\ 0.59938015846993 & \text{if } C \text{ contains } A_2 \text{ but neither } A_1 \text{ nor } A_3 \\ 0.54545454545454 & \text{if } C \text{ contains } A_3 \text{ but neither } A_2 \text{ nor } A_3 \\ 0.727492219605195 & \text{if } C \text{ contains } (A_1\cap A_2)\cup A_3 \\ & \text{but not its proper superset in } \Theta \\ 0.745386801780052 & \text{if } C \text{ contains } (A_1\cap A_3)\cup A_2 \\ & \text{but not its proper superset in } \Theta \\ 0.776651152111903 & \text{if } C \text{ contains } (A_2\cap A_3)\cup A_1 \\ & \text{but not its proper superset in } \Theta \\ 0.867059004538492 & \text{if } C \text{ contains } A_1 \text{ and } A_2 \text{ but not } A_3 \\ 0.849164422363635 & \text{if } C \text{ contains } A_1 \text{ and } A_3 \text{ but not } A_2 \\ 0.817900070734543 & \text{if } C \text{ contains } A_2 \text{ and } A_3 \text{ but no t} A_1 \\ 0.939572274790222 & \text{if } C \text{ contains } A_1, A_2 \text{ and } A_3 \text{ but } C\neq\Theta \\ 1 & \text{if } C=\Theta \end{cases} \tag{17}$$

for all $C\subseteq\Theta$.

6.2 Conflicting Evidences for 3D in Hyper Power Set

6.2.1 Evidences are pairwise disjoint

Suppose $A_i\cap A_j=\varnothing \quad i,j=1,2,3 \text{and} i\neq j$. From [1, 7], suppose that first body of evidence focused on A_1 with degree of support s_1 is probability obtained from binomial distribution $Binomial(10,0.7)$, second body of evidence focused on A_2 with degree of support s_2 is probability obtained from Poison distribution $Poison(2)$ and third body of evidence focused on A_3 with degree of support s_3 is probability obtained from discrete uniform distribution. By figure of conflicting evidences for three dimensions, the parallelopipeds corresponding to $A_1\cap A_2, A_1\cap A_3, A_2\cap A_3$ and $A_1\cap A_2\cap A_3$ are related to $\varnothing$. For application of Dempster's rule, we must eliminate corresponding parallelopipeds. Therefore decrease the measures of the remaining parallelopipeds by quantity reciprocal of $1-s_1(A_1)s_2(A_2)s_3(A_3)-s_1(A_1)s_2(A_2)(1-s_3(A_3))-s_1(A_1)s_3(A_3)(1-s_2(A_2))-s_2(A_2)s_3(A_3)(1-s_1(A_1))$. Here $A_1=\{1,2,3,8,9\}$, $A_2=\{0,6,10\}$ and $A_3=\{4,5,7\}$. Therefore $A_1\cap A_2=\varnothing$, $A_1\cap A_3=\varnothing$, $A_2\cap A_3=\varnothing$, and $A_1\cap A_2\cap A_3=\varnothing$. The *orthogonal sum* S of S_1, S_2 and S_3, have basic probability numbers

$$m(A_1)=s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))/K=0.27089215\,43877144,$$
$$m(A_2)=s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))/K=0.08149540\,0822394,$$
$$m(A_3)=s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))/K=0.17662905\,9350823,$$
$$m(\Theta)=(1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))/K=0.47103129\,4689857,$$
and $m(B)=0$ if B is none of the above mentioned subsets of Θ.

where $K=1-s_1(A_1)s_2(A_2)s_3(A_3)-s_1(A_1)s_2(A_2)(1-s_3(A_3))-s_1(A_1)s_3(A_3)(1-s_2(A_2))-s_2(A_2)s_3(A_3)(1-s_1(A_1)) = 0.83576054$. Also, sum of basic probability numbers corresponding to all parallelopipeds is one.

