Application of "Kuffi-Abbas-Jawad" KAJ Transform to the Solution of Stochastic Differential Equation

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Abstract: From a very long time differential equations have played very important and central role in every aspect of applied mathematics. Stochastic differential equations are used to model various phenomena. We use KAJ transform to obtain the solution of stochastic differential equations.

Key words: Stochastic differential equations, Integral transforms, KAJ transform.

I. Introduction:

The differential equation in which one or more of the terms is a stochastic process, resulting in a solution which is a stochastic process is a stochastic differential equation. These equations are useful in modeling. These equations are originated in the theory of Brownian motion, in the work of Albert Einstein and Smoluchowski. Stochastic modeling develops a mathematical model to derive all possible outcomes of a given problem or scenarios using random input variables. It focuses on the probability distribution of possible outcomes. This means stochastic differential equations are of much importance in mathematical modeling. We use integral transform to solve such type of equations.

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics.

Many researchers are attracted to this field, due to this important feature of the integral transforms and are engaged in introducing various integral transforms. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [2] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [3, 11, 31, 33, 36, 39] used Kushare transform for solving problems in various fields. D.P. Patil [4, 5, 7] also used Sawi transform in Bessel functions and to evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [6]. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [8].

D .P. Patil [1] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [10] also obtained dualities between double integral transforms. D. P. Patil [12] used Aboodh and Mahgoub transforms to solve boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by D. P. Patil [13] to solve parabolic boundary value problems.

Laplace, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [14]. D. P. Patil et al [15] obtained solution of Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [16] used Anuj transform for solving Volterra integral equations of first kind. Rathi sisters and D. P. Patil [17] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [18] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [19] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, et al [20] obtained the solution of parabolic boundary value problems by using double general integral transform. used Emad- Falih transform for solving problems based on Problems on Newton's law of cooling is solved by Dinkar Patil et al [21, 22, 24] by using various integral transforms.

Dinkar Patil et al [23, 44] used HY integral transform and ARA transform for handling growth and Decay problems, D. P. Patil et al [25] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [26] introduced double kushare transform. Recently, D. P. Patil et al [27] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by Patil et al [28]. Thete, et al [29] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [310] used, Kushare transform of error functions in evaluating improper integrals.

Wagh sisters and Patil used Soham [32] transform in chemical Sciences. Raundal and Patil [34] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [35] developed generalized double rangaig integral transform.

Kandekar et al [37] used new general integral equation to solve Abel's integral equations. Pardeshi, Shaikh and Patil [38] used Kharrat Toma transform for solving population growth and decay problems. Thakare and Patil [40] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [41]. Patil [42] used KKAT transform for solving growth and decay problems and Newton's law of cooling. Suryawanshi et al [43] used Soham transform for solving models in health sciences and biotechnology.

We organize this paper as follows. Introduction is included in first section. Second section is devoted for preliminary concepts. KAJ transform is used to the problems of Stochastic differential equations in third section.

II. **Preliminaries**:

In this section we state preliminaries concepts which are required for solving Stochastic Differential Equation, which contains KAJ Integral Transform, KAJ Integral Transform of some basic functions, KAJ Integral Transform of Derivatives and formulation of Stochastic Differential Equation.

a. KAJ Integral Transform[46]:

KAJ transform (denoted by S_m), as a modification on the SEE (Sadiq-Emad-Eman) Integral Transform, is given by:

$$S_m\{f(t)\} = K(v) = \frac{1}{v^n}\int_{t=0}^{\infty} f\left(\frac{t}{v}\right) e^{-t} dt, \qquad n \in \mathbb{Z},$$

$$0 < l_1 \le v \le l_2, where\ l_1\ and\ l_2\ are\ either\ finite\ or\ infinite.$$

b. KAJ Integral Transform for Some Basic Functions[46]:

Functions $f(t)$	KAJ Transform $S_m\{f(t)\} = K(v)$
1	$\frac{1}{v^n}$
t	$\frac{1}{v^{n+1}}$
t^m	$\frac{m!}{v^{n+m}}$
e^{at}	$\frac{v}{v^n\ (v-a)}$
$\sin(at)$	$\frac{a}{v^{n-1}\ (v^2-a^2)}$
$\cos(at)$	$\frac{1}{v^{n-2}\ (v^2-a^2)}$
$\sinh(at)$	$\frac{a}{v^{n-1}\ (v^2-a^2)}$
$\cosh(at)$	$\frac{1}{v^{n-2}\ (v^2-a^2)}$

c. KAJ Integral Transform of Derivatives[46]:

