SOME RESULTS ON SWITCHED GRAPHS

Mahadevaswamy.B.S.

Associate Professor of Mathematics
Department of Mathematics
Maharani's Science College for Women, Mysore, Karnataka, India -570 005

Abstract: We introduce new graphs by using minimum dominating set. We introduce the concept of partial switching of a graph, characterization of switched graphs are obtained. We also obtain some results on switched graphs and the relation between partial switching and switching on graphs.

Keywords: Minimum Dominating set. Switched graphs, partially switched graphs.

1. Introduction:

As in any branch of Mathematics the aim is how to get new mathematical structure using the existing ones. Analogously in this paper.We introduce new graphs by using minimum dominating sets, switching and partial switching.

Switched graphs are a natural extension to ordinary graphs and a natural abstract domain for representing concert problems. Many famous graphs are obtained by switching, for example, Shrikhandegraph is obtained by switching Clebsch graph with respect to some vertices. Also Change graphs which is obtained from triangular graph of order 28; that is T(8). We introduce the concept of partial switching of a graph, characterization of switched graphs are obtained. We also obtain some results on switched graphs and the relation between partial switching and switching on graphs.

In (36) switching is defined as follows:

Let G = (V, E) be a graph, for a given partition $\sigma = (V_1, V_2)$ of a set V we define the switched graph $S(G_\sigma) = (V, E_\sigma)$, where $\sigma = \{H, V - H\}$ by setting $E_\sigma = E_{11} \cup ((V_1 \times V_2) - E_{12}) \cup ((V_2 \times V_1) - E_{21}) \cup E_{22}$, where $E_{ij} = E \cap (V_i \times V_j)$. Sometimes we denote to the switched graph $S(G_\sigma)$ by $S_H(G)$.

If A (G) is adjacency of G then we can write a matrix in block form:

$$A(G) = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

With block determined by a partition σ,

$$A(G_\sigma) = \begin{pmatrix} A_{11} & J_{12} - A_{12} \\ J_{21} - A_{21} & A_{22} \end{pmatrix}$$

Where J_{ij} is $|V_i| \times |V_j|$ matrix whose entries are equal to one.

Definition 1.1. A partition $\sigma = (V_1, V_2)$ of the vertex set V of a regular graph G is called equitable partition of G, if for any pair (I, j) $\in \{1,2\}$ and any vertex $\upsilon \in V_i$ the number $m_{ij} = |G(\upsilon) \cap V_j|$ depends only on (i,j) (where G(v) in G adjacent to υ).

In this paper we study the switching of some graphs with respect to some subset, particularly dominating set.

Definition 1.2. Let G = (V, E) be a graph. For a given partition $\sigma = (V_1, V_2)$ of a set V. We define the partial switching graph $S_p(G_\sigma) = (E_\sigma^p)$ where $E_\sigma^p = (V_1 \times V_1) - E_{11} \cup (V_2 \times V_2) - E_{22} \cup E_{12}$, where $E_{ij} - E \cap (V_i \times V_j)$.

Example 1.1. With respect to the lebelling of the following figure 1, we can find the switching and partial switching with respect to the partition σ = ({2,5}, {1, 3, 4}).

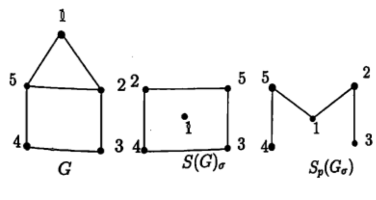

Figure 1

2. Elementary Results:

Observation 2.1 Let G be a graph and H is a subset of V (G). then the switching of a graph with respect to H is the same as switching of G with respect to $V(G) - H = \overline{H}$.

That is

$S_H(G) = S_{V-H}(G)$

Observation 2.2. Switching of any graph with respect to the whole vertex set V (G) is isomorphic to the original graph.

This is

$S_{V(G)}(G) \cong G$.