By using equation (9), the orthogonal sum $S=S_1\oplus S_2\oplus S_3$ is a separable support function and is given by

$$S(C)=\begin{cases} 0 & \text{If C contains neither } A_1 \text{ nor } A_2 \text{ nor} A_3 \\ 0.27089215\,43877144 & \text{If C contains } A_1 \text{ but neither } A_2 \text{ nor } A_3. \\ 0.08149540\,0822394 & \text{If C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3. \\ 0.17662905\,9350823 & \text{If C contains } A_3 \text{ but neither } A_1 \text{ nor } A_2. \\ 0.35232755\,5209538 & \text{If C contains } A_1 \text{ and } A_2 \text{ but not } A_3. \\ 0.44752121\,3737967 & \text{If C contains } A_1 \text{ and } A_3 \text{ but not } A_2. \\ 0.25806446\,0173217 & \text{If C contains } A_2 \text{ and } A_3 \text{ but not } A_1. \\ 0.52895661\,4560361 & \text{If Ccontains } A_1, A_2 \text{ and } A_3 \text{ but } C\neq\Theta. \\ 1 & \text{If } C=\Theta. \end{cases} \quad (18)$$

for all $C\subseteq\Theta$.

6.2.2 Intersection of all Evidences is $\varnothing$

Suppose $A_i\cap A_j=\varnothing \quad i,j=1,2,3 \text{and} i\neq j$. From [1, 7], suppose that first body of evidence focused on A_1 with degree of support s_1 is probability obtained from binomial distribution $Binomial(10,0.7)$, second body of evidence focused on A_2 with degree of support s_2 is probability obtained from Poison distribution $Poison(2)$ and third body of evidence focused on A_3 with degree of support s_3 is probability obtained from discrete uniform distribution. Here $A_1=\{1,2,3,5,6,9\}$, $A_2=\{0,1,6,8,10\}$ and $A_3=\{0,2,3,5,7,8\}$. Therefore $A_1\cap A_2=\{1,6\}$, $A_1\cap A_3=\{2,3,5\}$, $A_2\cap A_3=\{0,8\}$ and $A_1\cap A_2\cap A_3=\varnothing$. By figure of conflicting evidence for three dimensions, the parallelopiped is related to $A_1\cap A_2\cap A_3=\varnothing$. For application of Dempster's rule, we eliminate corresponding parallelopiped. Therefore decrease the measures of the remaining parallelopipeds by quantity $\dfrac{1}{1-s_1(A_1)s_2(A_2)s_3(A_3)}$. The *orthogonal sum* $S=S_1\oplus S_2\oplus S_3$ have basic probability numbers

$$
\begin{aligned}
m(A_1 \cap A_2) &= s_1(A_1)s_2(A_2)(1-s_3(A_3))/K = 0.091903748653322,\\
m(A_1 \cap A_3) &= s_1(A_1)s_3(A_3)(1-s_2(A_2))/K = 0.152966351591380,\\
m(A_2 \cap A_3) &= s_2(A_2)s_3(A_3)(1-s_1(A_1))/K = 0.143425470255328,\\
m(A_1) &= s_1(A_1)(1-s_2(A_2))(1-s_3(A_3))/K = 0.127471959668042,\\
m(A_2) &= s_2(A_2)(1-s_1(A_1))(1-s_3(A_3))/K = 0.119521225212776,\\
m(A_3) &= s_3(A_3)(1-s_1(A_1))(1-s_2(A_2))/K = 0.198933406160566,\\
m(\Theta) &= (1-s_1(A_1))(1-s_2(A_2))(1-s_3(A_3))/K = 0.165777838467142,
\end{aligned}
$$

and $\quad m(B) = 0$ if B is none of the above mentioned subsets of Θ.

where $K = 1 - s_1(A_1)s_2(A_2)s_3(A_3) = 0.90067005479721$.

Also, sum of basic probability numbers corresponding to all parallelopipeds is one.

