Probing Nucleon Structure with Deep Inelastic Scattering

Apoorv Indrajit Belgundi

Greenwood High School,
Bangalore, India

Abstract: **This paper reviews lepton-hadron deep inelastic scattering and its contribution to finding the internal structure of nucleons. This paper focuses on the contribution of DIS's momentum transfers in calculating a proton's internal structure using Parton distribution functions and structure functions of the Parton model. The level of discussion is moderate, and the reader should have some prior knowledge of particle physics and quantum mechanics.**

Index Terms: **Deep Inelastic Scattering, Parton Distribution Functions, Nucleon Structure, Bjorken Scaling**

I. INTRODUCTION

Quantum Chromodynamics has been firmly established as the theory of strong interactions [1]. It was initially devised to describe nuclear physics and ordinary matter. However, it has successfully explained a vast array of phenomena. It is now being used to describe high-energy accelerator physics [2]. In the 1960s, protons were smashed with other protons in particle accelerators. M. Gell-Mann and Y. Neeman showed that these particles could fit into an octet (forming mesons or baryons) or decuplet representations of the SU(3) group [3]. Deep inelastic lepton-hadron scattering is one of the best ways to investigate the nucleon structure at short distances as both charged leptons (e±,μ±) and neutrinos (ν,ν-) may be used as probes in the deep inelastic scattering of protons and neutrons. Further, various scattering cross sections can be measured containing different flavour combinations of quarks [4].

II. QUARK PROPERTIES

We will be dealing with lepton-hadron scattering, so it is best to have a quick review of relevant quark properties. Up quarks (symbol u) have a charge of $+\frac{2}{3}$ and are the "lightest" of all quarks, having a mass of 2.2 MeV. Down quarks (symbol d) have a charge of $-\frac{1}{3}$ and a mass of 4.7 MeV. Strange quarks (symbol s) have a charge of $=\frac{1}{3}$ and are much heavier than up and down quarks, each having a mass of around 101 MeV.

III. DEEP INELASTIC SCATTERING

Deep inelastic scattering refers to large momentum transfer collisions between a lepton and a hadron [5]. In DIS, a lepton l = e±, μ±, νᵢ($\overline{ν_1}$) (usually an electron e^-) with momentum k scatters off a nucleon N by exchanging an electroweak gauge boson V = γ, Z^0, W±. The lepton transforms to l' with momentum $k' = k - q$ [6–8]. From the study of inelastic lepton-hadron scattering, we aim to obtain detailed information about the structure and any fundamental constituents of hadrons [9]. Let us look at the distribution of outgoing electrons in the process e⁻ + N → e⁻ + X [1, 10], where N is usually a nucleon and X is the final state (some hadron). As shown in Fig. (1), the process involves the reaction of a virtual photon between the electron and the target constituent (Parton). The virtual photon transfers the 4-momentum to the hadron by [12]

$$q = k - k' \tag{1}$$

The 4-momentum is such that the incoming electron moves in the z direction and k = E [1 0 0 1]. The outgoing electron has an arbitrary direction, and in the rest frame of the hadron, P = M [1 0 0 0] [1]. The Bjorken variable x is [1, 13–15]:

$$x = -\frac{q^2}{2P \cdot q} \tag{2}$$

and y measures the energy of the photon as given by

$$y = \frac{2P \cdot q}{2P \cdot k}. \tag{3}$$

In equations (2) and (3),

$$q \cdot P = M(E' - E) \tag{4}$$

And

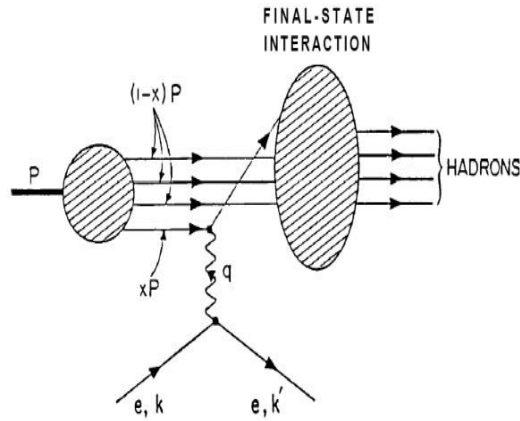$$q^2 = -2k \cdot k' = -2EE'(1 - cos\theta) < 0$$

FIG. 1. Deep Inelastic eN Scattering
This figure shows all the kinematic variables for deep inelastic scattering

Note that q^2 is the square of the momentum transferred, M is the mass of the nucleon and q. P is related to the scattering angle [6, 12]. q^2 is negative; therefore, it is customary to introduce a positive variable $Q^2 \equiv -q^2 > 0$ [7]. Note that the shape and magnitude of the cross-section are determined by the Parton and antiparton distributions measured in deep inelastic lepton scattering [3] as given by

$$\frac{d\sigma}{dQ^2 dy} \cong \sim \frac{4\pi\alpha^2}{3Q^4} \frac{1}{N_c} \sum_p x_1 \, fp(x_1) x_2 f\bar{p}(x_2) \tag{5}$$