Observation 2.3. Switching successively with respect to H_1 and H_2 is the same as switching with respect to the symmetric difference $H_1 \Delta H_2$,

Where $H_1 \Delta H_2 = H_1 \cup H_2 / H_1 \cap H_2$. That is
$\mathbf{S_{H2} (S_{H1} (G)) = S_{H1\Delta H2}} (G)$.
Example 2.4. Let $G = C_5$ as in Figure 2 and $H_1 = \{2,5\}$ and
$H_2 = \{1\}$. Then S_{H1} (G) is the switching with respect to H_1 is shown in Figure 2.

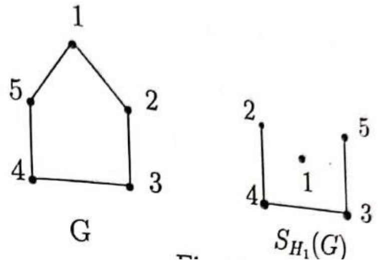

Figure 2

Now switching the resulting graph with respect to H_2 we get S_{H2} (S_{H1}(G)) which is equal to $S_{H1\Delta H2}$(G) as shown in Figure 3.

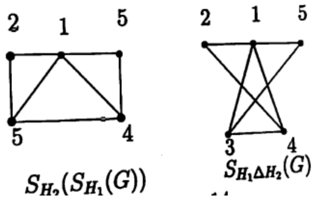

Figure 3

Theorem 2.5. Let $G = K_p$ be a complete graph. For any vertex
$\upsilon \in V(G)$, $S_{\{\upsilon\}}$ $(K_p) \cong K_1 \cup K_p$.

Proof. Let K_p be a complete graph with p vertices with the vertex set V(G) with the cardinality p. Let υ be any vertex in the complete graph and we make a switching of K_p with respect to $\{\upsilon\}$ such that the partition $\sigma = (\{\upsilon\}, V - \{\upsilon\})$. All the edges between the $\{\upsilon\}$ and V - $\{\upsilon\}$ become non edges. That is $\{\upsilon\}$ is isolated in switched graph that is K_1. On the other hand every edges in $<\{\upsilon\}, V - \{\upsilon\}>$ will be as it is. Since we have (p – 1) vertices in ($\{\upsilon\}$, V - $\{\upsilon\}$) and also edge vertex is joined to all the other vertices that is K_{p-1}. Hence

$S_{\{\upsilon\}}$ $(K_p) \cong K_1 \cup K_p$.

Corollary 2.6. For any graph K_p we have, $\gamma(S_{\{\upsilon\}}\ (K_p)) = 2$.
Observation 2.7. $S_H (W_p) = C_{p-1}$, where H is a minimum dominating set, and so we have,
$\gamma(S_H (W_p)) = 1 + \lceil (p - 1)/3 \rceil$.

3. Main Results:

Proposition 3.1. For any graph G = (V, E) and $H \subseteq V$ with σ = (V1, V2), $\overline{S_H(G)} \cong S_H(\overline{G})$.

Proof. Let $\overline{S_H(G)}$ and $S_H(\overline{G})$ be two graphs with the same vertices.

S_H (G) replaces all the edges between V_1 and V_2 with non-edge and vice versa. By taking complement of G = (V,E) with respect to σ .

$\overline{S_H(G)}$ replaces all edges and non edgesbetweenV_1 and V_2 and vice versa to become as original graph. But edge which lies completely inside V_1 will be replaced by non-edge and non-edge by edge and also the same of V_2.

Also for the graph $S(\overline{G})_\sigma$ we get the same.

Corollary 3.2. For any self complementary graph,

$\gamma\, \overline{S_H(G)} \cong \gamma\, (S(\overline{G})\, \sigma)$.