$If\ S_m\{f(t)\} = K(v)$:

- $S_m\{f'(t)\} = \left[vK(v) - \frac{v}{v^n} f(0)\right].$
- $S_m\{f''(t)\} = \frac{1}{v^n}[-v^2 f(0) - v f'(0)] + v^2 K(v).$

d. **Stochastic Differential Equation:**

A Stochastic differential equation (SDE) is a differential equation in which one or more of the terms are a process, resulting in a solution which is also a stochastic process. Therefore, the following is the most general class of SDEs.

$$\frac{dx(t)}{dt} = F\big(x(t)\big) + \sum_{\alpha=1}^{n} g\alpha\big(x(t)\big)\,\xi^{\alpha} \qquad (1)$$

Where is the position in the system in its phase (or state) space, X, assumed to be a differentiable manifold.

e. **Formulation for Stochastic Process on Option Pricing Black – Scholes PDE[1] :**

Given a finite time horizon T > 0, we shall consider a complete probability space(Ω, F, P) equipped with Standard Brownian motion $W = (\{W_t^1, \dots, W_t^d\}, 0 \le t \le T)$ valued in Γ^d, and generating the (p-augmented) filtration. The financial market consists of a non-risky asset S^0 normalized to unity, that is $S^0 = 1$, and a risky assets with price process

$S = (S_t^1, \dots, S_t^d)$ whose dynamics is defined by a stochastic differential equation,

$$dS_t = \alpha S_t dt + \sigma S_t dW_t. \qquad (2)$$

Starting from S_0 at time 0, we can obtain the solution of equation (2) as,

$$S_t = S_0 \exp\left\{\sigma W_t + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\} \qquad \forall\, t \in [0, T] \qquad (3)$$

Considering a short trading period where new dividends will not have been declared and no new assets have been purchased then the stock price follows the process,

$$dS_t = \hat{\alpha} S_t dt + \sigma S_t dW_t\,, \hat{\alpha} = \alpha + \lambda \qquad (4)$$

Here λ is the market price of risk. The stock pricing PDE is then the backward Black-Scholes PDE given by:

$$\frac{1}{2}\sigma^2 S_t^2 \frac{\partial v}{\partial S_t^2} + \alpha \frac{\partial v}{\partial S_t} - rV = -\frac{\partial v}{\partial t}, \qquad (5)$$

where $V = V(S_t, K_t)$is investment worth, S_t is the stock price at time t and K_t is the investment output over a period of time t.

Let the rate of change of worth V with respect to time t, be given as:

$$\frac{\partial v(S_t, K_t)}{\partial t} = P(S_t, K_t) - C(S_t, K_t). \qquad (6)$$

Where P is the proportion rate, C is the consumption rate and K_t is the total output. For a relatively short time, the rate of consumption is very small so that, $C(S_t, K_t) \to 0 \; as \; t \to 0$.

We then write (6) as, $\frac{\partial v}{\partial t} = P(S_t, K_t).$ (7)

Since the rate of change of worth of the firm depends on the investment output at present time t, we write (7) as,

$$\frac{dv}{dt} = P(K_t) = S_t, \qquad (8)$$

for short selling period. Since the difference of V on the left hand side of (8)

is with respect to S (and not K), we can denote $V(S, K) \; by \; V(S)$ so that (8) reduces to an ODE of the form,

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2V(S)}{dS^2} + \alpha S \frac{dV(S)}{dS} - rV(S) = -P(K) = -S, \qquad (9)$$

III. **KAJ transform to solve Stochastic differential equations**

Now we state the following theorem:

Theorem: Let the second order linear Black-Schole's differential equation be given in the form ,

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + \alpha S \frac{dV}{dS} - rV = -S, \qquad (10)$$

With initial conditions $V(0) = a, V'(0) = b.$

Then the solution of this initial value problem is given by

$$V(S) = \frac{-S}{\gamma\beta} + \frac{a\gamma^3 - 1}{\gamma(\beta - \gamma)} e^{\gamma S} + \frac{a\beta^3 - 1}{\beta(\gamma - \beta)} e^{\beta S} \qquad (11)$$

Proof: We prove this theorem by using KAJ transform.

