Fluid Analogy: Acoustic Wormhole

Amrita Bhattacharya

Assistant Professor
Department of Mathematics
Kidderpore College, Kolkata-700023, West Bengal, India

Abstract: Einstein and Rosen conceived charged particles in the form of a bridge (wormhole throat) that are simulated here in terms of fluid analogy. It is found that the first kind of modelling requires a thin layer of exotic matter at the bridge. An acoustic equation is also derived which has characterised the model. Using a second kind of model, it is already demonstrated that the Einstein- Rosen charge has a sonic Hawking- Unruh temperature. Present work suggests that the Unruh's fluid model can be consisted with the wormhole geometry too.

Key Words: Acoustic Wormhole, Exotic Matter, Einstein-Rosen charge.

I. INTRODUCTION:

A class of static spherically symmetric acoustic wormholes has been recently advanced as a natural completion of the acoustic analogue of black holes [1]. (The work contains the acoustic analogues of Morris-Throne wormholes with a minimally coupled scalar field [2]). Acoustic model of black hole geometry have originated with Unruh's novel discovery that a sonic horizon in transonic flow could give out a thermal spectrum of sound waves mimicking Hawking's general relativistic black hole evaporation [3]. The radiation of such sound waves is now commonly known as Hawking-Unruh radiation [4]. This basic acoustic analogy has been further explored under different physical circumstances by several authors [5]. For a good discussion of acoustic geometries including the Hawking – Unruh temperature, see Ref. [6].The works surrounding the analogy have engendered the practical possibility of detecting Hawking – Unruh radiation in fluid (particularly in super fluid) models under appropriately simulated conditions [7]. For this reason, the analogue models have attracted widespread attention as they open up alternative windows to look for many unknown effects of quantum gravity on black holes in the laboratory.

II. OUTLINE OF THE WORK:

However, the work reported here is somewhat limited in its scope but the result could still be curious and useful. We shall begin by recalling a history. Einstein was not satisfied with the representation of ponderable matter appearing in the general relativity field equations: "But it (general relativity) is similar to a building, one wing of which is made of fine marble (the left side of the equation), but the other wing of which is built of low grade wood (the right side of the equation). The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter" [8]. In particular, Einstein was averse to accepting material particles as singularities of the field. In an effort to avoid these singularities, Einstein and Rosen [9] geometrically modelled massive neutral elementary particles as "bridges" connecting two sheets of space-time. These objects are similar to what are now generally dubbed as Lorentzian wormholes and the connecting bridges as wormhole throats. They also represent massless charged particles as bridges in a similar fashion. The physicality of such mathematical representations could always be arguable and my intention is not to discuss here. Instead, in the present letter, we shall develop the acoustic analogue of the massless electrical particle in two inequivalent ways and show that sonic Hawking-Unruh temperature could be assigned to the particle in a purely formal manner.

III. THE ACOUSTIC WORMHOLE:

The construction of the acoustic analogue is based on the identification of a given general relativity metric with the acoustic metric derived from the standard set of irrational, non-relativistic inviscid, baroscopic fluid equations given by

$\vec{\nabla} \times \vec{v} = 0 \rightarrow \vec{v} = \vec{\nabla}\psi$ (1)

$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\vec{v}) = 0$(2)

$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{v}\right] = -\vec{\nabla}p$(3)

$p = p(\rho)$(4)

In which all relevant terms have their own meanings. Now linearize the equations around a background exact solution set $[p_0(t,\vec{x}), \rho_0(t,\vec{x}), \psi_0(t,\vec{x})]$ such that

$p = p_0 + \delta p_0 + O(\delta p_0)^2, \rho = \rho_0 + \delta\rho_0 + O(\delta\rho_0)^2, \psi = \psi_0 + \delta\psi_0 + O(\delta\psi_0)^2$ (5)

Whereδ denotes a small perturbation to the relevant quantities. Then the perturbation satisfies the equation [3].