Example 3.3. C_3, C_5 and Paley graph.
Theorem 3.4. Let G be a graph of order p, with $H \subseteq V(G)$, |H| = m. Then G is complete bipartite graph $K_{m,n}$ if and only if $S(G)_H$ is an empty graph or totally disconnected graph.
Proof. If $G = K_{m,n}$ and $H \subseteq V$ (G), |H| = m, then the $S(K_{m,n})$ is totally disconnected. If G is complete bipartite graph $K_{m,n,}$ then the switching with respect to any of its partite set is totally disconnected graph.
Conversely, suppose G is a graph of order m and $H \subseteq V(G)$,|H| = m. We prove that G is complete bipartite graph. Since the switching with respect to H is totally disconnected, so that H and V – H are independent on $S(G)_H$ and every element on edge is adjacent with every element in V – H, implies G is bipartite graph.
Theorem 3.5. Let G be a graph. Then the partial switching of a switching graph with respect to some subset H is isomorphic to the complement of a graph G. That is

$S_p(S(G)_H)_H\, \overline{G} =$.

Proof. From the definition of switching and partial switching. It is clear that the vertices in $S_p(S(S)_H)_H$ are the same in $\overline{G}$.

We prove that the graphs have the same edges. So we can make a partition for the edges as E_{11}, E_{12} and E_{22} where E_{ij} is as the same in definition of switching graph.

In the first switching every edge in E_{11} and E_{22} will be the same and we replace every edge by non-edge and vice versa. In the graph $S_p(S(G)_H)_H$ we get every edge in E_{11}, E_{22} and E_{12} replaced edge by non-edge and vice versa, that is any two vertices in $S_p(S(G)_H)_H$ are adjacent if they are non adjacent in G. Therefore, $\overline{G} = S_p(S(G)_H)_H$.

Theorem 3.6. The partial switching of $K_{m,n}$ with respect to one of the partite set is a complete graph K_{m+n}.

Proof. Let G = (V, D_1, D_2) be a complete bipartite graph. We prove the theorem by contradiction.

Let $S_p(G)D_1$ be the partial switching of G with respect to the partite set D_1. Suppose $S_p(G)D_1$ is not complete then there exists at least two points say *u* and *v* are not adjacent, that is u, v $\notin$ E($S_p(G)D_1$). i.e.: $(u,v) \notin E_{ij}(G)$ and this contradicts our assumption that G is not complete. Hence G is complete. Similarly with the partite set D_2.

Corollary 3.7. The graph G is complete if and only if G is the partial switching of switching of complete bipartite graph.

4. Switched Neighbourhood Graphs:

A subset S of V (G) is a neighbourhood set of G if $G = \cup_{\upsilon \in S}\langle N(v)\rangle$, where $\langle N(v)\rangle$ is the subgraph of G induced by N[*v*[. The neighbourhood number $\eta(G)$ of G is the minimum cardinality of a neighbourhood set of G. A subset set S of V is called global neighbourhood set of the graph G if it is neighbourhood for both G and $\overline{G}$. For more details we refer 40.

Theorem 4.1. Let D be a neighbourhood set of a graph G. Then D is also a neighbourhood set of $S(G)_D$ if and onlyif D is global.

Proof. To prove that D s also a neighbourhood of $S(G)_D$, we prove that $S(G)_D = \cup_{\upsilon \in D}\langle N(v)\rangle$,

The edges on S(G) either belongs to E_{11}, E_{12} or E_{22}. Let D be a global neighbourhood set of G i.e.; G = $\langle N(v)\rangle$ and also $G = \cup_{\upsilon \in D}\langle N(v)\rangle$, Every edge in E_{11} and E_{22} will be covered by D. (since D is neighbourhood set of G).

Similarly every edge inside E_{12} is covered by D (since D is neighbourhood of $\overline{G}$)

Hence every edge on $S(G)_D$ will be covered by D. That is, D is a neighbourhood set of $S(G)_D$.

Theorem 4.2. For any graph G, $\eta(S_p(G)D) = 1$ if and only if one of the following condition is satisfied:

(i) $\langle D\rangle$ contains an isolated vertex adjacent to every vertex in (V – D);

(ii) $\langle V - D\rangle$ contains an isolated vertex adjacent to every vertex in D.

Proof. Suppose that $\eta(S_p(G)D) = 1$ and any one of the above condition is not satisfied. Then either both $\langle D\rangle$ and $\langle V - D\rangle$ have no isolated vertex for every isolated vertex adjacent to every vertex in D and D and vice versa. Thus in any case $\Delta(S_p(G)_D) \leq n - 2$ and hence $\eta(S_p(G)D) \geq 2$ this contradicts.