Let $\lambda_1 \; and \; \lambda_2$ be the roots of the homogeneous part of (10), then,

$$\lambda_1 + \lambda_2 = \frac{-2\alpha}{S\sigma^2}, \qquad \lambda_1\lambda_2 = \frac{-2(S+\alpha)}{\sigma^2 S^2} \qquad (12)$$

where $r = (S + \alpha)$we assume that r a linear function price instead of a linear function of time .

$$We\ now\ have,\ \frac{d^2V}{dS^2} + (\lambda_1 + \lambda_2)\frac{dV}{dS} - \lambda_1\lambda_2 V = -S \qquad (13)$$

Applying KAJ transform to the (13)

$$S_m\left\{\frac{d^2V}{dS^2}\right\}+(\lambda_1+\lambda_2)S_m\left\{\frac{dV}{dS}\right\}-\lambda_1\lambda_2 S_m\{V\}=-S_m\{S\}$$

$$\left[\frac{1}{p^n}(-p^2V(0)-pV'(0)+p^2K(p)\right]+(\lambda_1+\lambda_2)\left[pK(p)-\frac{p}{p^n}V(0)\right]-\lambda_1\lambda_2K(p)=-\frac{1}{p^{n+1}}$$

$$(p^2+(\lambda_1+\lambda_2)p-\lambda_1\lambda_2)K(p)=-\frac{1}{p^{n+1}}+\frac{ap^2}{p^n}+\frac{bp}{p^n}+\frac{(\lambda_1+\lambda_2)ap}{p^n}$$

$$K(p)=-\frac{1}{p^{n+1}(p^2+(\lambda_1+\lambda_2)p-\lambda_1\lambda_2)}+\frac{ap^2}{p^n(p^2+(\lambda_1+\lambda_2)p-\lambda_1\lambda_2)}+\frac{bp}{p^n(p^2+(\lambda_1+\lambda_2)p-\lambda_1\lambda_2)}+\frac{(\lambda_1+\lambda_2)ap}{p^n(p^2+(\lambda_1+\lambda_2)p-\lambda_1\lambda_2)}$$

$$K(p)=-\frac{1}{p^{n+1}(p-\gamma)(p-\beta)}+\frac{ap^3}{p^{n+1}(p-\gamma)(p-\beta)}+\frac{bp}{p^n(p-\gamma)(p-\beta)}+\frac{(\lambda_1+\lambda_2)ap}{p^n(p-\gamma)(p-\beta)}$$

where γ *and* β *are roots of quadriatic* $eq^n(p^2+(\lambda_1+\lambda_2)p-\lambda_1\lambda_2)$

$$K(p)=\frac{ap^3-1}{p^{n+1}(p-\gamma)(p-\beta)}+\frac{(b+(\lambda_1+\lambda_2)a)p}{p^n(p-\gamma)(p-\beta)}$$

$$K(p)=\frac{1}{p^n}\frac{ap^3-1}{p(p-\gamma)(p-\beta)}+\frac{1}{p^n}\frac{(b+(\lambda_1+\lambda_2)a)p}{(p-\gamma)(p-\beta)}$$

Now by computing partial fraction,

$$K(p)=\frac{-1}{p^n}\left[\frac{1}{\gamma\beta p}+\frac{a\gamma^3-1}{\gamma(\gamma-\beta)(p-\gamma)}+\frac{a\beta^3-1}{\beta(\beta-\gamma)(p-\beta)}\right]+\frac{1}{p^n}\left[\frac{(b+(\lambda_1+\lambda_2)a)\gamma}{(\gamma-\beta)(p-\gamma)}+\frac{(b+(\lambda_1+\lambda_2)a)\beta}{(\beta-\gamma)(p-\beta)}\right]$$

$$K(p)=\frac{-1}{\gamma\beta p^{n+1}}+\frac{a\gamma^3-1}{\gamma(\beta-\gamma)}\left(\frac{1}{p^n(p-\gamma)}\right)+\frac{a\beta^3-1}{\beta(\gamma-\beta)}\left(\frac{1}{p^n(p-\beta)}\right)+\frac{(b+(\lambda_1+\lambda_2)a)\gamma}{(\gamma-\beta)}\left(\frac{1}{p^n(p-\gamma)}\right)+\frac{(b+(\lambda_1+\lambda_2)a)\beta}{(\beta-\gamma)}\left(\frac{1}{p^n(p-\beta)}\right)$$