By using equation (10), the orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is a separable support function and is given by

$$
S(C) = \begin{cases}
0 & \text{If C contains neither } A_1 \cap A_2 \text{ nor } A_1 \cap A_3 \\
 & \text{nor } A_2 \cap A_3 \\
0.091903748653322 & \text{if C contains } A_1 \cap A_2 \text{ but neither} \\
 & A_1 \cap A_3 \text{ nor } A_2 \cap A_3 \\
0.152966351591380 & \text{if C contains } A_1 \cap A_3 \text{ but neither} \\
 & A_1 \cap A_2 \text{ nor } A_2 \cap A_3 \\
0.143425470255328 & \text{if C contains } A_2 \cap A_3 \text{ but neither} \\
 & A_1 \cap A_2 \text{ nor } A_1 \cap A_3 \\
0.244870100244702 & \text{if C contains } A_1 \cap A_2 \text{ and} \\
 & A_1 \cap A_3 \text{ but not } A_2 \cap A_3 \\
0.235329218908650 & \text{if C contains } A_1 \cap A_2 \text{ and} \\
 & A_2 \cap A_3 \text{ but not } A_1 \cap A_3 \\
0.296391821846708 & \text{if C contains } A_1 \cap A_3 \text{ and} \\
 & A_2 \cap A_3 \text{ but not } A_1 \cap A_2 \\
038829557050003 & \text{if C contains } A_1 \cap A_2, A_1 \cap A_3 \text{ and} \\
 & A_2 \cap A_3 \text{ but not their proper superset in } \Theta \\
0.372342059904191 & \text{if C contains } A_1 \text{ but neither } A_2 \text{ nor } A_3 \\
0.35485044412428 & \text{if C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3 \\
0.495325228007276 & \text{if C contains } A_3 \text{ but neither } A_2 \text{ nor } A_3 \\
0.5872228976660599 & \text{if C contains } (A_1 \cap A_2) \cup A_3 \\
 & \text{but not its proper superset in } \Theta \\
0.507816795712808 & \text{if C contains } (A_1 \cap A_3) \cup A_2 \\
 & \text{but not its proper superset in } \Theta \\
0.515767530159519 & \text{if C contains } (A_2 \cap A_3) \cup A_1 \\
 & \text{but not its proper superset in } \Theta \\
0.635288755372295 & \text{if C contains } A_1 \text{ and } A_2 \text{ but not } A_3 \\
0.714700936320086 & \text{if C contains } A_1 \text{ and } A_3 \text{ but not } A_2 \\
0.706750201873376 & \text{if C contains } A_2 \text{ and } A_3 \text{ but not } A_1 \\
0.834222161532863 & \text{if C contains } A_1, A_2 \text{ and } A_3 \text{ but } C \neq \Theta \\
1 & \text{if } C = \Theta
\end{cases}
\qquad (19)
$$

for all $C \subseteq \Theta$.

6.3 Mixing of Above Two Cases

In this case, any two evidences may be heterogeneous or conflicting. If given two evidences are heterogeneous then their intersection is not related to $\varnothing$. If given two evidences are conflicting then their intersection is related to $\varnothing$. Such combinations of evidences should be done for all possible pairs of given evidences. The effect of conflicting evidences is taken into account while calculating the value of K as in above cases. Also eliminate cases in $S(C)$ whose intersection is related to $\varnothing$. As Dempster's rule of combination is associative and commutative hence Bernaulli's rule of combination, the order of combination of support functions is not important.

Suppose $A_1 \cap A_2 \cap A_3 = \varnothing$ and $A_2 \cap A_3 = \varnothing$. From [1, 7], suppose that first body of evidence focused on A_1 with degree of support s_1 is probability obtained from binomial distribution $Binomial(10,0.7)$, second body of evidence focused on A_2 with degree of support s_2 is probability obtained from Poison distribution $Poison(2)$ and third body of evidence focused on A_3 with degree of support s_3 is probability obtained from discrete uniform distribution. By figure of conflicting evidence for three dimensions, WOLOG, assume that the parallelopiped is related to $A_2 \cap A_3 = \varnothing$ hence $A_1 \cap A_2 \cap A_3 = \varnothing$. For application of Dempster's rule, we eliminate corresponding parallelopiped. Therefore decrease the measures of the remaining parallelopipeds by quantity $\dfrac{1}{1 - s_2(A_2)s_3(A_3)}$. Here we need at least one pair of subsets of Θ among the subsets A_1, A_2 and A_3 of Θ, is disjoint.

Here $A_1 = \{1,2,3,5,6,8,9\}$, $A_2 = \{0,1,3,6,10\}$ and $A_3 = \{2,5,7,9\}$.