$-\partial_t\left[\frac{\partial\rho}{\partial p}\rho_0\left(\partial_t\psi_1 + \vec{v_0}.\vec{\nabla}\psi_1\right)\right] + \vec{\nabla}.\left[\rho_0\vec{\nabla}\psi_1 - \frac{\partial\rho}{\partial p}\rho_0\vec{v_0}\left(\partial_t\psi_1 + \vec{v_0}.\vec{\nabla}\psi_1\right)\right] = 0$ (6)

This equation can nearly be rewritten as the minimally coupled wave equation [6]

$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-\bar{g}\bar{g}^{\mu\gamma}}\frac{\partial\psi_1}{\partial x^\gamma}\right) = 0$ (7)

Where$\bar{g}^{\mu\gamma}$ is the acoustic metric, $\bar{v}_0 = \vec{\nabla}\psi_0, \quad \psi_1 = \delta\psi_0, \quad \bar{g} = det|\bar{g}_{\mu\gamma}|$, and the stationary metric form is given by

$ds^2_{acoustic} = \bar{g}_{\mu\gamma}dx^\mu dx^\gamma = \frac{\rho_0}{c_s}\left[c_s^2dt^2 - \left(dx^i - v_0^i dt\right)\delta_{ij}\left(dx^j - v_0^j dt\right)\right]$ (8)

In which $i, j = 1,2,3$ and the local speed of the sound is given by $c_s^2 = \frac{\delta p_0}{\delta \rho_0}$. Before closing this part, we note that in a physical situation where $\vec{v}_0 = 0$,and $\frac{\rho_0}{c_s} = constant$, we have an exact Minkowski space-time where the role of the signal speed is delayed by that of sound waves. On a scale larger than interatomic distance and in a reasonably local neighbourhood inside the fluid, this geometry underlies the basic ingredient of what one might call sonic special relativity where the observers are equipped with only sound waves, that is, a word where the observers can only "hear" and "see". It might be of some interest to note that a similar framework of sonic relativity was conceived decades ago in which the subsonic and the supersonic Lorentz like transformations containing c_s as the invariant speed were employed in the treatment of many acoustical problems in a very profitable manner [10].

Proceeding further, note that the signature convention in use in this paper is $(+,-,-,-)$ and unless otherwise noted, we shall use units$8\pi G = c = 1$. Consider now the metric form used by Einstein and Rosen in standard coordinates(t, R, θ, φ) :

$$ds^2 = \left(1 - \frac{2m}{R} - \frac{\epsilon^2}{2R^2}\right) dt^2 - \left(1 - \frac{2m}{R} - \frac{\epsilon^2}{2R^2}\right)^{-1} dR^2 - R^2 d\theta^2 - R^2 sin^2\theta d\varphi^2 \quad (9)$$

$$\varphi_0 = \frac{\epsilon}{R} \quad (10)$$

whereφ_0 is the electrostatic potential while the magnetic potentials $\varphi_i$are set to zero. An elementary electrical particle without mass $(m = 0)$is represented by the singularity free solution obtained by the radial transformation $(R \to u)$ as follows:

$$u^2 = R^2 - \frac{\epsilon^2}{2} \quad (11)$$

The bridge (or the particle) is located at $u = 0$ or $R_b = {}^{\epsilon}/_{\sqrt{2}}$ connecting two sheets $u > 0$ and $u < 0$, (Similarly, a neutral particle corresponds to $\epsilon = 0$ with the radial transformation $R^2 = R - 2m$ so that the particle is represented by a bridge $u = 0$.) For our purposes, we shall rewrite the solution in isotropic coordinates (t, r, θ, φ) :

$$ds^2 = g_{\mu\gamma} dx^\mu dx^\gamma = \Omega^2(r) dt^2 - \Phi^{-2}(r)[dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2] \quad (12)$$

$$\Omega^2(r) = \left[1 - \frac{m^2+\beta^2}{4r^2}\right]^2 \left[1 + \frac{m}{r} + \frac{m^2+\beta^2}{4r^2}\right]^{-2} \quad (13)$$