By using inverse KAJ transform to above equation we get the required solution as follows, $V(S)=\frac{-S}{\gamma\beta}+\frac{a\gamma^3-1}{\gamma(\beta-\gamma)}e^{\gamma S}+\frac{a\beta^3-1}{\beta(\gamma-\beta)}e^{\beta S}$

Example: Solve the second order linear Black-Scholes's Equation with a specified initial condition given as:

$$\frac{1}{2}\sigma^2S^2\frac{d^2V}{dS^2}+\alpha S\frac{dV}{dS}-rV=-S \tag{14}$$

$$V(0)=0, V'(0)=1 \tag{15}$$

Solution: Let λ_1 *and* λ_2 be as in previous theorem,

$$\therefore \frac{d^2V}{dS^2}+(\lambda_1+\lambda_2)\frac{dV}{dS}-\lambda_1\lambda_2V=-S \tag{16}$$

Now applying KAJ Integral Transform to above equation,

$$S_m\left\{\frac{d^2V}{dS^2}\right\}+(\lambda_1+\lambda_2)S_m\left\{\frac{dV}{dS}\right\}-\lambda_1\lambda_2 S_m\{V\}=-S_m\{S\}$$

$$\left[\frac{1}{p^n}(-p^2V(0)-pV'(0)+p^2K(p)\right]+(\lambda_1+\lambda_2)\left[pK(p)-\frac{p}{p^n}V(0)\right]-\lambda_1\lambda_2K(p)=-\frac{1}{p^{n+1}}$$

$$\left[\frac{1}{p^n}(0-p)+p^2K(p)\right]+(\lambda_1+\lambda_2)[pK(p)-0]-\lambda_1\lambda_2K(p)=-\frac{1}{p^{n+1}}$$

$$\frac{p}{p^n} + p^2K(p) + (\lambda_1 + \lambda_2)\,pK(p) - \lambda_1\lambda_2K(p) = -\frac{1}{p^{n+1}}$$

$$(p^2 + (\lambda_1 + \lambda_2)p - \lambda_1\lambda_2)K(p) = -\frac{1}{p^{n+1}} - \frac{p}{p^n}$$

$$K(p) = -\frac{1}{p^{n+1}(p^2 + (\lambda_1 + \lambda_2)p - \lambda_1\lambda_2)} - \frac{p}{p^n(p^2 + (\lambda_1 + \lambda_2)p - \lambda_1\lambda_2)}$$

$$K(p) = \frac{1}{p^n}\left[\frac{-1}{p(p-\gamma)(p-\beta)}\right] - \frac{1}{p^n}\left[\frac{p}{(p-\gamma)(p-\beta)}\right]$$

where γ and β are roots of the quadratic equation $(p^2 + (\lambda_1 + \lambda_2)p - \lambda_1\lambda_2)$

Now by computing partial fractions we get,

$$K(p) = \frac{1}{p^n}\left[\frac{-1}{p\gamma\beta} - \frac{1}{\gamma(\gamma-\beta)(p-\gamma)} - \frac{1}{\beta(\beta-\gamma)(p-\beta)}\right] - \frac{1}{p^n}\left[-\frac{\gamma}{(\gamma-\beta)(p-\gamma)} - \frac{\beta}{(\beta-\gamma)(p-\beta)}\right]$$

$$K(p) = \frac{-1}{\gamma\beta p^{n+1}} + \frac{\gamma^2-1}{\gamma(\gamma-\beta)}\left(\frac{1}{p^n(p-\gamma)}\right) + \frac{\beta^2-1}{\beta(\beta-\gamma)}\left(\frac{1}{p^n(p-\beta)}\right)$$

Now, by applying inverse KAJ transform,

$$K^{-1}\{K(p)\} = K^{-1}\left\{\frac{-1}{\gamma\beta p^{n+1}}\right\} + K^{-1}\left\{\frac{\gamma^2-1}{\gamma(\gamma-\beta)}\left(\frac{1}{p^n(p-\gamma)}\right)\right\} + K^{-1}\left\{\frac{\beta^2-1}{\beta(\beta-\gamma)}\left(\frac{1}{p^n(p-\beta)}\right)\right\}$$

We get the solution of given PDE is,

$$V(S) = \frac{-S}{\gamma\beta} + \frac{\gamma^2-1}{\gamma(\gamma-\beta)}e^{\gamma S} + \frac{\beta^2-1}{\beta(\beta-\gamma)}e^{\beta S}$$

Conclusion: we have successfully used KAJ integral transform to solve the stochastic differential equations.

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