Therefore $A_1 \cap A_2 = \{1,3,6\}$, $A_1 \cap A_3 = \{2,5\}$, $A_2 \cap A_3 = \varnothing$,

and $A_1 \cap A_2 \cap A_3 = \varnothing$. The *orthogonal sum* $S = S_1 \oplus S_2 \oplus S_3$ have basic probability numbers

$$m(A_1 \cap A_2) = s_1(A_1)s_2(A_2)(1 - s_3(A_3))/K = 0.325283340866,$$
$$m(A_1 \cap A_3) = s_1(A_1)s_3(A_3)(1 - s_2(A_2))/K = 0.124683050302971,$$
$$m(A_1) = s_1(A_1)(1 - s_2(A_2))(1 - s_3(A_3))/K = 0.218195338030203,$$
$$m(A_2) = s_2(A_2)(1 - s_1(A_1))(1 - s_3(A_3))/K = 0.161549901223068,$$
$$m(A_3) = s_3(A_3)(1 - s_1(A_1))(1 - s_2(A_2))/K = 0.061923043482520,$$
$$m(\Theta) = (1 - s_1(A_1))(1 - s_2(A_2))(1 - s_3(A_3))/K = 0.108365326094412,$$
$$\text{and} \qquad m(B) = 0 \text{ if B is none of the above mentioned subsets of } \Theta,$$

where $K = 1 - s_2(A_2)s_3(A_3) = 0.782356041152609$.

Also, sum of basic probability numbers corresponding to all parallelopipeds is one.

By using equation (11), the orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ is a separable support function and is given by

$$S(C)=\begin{cases} 0 & \text{If C contains neither } A_1\cap A_2 \text{ nor } A_1\cap A_3 \\ 0.325283340866825 & \text{if C contains } A_1\cap A_2 \text{ but not } A_1\cap A_3 \\ 0.124683050302971 & \text{if C contains } A_1\cap A_3 \text{ but not } A_1\cap A_2 \\ 0.449966391169796 & \text{if C contains } A_1\cap A_2 \text{ and } A_1\cap A_3 \text{ but not its proper subset in } \Theta \\ 0.668161729200001 & \text{if C contains } A_1 \text{ but neither } A_2 \text{ nor } A_3 \\ 0.486833242089893 & \text{if C contains } A_2 \text{ but neither } A_1 \text{ nor } A_3 \\ 0.186606093785492 & \text{if C contains } A_3 \text{ but neither } A_2 \text{ nor } A_3 \\ 0.511889434652318 & \text{if C contains } (A_1\cap A_2)\cup A_3 \text{ but not its proper superset in } \Theta \\ 0.611516292392866 & \text{if C contains } (A_1\cap A_3)\cup A_2 \text{ but not its proper superset in } \Theta \\ 0.829711630423069 & \text{if C contains } A_1 \text{ and } A_2 \text{ bu tnot } A_3 \\ 0.730084772682521 & \text{if C contains } A_1 \text{ and } A_3 \text{ but not } A_2 \\ 0.673439335875385 & \text{if C contains } A_2 \text{ and } A_3 \text{ but not } A_1 \\ 0.861488477886919 & \text{if C contains } A_1, A_2 \text{ and } A_3 \text{ but } C\neq\Theta \\ 1 & \text{if } C=\Theta \end{cases} \quad (20)$$

for all $C\subseteq\Theta$.

6.4 Combination of Evidences in 2D in Super Power Set

6.4.1 Heterogeneous Evidences in 2D in Super Power Set

Suppose $A\cap B\neq\varnothing$. From [1, 7], let S_1 be a simple support function focused on A_1 with $S_1(A)=s_1$ following binomial distribution $Binomial(10,0.7)$. Let S_2 be another simple support function focused on B with $S_2(B)=s_2$ following Poison distribution $Poison(2)$. Here $A=\{1,2,3,5,6,8,9\}$ and $B=\{0,1,3,6,8,10\}$. Therefore $A\cap B=\{1,3,6,8\}$. Here $s_1=p(A)=0.6681172922$ and $s_2-p(B)=0.59938015846993$. The orthogonal sum $S=S_1\oplus S_2$ focused on $A\cup B$, have basic probability numbers $m(A\cap B)=s_1s_2=0.400482883131438$, $m(A)=s_1(1-s_2)=0.267878846068561$, $m(B)=s_2(1-s_1)=0.198897268701726$ and $m(\Theta)=(1-s_1)(1-s_2)=0.132940995461508$. Therefore by using equation (12),