$$\Phi^{-2}(r) = \left[1 + \frac{m}{r} + \frac{m^2+\beta^2}{4r^2}\right]^2 \quad (14)$$

$$\varphi_0 = \frac{4\sqrt{2}\,\beta r}{(m+2r^2)^2+\beta^2} \quad (15)$$

The massless bridge is now located at $r_b = {}^{\beta}/_{2}$. For the sake of generality, we keep $m \neq 0$ for the moment. The factor $\sqrt{2}$is imported in to Eq. (15) as the constant $\epsilon$is slightly redefined in Eq. (9) as $\epsilon^2 = 2\beta^2$ for the ease of calculations. The arbitrary constants m and β represent, respectively, the mass and the electric charge of the configuration with the solutions satisfying the field equations without denominators given by [9]

$$\varphi_{\mu\gamma} = \varphi_{\mu,\gamma} - \varphi_{\gamma,\mu} \quad (16)$$

$$g^2 \varphi_{\mu\gamma,\sigma} g^{\gamma\sigma} = 0 \quad (17)$$

$$g^2 \left(R_{\alpha\beta} + \varphi_{\alpha\varepsilon}\varphi_\beta^\varepsilon - \frac{1}{4} g_{\alpha\beta}\varphi_{\omega\delta}\varphi^{\omega\delta}\right) = 0 \quad (18)$$

in which $g = det|g_{\mu\gamma}|, \varphi_\mu = (\varphi_0, \varphi_i)$ and $R_{\alpha\beta}$ is the Ricci Tensor. The metric (9) above with the term $-\epsilon^2$ $(\epsilon > 0)$ is not quite the familiar Reissner- Nordström metric which has $+\epsilon^2$in its place. Correspondingly, there appears an overall negative sign before the stress energy tensor of the electrostatic field appearing at the right hand side of the field equations. This, in turn, indicates that the electrical stress violate the energy conditions, as long as we hold ϵ^2 (or β^2) > 0 in the metric (9), thereby providing materials necessary for the construction of the Einstein-Rosen bridge making up charged particles. This violation (of energy conditions) is precisely the price that one has to pay in order to build the bridge.

Let us return to the acoustic metric (8) and consider a preassigned velocity profile $\vec{v_o} \neq 0$. If the vector $\vec{v_0}/(c_s^2 - v_0^2) = 0$, then defining a new time coordinate $d\tau = dt + \left[\vec{v_0}.\vec{dx}/(c_s^2 - v_0^2)\right]$, it is possible to write the acoustic metric in the static form as [3b,6]

$$ds^2_{acoustic} = \bar{g}_{\mu\gamma} dx^\mu dx^\gamma = \frac{\rho_0}{c_s}\left[(c_s^2 - v_0^2)d\tau^2 - \left(\delta_{ij} + \frac{v_0^i v_0^j}{c_s^2 - v_0^2}\right) dx^i dx^j\right] \quad (19)$$

The first kind of acoustic model is obtained by choosing the simplest velocity profile of the background fluid: $\vec{v_0} = 0$. This choice allows us to directly build the analogy in the fashion suggested by Visser and Weinfurtner [11] : Replace the vacuum speed of light c in (12) [which we have set to unity] by the asymptotic speed of sound c_∞ corresponding to a linear medium. Just a formal replacement; it does not mean that numerically$c \equiv c_\infty$. Identify t with τ and the metric (12) with the metric (19), that is, set $g_{\mu\gamma} \equiv \bar{g}_{\mu\gamma}$. Immediately one obtains the density and sound speed profiles respectively as

$$\rho_0(r) = \rho_\infty \left(1 - \frac{m^2+\beta^2}{4r^2}\right) \quad (20)$$

$$c_s(r) = c_\infty \left[1 - \frac{m^2+\beta^2}{4r^2}\right]^2 \left[1 + \frac{m}{r} + \frac{m^2+\beta^2}{4r^2}\right]^{-2} \quad (21)$$