Conversely, suppose the two conditions are satisfied. Then clearly there exists a vertex of degree (n-1) which covers all the edges in $(S_p(G)_D)$. Hence $\eta(S_p(G)_D) = 1$.

Theorem 4.3. For any independent neighbourhood set D of a graph G. $\eta(G) = \eta(S(G)_D)$.

Proof. Let G be a graph and D be a neighbourhood set of G, and *v* be a vertex in D then all the points N(*v*) are in (V – D), since D is independent neighbourhood set. So every element in (V – D) is adjacent to some element in V. That is D is global neighbourhood set for G. Hence by Theorem (3.5)

$$\eta(G) = \eta(S(G)_D).$$

SOME SWITCHED GRAPHS

Some switched graphs with respect to the minimum dominating sets are not isomorphic.

Example 5.1

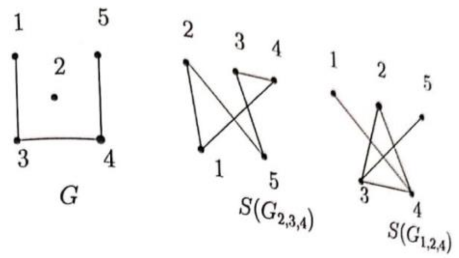

Figure 4

Some switched graphs for example if we suppose G = C_5, as labelled in the figure 5, with the minimum dominating sets {1,4}, {2,4}, {2,5}, {1,3} and {3,5} are isomorphic.

Example 5.2.

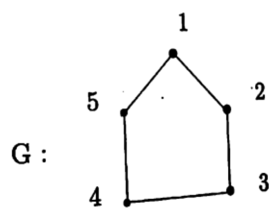

Figure 5

Switching with respect to {2,5} we have switching graph and partial switching as in figure 6.

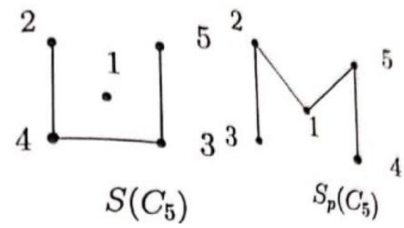

Figure 6

Switching with respect to {1,4} we have switching graph and partial switching as in figure 7.

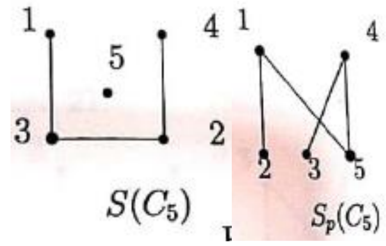

Figure - 7

Switching with respect to {1,3} we have switching graph and partial switching as in figure 8

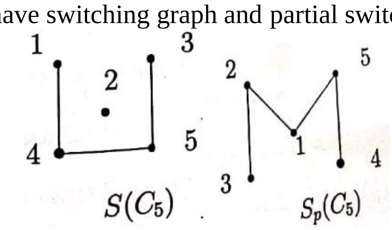

Figure 8

Switching with respect to {2, 4} we have switching graph and partial switching as in figure 9.

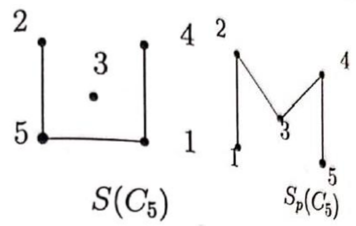

Figure 9

Switching with respect to {3, 5}we have switching graph and partial switching as in figure 21.