$$S(C)=\begin{cases} 0 & \text{if C does not contain } A\cap B \\ 0.400482883131438 & \text{if C contains } A\cap B \text{ but neither A nor B} \\ 0.6681172922 & \text{if C contains A but not B} \\ 0.59938015846993 & \text{if C contains B bu tnot A} \\ 0.86705900453849 & \text{if C contains both A and B and } C\neq\Theta \\ 0.132940995461508 & \text{if C contains } c(A\cup B) \text{ but not its proper superset in } \Theta \\ 0.3318382708 & \text{if C contains } c(A) \text{ but not its proper superset in } \Theta \\ 0.400619841530070 & \text{if C contains } c(B) \text{ but not its proper superset in } \Theta \\ 0.59951711686856 & \text{if C contains } c(A\cap B) \text{ but not its proper superset in } \Theta \\ 1 & \text{if } C=\Theta \end{cases} \quad (21)$$

for all $C\subset\Theta$.

6.4.2 Conflicting Evidences in 2D in Super Power Set

If $A \cap B = \varnothing$ then the combination of evidences pointing to A and evidences pointing to B is simple as compared to $A \cap B = \neq \varnothing$ i.e. the bodies of evidence are conflicting and effect of one body of evidence decreases the effect of other body of evidence. While applying Dempster‘s rule of combination, we eliminate lower-left rectangle (see the figure of conflicting evidences) and increase the measures of remaining rectangles by multiplying factor $\frac{1}{1-s_1 s_2}$. From [1, 7], let S_1 be a simple support function focused on A with $S_1(A) = s_1$ which follows binomial distribution $Binomial(10,0.7)$. Let S_2 be a simple support function focused on B with $S_2(B) = s_2$ which follows poison distribution $Poison(2)$. Here $A = \{1,2,3,5,6,8,9\}$ and $B = \{0,3,7,10\}$ hence $A \cap B = \varnothing$ $s_1 = pA) = 0.6681617292$ and $s_2 = p(B) = 0..3192576039 1133$. The **orthogonal sum** $S = S_1 \oplus S_2$ focused on $A \cup B$ have basic probability numbers:

$$m(C) = \begin{cases} 0 & \text{if } C = \varnothing \\ \dfrac{s_1(1-s_2)}{1-s_1 s_2} = 0.5781138015427 & \text{if } C = A \\ \dfrac{s_2(1-s_1)}{1-s_1 s_2} = 0.134668879182659 & \text{if } C = B \\ \dfrac{(1-s_1)(1-s_2)}{1-s_1 s_2} = 0.287149982801223 & \text{if } C = \Theta. \end{cases} \tag{22}$$

By using equation (13), the orthogonal sum S of simple support functions S_1 and S_2 focused on $A \cup B$, is given by

$$S(C) = \begin{cases} 0 & \text{if C contains neither A nor B} \\ 0.5781138015427 & \text{if C contains A but not B} \\ 0.134668879182659 & \text{if C contains B but not A} \\ 0.712850017198086 & \text{if C contains both A and B and } C \neq \Theta \\ 0.28714998280191 & \text{if Ccontains } c(A \cup B) \text{ but not its proper superset in } \Theta \\ 0.421818861984573 & \text{if C contains } c(A) \text{ but not its proper superset in } \Theta \\ 0.86533112081734 & \text{if C contains } c(B) \text{ but not its proper superset in } \Theta \\ 1 & \text{if } C = \Theta \end{cases} \tag{23}$$

for all $C \subset \Theta$.

7 CONCLUSION

Usually it is beneficial to directly point out subset A_i of Θ which is concerned to our interest. Such set is always assigned basic belief assignment $m_i(A_i)$ greater than 0.5 and $m_i(\Theta) = 1 - m_i(A_i)$, by choosing appropriate probability distribution. In this paper, we have obtained support functions for heterogeneous and conflicting evidences in hyper power set and super power set. These are illustrated by an example, considering simple and commonly used probability distributions viz. discrete uniform distribution, Binomial distribution and Poison Distribution.

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