The main aim is to look for an acoustic analogue to the Einstein- Rosen charge which is essentially a wormhole. Consequently the analogue fluid energy density $\rho_0(\geq 0)$ is not the actual energy density of wormhole matter provided by the electrostatic field φ_0 , the latter density being strictly negative (exotic) for $\beta^2 > 0$, as already mentioned. Returning to the fluid description, the Euler Equation (3) can be rewritten for $p_0 = p_0(r)$ by explicitly displaying the force required to hold the fluid configuration in place against the pressure gradient:

$\vec{f} = \rho_0(\vec{v_0}.\vec{\nabla})\vec{v}_0 + c_s^2\partial_r\rho_0\hat{r}$(22)

It is now possible to find the pressure profile by integrating the equation:

$f_{\hat{r}} = \frac{dp_0}{dr} = c_s^2\frac{d\rho_0}{dr}$ (23)

which gives, under the physical condition that $p_\infty \to 0$,

$p_0(r) = \frac{1}{12}p_\infty c_\infty^2(m^2+\beta^2)\left[\frac{16(m^4+2m^3r-6mr\beta^2-\beta^4)}{\{(m+2r)^2+\beta^2\}^3} - \frac{4(m^4+2m^3r+3m^2\beta^2-18mr\beta^2-6\beta^4)}{\beta^2\{(m+2r)^2+\beta^2\}^2} - \frac{6(m^4+2m^3r+m^2\beta^2+2mr\beta^2+2\beta^4)}{\beta^4\{(m+2r)^2+\beta^2\}} + \frac{3m(m^2+\beta^2)}{\beta^5}\left(\pi - 2arctan\left\{\frac{m+2r}{\beta}\right\}\right)\right]$ (24)

The barotropic equation of state can be found by eliminating r from Eqs. (20) and (24), but this is not displaying here. Instead let us concentrate on the $m = 0$ case in accordance with the Einstein-Rosen model. Then $p_0(r) = -\frac{\rho_\infty c_\infty^2\beta^2(48r^4+\beta^4)}{3(4r^2+\beta^2)^3}$ (25)

Using the relation$r = \frac{\beta}{2\sqrt{1-\rho_0/\rho_\infty}}$, we can write the equation of state as

$p_0(\rho_0) = -\rho_\infty c_\infty^2\frac{(1-\rho_0/\rho_\infty)\left(4-2\rho_0/\rho_\infty+\rho_0^2/\rho_\infty^2\right)}{3\left(2-\rho_0/\rho_\infty\right)^3}$ (26)

The radial force required to maintain the acoustic configuration is

$f_{\hat{r}} = \rho_\infty c_\infty^2 \times \frac{\beta^2}{2r^3} \times \frac{\left(1-\frac{\beta^2}{4r^2}\right)^2}{\left(1+\frac{\beta^2}{4r^2}\right)^4}$ (27)

which is finite for all values of $r \neq 0$. It further turns out that $p_0(\rho_0)$ blows up at $\rho_0/\rho_\infty = 2$, but that is of no concern as the radial variable r is defined only for $\rho_0/\rho_\infty < 1$. At the bridge $r_b = \beta/2$, we have $\rho_0 = 0, c_s = 0$ and $p_0 = -\rho_\infty c_\infty^2/6$. The Eqs. (20) , (21) with $m = 0$ and Eqs.(26), (27) represent the exact acoustic model for the Einstein- Rosen massless electrical particle that was the actual aim to look for. The following characteristics are observed. From the expression (25), it follows that the pressure drops from zero at infinity to negative values as one proceeds to the bridge radius. However, with regard to the overall energy condition, normalizing $p_\infty = c_\infty = 1$,we can see that $\rho_0(r_b) + p_0(r_b) < 0$ implying that the (dominant) energy condition is violated at the bridge. Taking further $\beta = 1$, that is,$r_b = 0.5$, it is graphically found that at $r \approx 0.55$, $\rho_0 + p_0 = 0$ and for $r > 0.55$, we have $\rho_0 + p_0 > 0$. Thus, there is a thin layer of exotic acoustic material of the order of thickness $\delta \approx 0.05$ wrapping up the bridge radius. Finally, we find that there is a relation between the sound speed and density is given by