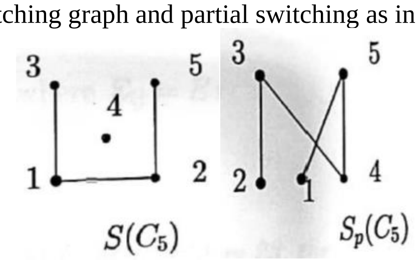

Figure 10

Let G = (V, E) be a graph for given partition σ= (V_1, V_2) of set V. We define the various partial switching graphs as follows:

(1). The partial switching $Sp_1(G_\sigma)$ = (V, $E_\sigma^{p_1}$), with = $E_\sigma^{p_1}$ = E_{11} U E_{12} U ((V_2 x V_2) – E_{22}, (where E_{ij} – E ∩ (V_i x V_j).

Example 5.3. Let G be a graphs in Figure 112, and the partial switched graph with respect to σ= {1,5}

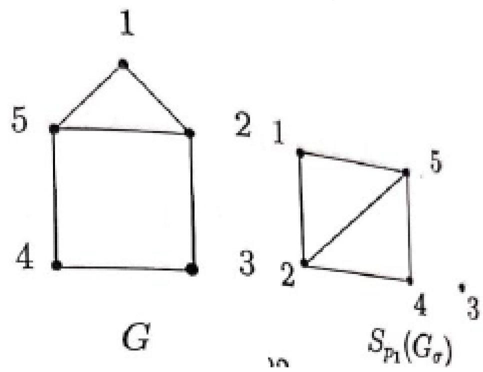

Figure 11

2) The partial switching $S_{p2}(G_\sigma) = (V_1 E_\sigma^{p_2})$, with $E_\sigma^{p_2} = E_{11} \cup E_{22} \cup (V_1 \times V_2) - E_{12})$, where $E_{ij} = E \cap (V_i \times V_j)$.

Example 5.4. Let G be a graph as in Figure 12, the partial switched graph with respect to $\sigma = \{1, 5\}$

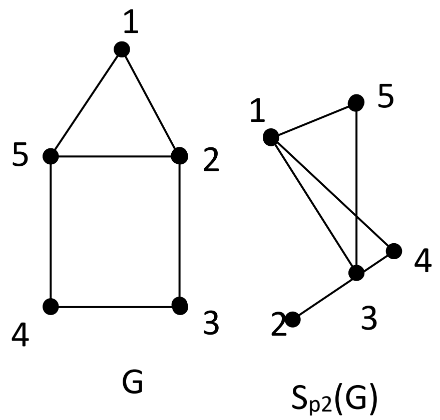

Figure 12

3) The partial switching $E_\omega^{p_3} = (V, E_\omega^{p_3})$, with $E_\omega^{p_3} = E_{12} \cup E_{22} \cup$ $(V_1 \times V_1) - E_{11})$, where $E_{ij} = E \cap (V_i \times V_j)$.

Example 5.5. Let G be a graph as in Figure 24, the partial switched graph with respect to $\sigma = \{1,5\}$

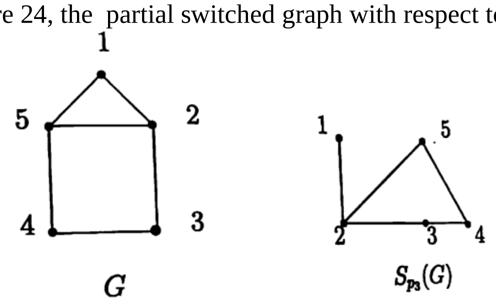

Figure 13

4) The partial switching $E_\sigma^{p_4} = (V, E_\sigma^{p_4})$, with $E_\sigma^{p_4} = E_{11} \cup ((V_1 \times V_2) \cup$ $(V_2 \times V_2) - E_{22})$, where $E_{ij} = E \cap (V_i \times V_j)$.

Example 5.6. Let G be a graph as in Figure 13, the partial switched graph with respect to $\sigma = \{1,5\}$

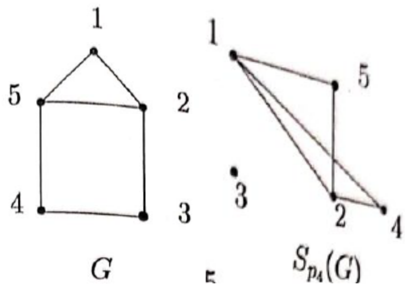

Figure 14

5) The partial switching $E_\sigma^{p_5} = (V, E_\sigma^{p_5})$, with $E_\sigma^{p_5} = E_{12} \cup ((V_1 \times V_1) \cup$ $(V_2 \times V_2) - E_{22})$, where $E_{ij} = E \cap (V_i \times V_j)$.

Example 5.7. Let G be a graph as in Figure 15, the partial switched graph with respect to $\sigma = \{1,5\}$

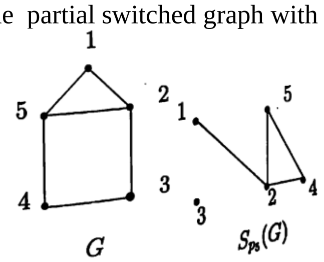

Figure 15

6) The partial switching $E_\omega^{p_6} = (V, E_\sigma^{p_5})$, with $E_\sigma^{p_6} = E_{22} \cup ((V_1 \times V_1) \cup$ $(V_2 \times V_2) - E_{11})$, where $E_{ij} = E \cap (V_i \times V_j)$.

Example 5.8. Let G be a graph as in Figure 16, the partial switched graph with respect to $\sigma = \{1,5\}$

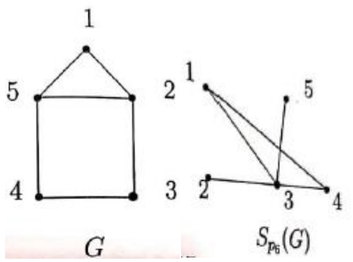

Figure 16

7) The partial switching $E_{\sigma}^{p_7}$ = (V, $E_{\sigma}^{p_7}$), with $E_{\sigma}^{p_7}$ = (V_1 x V_1) – E_{11} ∪ (V_2 x V_2) ∪ (V_1 x V_2) – E_{12}, where E_{ij} = E ∩ (V_i x V_j).

Example 5.9. Let G be a graph as in Figure 17, the partial switched graph with respect to σ = {1,5}

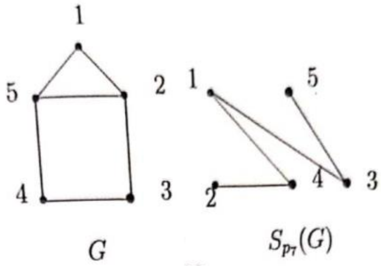

Figure 17

Theorem 5.10. For any graph G = (V, E) and H ⊆V(G),

$S_H(S_H(G))$ = G

Proof. Any edge in V(G) ∈ E_{12} will be non-edge and non edge will be edge. The other edges are same in E_{11} and E_{22} in ($S_H(G)$).

Again in $S_H(S_H(G))$ any edge in V(G) ∈E_{12} will be non edge and vice versa. And the other edges are same in E_{11} and E_{12}, which is isomorphic to G.

That is $S_H(S_H(G))$ = G

Example 5.11.

Fig 18a

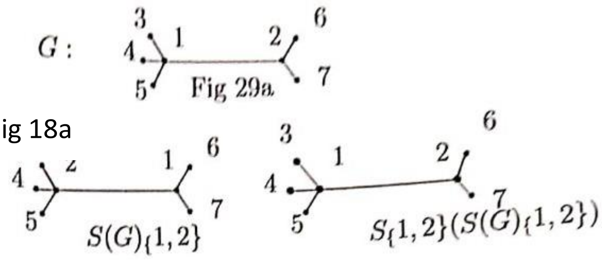

Figure 18 b

Theorem 5.12. Let G be a graph G = K_r∪K_1. Then $S_{K1}(K_r \cup K_1) \cong K_{r+1}$

Proof. Obviously switching with respect to K_1, every vertex in $K_r$will be adjacent to the single point K_1 in the switched graph of $S_{K1}(K_r \cup K_1)$. Therefore, $S_{K1}(K_r \cup K_1) \cong K_{r+1}$.

Example 5.13.