$\frac{c_s}{c_\infty} = \frac{\rho_0}{\rho_\infty}\left(2-\frac{\rho_0}{\rho_\infty}\right)^{-2}$(28)

which holds at every point in the medium characterizing the massless charge in its acoustic model.

The second acoustic model may be formed as follows. Consider the $\vec{v}_0 \neq 0$ situation together with the metric (19) and note Unruh's derivation [3]. The key assumptions were the constancy of the sound speed c_s (which is normalized to unity, but it can be set to any other value as well) and a radial flow with a particular form of the fluid velocity profile

$v_0^R = -1 + \alpha(R-R_0) + O(R-R_0)^2$(29)

in which $\alpha[\equiv(\partial v_0^R/\partial R)$ evaluated at $R = R_0$ is a constant proportional to sonic Hawking-Unruh temperature $[T_H = (h_\alpha/2\pi\kappa)]$, where κ is the Boltzmann constant and R_0 is the sonic horizon defined as the surface where $v_0^R = \pm 1$, the minus denoting the opposite motion of the fluid and the sound. Plugging Eq.(29) in to the metric (19) , and after dropping the angular parts, it has the form

$ds^2 \approx \frac{\rho_0}{c_s}\left[2c_s\alpha(R-R_0)d\tau^2 - \frac{dR^2}{2\alpha(R-R_0)}\right]$ (30)

which compares the black hole metric near the horizon

$ds^2 \approx \left[\frac{(\hat{r}-2M)}{2M}\right]d\hat{t}^2 - 2Md\hat{r}^2/(\hat{r}-2M)$(31)

provided one identifies $\alpha \equiv \frac{1}{4M}$.

IV. THE WORMHOLE SPACETIME:

Our acoustic configuration, on the other hand, has modelled a wormhole space-time given by (9) instead of Schwarzschild black hole (for which $\beta = 0$) and the two physical situations are entirely different [12]. For instance, black holes are the possible end results of a collapse due to Hawking-Penrose singularity theorems while these theorems do not apply when energy conditions are violated. Nonetheless, using the assumption of constancy of sound aped, and comparing the matrices (9) and (19), we get profiles $(v_0^R)^2 = \frac{2m}{R} + \frac{\beta^2}{R^2}$, $\rho_0 = 1, p_0 = 0$. This implies that we now have a dust-like fluid. The fluid horizon occurs at $R = R_0 = m \pm \sqrt{m^2+\beta^2}$where $v_0^R = \pm 1$. Note from the metric (9) that the wormhole is non traversable due to the fact that the throat radius R_{th} coincides with the geometric horizon R_h and that $R_{th} = R_h = R_0$. By the Taylor Expansion of v_0^Raround $R = R_0$asin Eq.(29), we find that $\alpha = \mp\frac{1}{R_0}\left(1-\frac{m}{R_0}\right)$.We can keep the sign open knowing that wormholes can have negative Hawking temperatures as speculated by Hong and Kim [13] on different grounds. The constants m, β appearing in α and R_0 of the fluid configuration must be treated as mere constants bereft of their original physical meanings and it signifies the limitations of the present approach. The main thing however is that α is finite at the throat and it can be related to the temperature of a massive electric charge ($m \neq 0, \beta \neq 0$). Within the framework embodied in Eqs. (30) and (31), an equivalent description can be assigned to the wormhole in terms of a

fictitious Schwarzschild horizon for mass M given by $M = \pm\frac{R_0^2}{4(R_0-m)}$. In the Einstein-Rosen model of neutral particle ($\beta = 0$), the value of the sonic T_H is proportional to $\mp 1/4m$, as expected , while for the massless ($m = 0$) electrical particle, it is proportional to $\mp 1/\beta$. All these results have followed from direct application of Unruh's method with the only physical difference that the metric (9) represents a wormhole, not s black hole. If , on the other hand, we had $\beta^2 < 0$ (that is, the usual Reissner- Nordström case with positive stresses), then the R_0 or the sonic T_H would have been imaginary for $m = 0$. This indicates that the existence of the real sonic T_H is consistent more with the exotic matter present in spacetime than the ordinary matter at least in the case under discussion here.

Finally, one might write down the acoustic equations corresponding to the general Morris-Throne wormhole given by

$$ds^2 = e^{2\Phi(R)}dt^2 - \frac{dR^2}{1-\frac{b(R)}{R}} - R^2 d\theta^2 - R^2 sin^2\theta d\varphi^2 \quad (32)$$

where$\Phi(R)$ and $b(R)$ are the redshift and the shape functions , respectively, satisfying the appropriate conditions. The throat is defined at $R = R_0$by $b(R_0) = R_0$. A procedure similar to that in the above paragraph leads to the exact expressions for density, radial velocity and gradient as follows:

$$\rho_0(R) = e^{\Phi}\left(1-\frac{b}{R}\right)^{-1/2} (33)$$

$$(v_0^R)^2 = 1 - e^{\Phi}\left(1-\frac{b}{R}\right)^{1/2} (34)$$

$$\frac{dv_0^R}{dR} = -\frac{1}{2v_0^R}\left[\left(1-\frac{b}{R}\right)^{1/2}\frac{d(e^{\Phi})}{dR} + e^{\Phi}\frac{d}{dR}\left(1-\frac{b}{R}\right)^{1/2}\right] (35)$$

This shows that at $R = R_0$, we have a fluid horizon defined by $v_0^R = \pm 1$, but the gradient $\alpha \equiv \frac{dv_0^R}{dR} \to \infty$ due to the last term thereby rendering the notion of sonic Hawking radiation meaningless. It might be possible to choose Φ and b in such a way as to offset the divergence like Einstein-Rosen case.

V. CONCLUSION:

The result in this paper have been derived in a quite straightforward way using a "back door" approach that has its own limitations. Nonetheless, the analyses above have attempted to provide some curious insights as to how acoustic analogue to Einstein-Rosen charge (actually wormhole) would look like. The conclusions are essentially of academic interest at present knowing that the corresponding sonic configurations may even be physically absurd. The important lesson seems to be that the Hawking-Unruh radiation (involving the existence of sonic horizon) need not emanate from dumb hole alone; it can, in principle, emanate also from the throat of a wormhole threaded by nontrivial exotic matter reinterpreted as a fictitious Schwarzschild mass M. that is, worms can squeal- acoustically, and that is! However few words of caution are necessary. As shown recently, the possibility of actual black hole evaporation itself depends on several crucial assumptions if has be independent of the laws of Planck scale physics [14]. The situation in the context of wormholes seems to be worse. The Ford-Roman quantum inequality [15] suggests that the wormhole sizes could be tiny but the mechanism of any wormhole evaporation is altogether unknown as these subjects are not formed in a collapse process. All these circumstances considerably complicate, if not nullify, the very foundation of the acoustic reincarnations of Hawking radiation from wormholes. Thus the ultimate validity of acoustic analogies playing the role of alternative black or wormholes must come from the success of appropriately designed acoustic/optics experiments in the laboratory. At any rate, two inequivalent acoustic models of Einstein-Rosen charge are provided here also in the first case($\overrightarrow{v_0} = 0$), one needs a thin layer of exotic fluid for its construction. This exotic behaviour together with the Eqs.(28) seem to characterize the model in an interesting way. The second analogue of the charge $\overrightarrow{v_0} \neq 0$ shows up a sonic Hawking temperature proportional to$\pm 1/\beta$. Work is underway to further redefine the results.

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