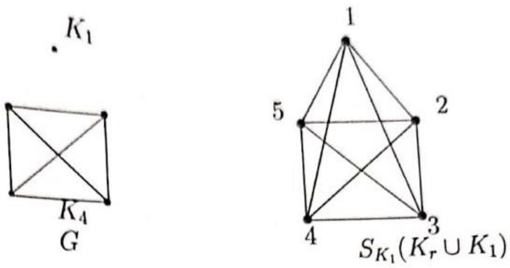

Figure 19

Theorem 5.14. Let G = $2K_r$, and S ⊆ G for any subset S in any copy of K_r, such that |H| = 1. Then $S_H(2K_r)$ – K_{r-1}∪ K_{r+1}.

Proof. Let G = 2 K_r with |H| = 1. Switching $2K_r$ with respect to H is equivalent to the graph which can be obtained by deleting one point from $2K_r$ and joining this point to all the vertices of the second copy. That is

$S_H(2K_r)$ – K_{r-1}∪ K_{r+1}.

Example 5.15.

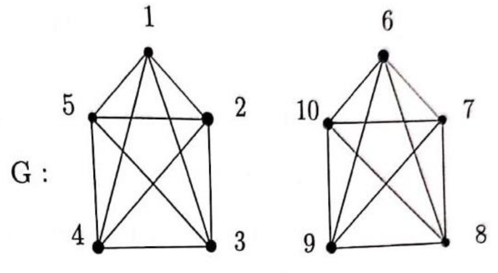

Figure 20a

Let |H| =1, then switching with respect to {1}, we get

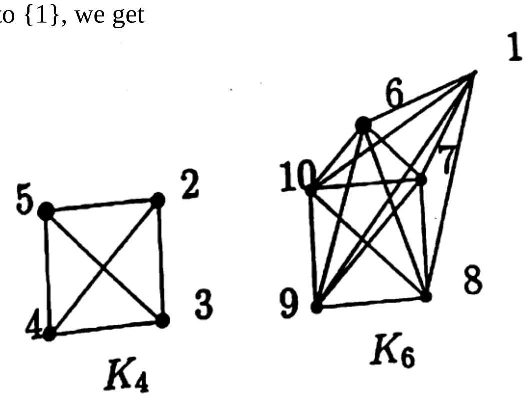

Figure 20b

That is

$S\{1\}\ (K_5 \cup K_5) = K_4 \cup K_6$.

Corollary 5.16. Let $G = 2K_r$ be a graph. Then $\gamma(S_H(2K_r)) = \eta(S_H(2K_r))$.

Theorem 5.17. Let $G = mK_r$ and H is any subset of any copy K_r, such that |H| = t. Then $S_H(mK_r) = K_{r-t} \cup (H + (m-1) K_r)$.

Proof. Let $G = mK_r$ be a graph with |H| = t. Switching mK_r with respect to H is equivalent to the graph which can be obtained by deleting edges between points inside H and V – H from one copy and joining these points to all the vertices of the remaining (m – 1) K_r. So we get the graph $K_{r-t} \cup (H + (m-1) K_r)$.

Hence

$S_H(mK_r) = K_{r-t} \cup (H + (m-1) K_r)$

Corollary 5.18. Let G be a graph. Then $\gamma(S_H(mK_r)) = \eta(S_H(mK_r))$.

Theorem 5.19. Let H = (V,E) and G = mH such that $S \subseteq V(H)$, for any copy then, $S_S(G) = \langle V\text{-}H\rangle \cup (\langle S\rangle + (m\text{-}1) H)$.

Proof. Suppose G = mH with $S \subseteq V(H)$ where H = (V,E). By switching the graph G with respect to a set S, we obtain a graph by deleting the edges between S and V-S in one copy of H and joining every vertex of H to every vertex of the remaining copies. That is

$S_S(G) = \langle V\text{-}H\rangle \cup (\langle S\rangle + (m\text{-}1)H)$.

Corollary 5.20. Let H = (V, E) and G = mH such that $S \subseteq V(H)$. Then $\gamma(S_S(G)) \leq \gamma\langle V\text{-}S\rangle + t$.

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