To find an interesting relation between the Bjorken variable x and partonic momentums, let us introduce Mandelstam[1] variables:

$$s \cong (p_1 + p_2)^2 \tag{6}$$
$$t = (k_1 - p_1)^2 \tag{7}$$
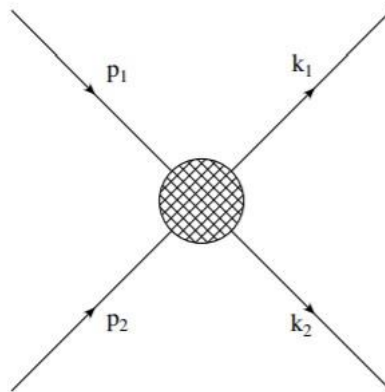
FIG. 2. Mandelstam variables defined for the process of type $1 + 2 \rightarrow 1^0 + 2^0$

The direction of travel of the partons is the same as their parent hadron, so their 4-momentum is proportional to the 4-momentum of their parent hadron. As shown in Fig. (1), the momentum of the participating Parton is p. This momentum is collinear to the proton momentum P and thus,

$$p = \xi P \tag{9}$$

where $0 \le \xi \ge 1$. Note that ξ is the collinear momentum fraction of the Parton [1]. From this,

$$\hat{s} = (p + k)^2 = 2p \cdot k = 2\xi P \cdot k = \xi s \tag{10}$$

and ˆs = ξs. The outgoing Parton carries negligible mass, and its momentum is very small (rather negligible), so $(p + k) = 0$ and from Equation (10), $2\xi P . q - Q^{\{2\}} = 0$. We now have an interesting relation between the momentum fraction of the interacting Parton ξ and the dimensionless kinematic variable x!

$$\xi = \frac{Q^2}{2P \cdot q} = x \tag{11}$$

IV. BJORKEN SCALING

An essential step in computing the results from the inelastic scattering experiment is calculating the probability of a particular hadron scattering. To do this, we introduce Parton densities, where $f_i^H(\xi) d\xi$ gives the probability of finding the Parton in the hadron H having a momentum fraction in the range $d\xi$ and ξ [1]. The probability of scattering the hadron is the sum of probabilities of scattering hadron components [1, 14].

$$\frac{d\sigma}{d\hat{t}}(e(k)H(P) \rightarrow e(k')X) \sum_i \int d\xi f_i(\xi) \frac{d\hat{\sigma}}{d\hat{t}}(e)(k)q_i(p) \rightarrow e(k')q_i \tag{12}$$

Here, $\frac{d\hat{\sigma}}{d\hat{t}}$ is the cross-section for Parton e^- scattering. If e_i denotes charge, we have

$$\frac{d\sigma}{dQ^2 dx}\sum_i f_i(x) e_i^2 \frac{2\pi\alpha^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right] \tag{13}$$

Up to the explicit factor $\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]$ the cross-section is independent of Q^2 [1, 6]. This phenomenon is known as Bjorken scaling. The dependence of x is from $\sum_i e_i^2 f_i(x)$ which depends upon the target structure. If the structure-function is known at some value of Q^2, we can calculate, with reasonable accuracy, the structure-function at all values of Q^2 (thanks to Bjorken scaling) by using the DGLAP equations. The DGLAP equation for QCD is [14]:

$$\frac{\partial f}{\partial ln\mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi}(P \otimes f) \tag{14}$$

where

$$P \otimes f = \int_x^1 \frac{dy}{y} P(y) f\left(\frac{x}{y}\right) \tag{15}$$

Furthermore, $P(y)$ is a splitting function calculated using perturbative QCD equations.

V. ANALYSIS AND APPLICATIONS OF THE RESULTS FROM DEEP INELASTIC LEPTON-HADRON SCATTERING

Parton Distribution Functions

The probability of a parton of type i having a fraction x of the nucleon energy is the Parton distribution function (pdf) and is denoted by $f_i(x)$ [16, 17]. A proton structure function is a sum of (various) Parton distribution functions as given by:

$$F_1^p(x) = \frac{1}{2}\sum_i Q_i^2 f_i(x) \tag{16}$$

$$F_2^p(x) = \sum_i xQ_i^2 f_i(x) \tag{17}$$

This indicates that spin ½ quarks have $2xF_1^p(x) = F_2^p(x)$ [16]. This relation is called the 'Callan-Gross Relation' [6].

Valence Quark Fractions

The proton structure function from the valence quarks *uud* is as follows [14, 16, 18–20]:

$$F_2^p(x) = \sum_q xe_q^2 q^p(x) = x\left(\frac{4}{9}\left(\overline{u_p} + u_p\right) + \frac{1}{9}\left(\overline{d_p} + d_p\right)\right) \tag{18}$$

The neutron has valence quarks *udd* so we can interchange $u(x)$ and $d(x)$.

$$F_2^n(x) = \sum_q xe_q^2 q^n(x) = x\left(\frac{4}{9}\left(\overline{d_n} + d_n\right) + \frac{1}{9}\left(\overline{u_n} + u_n\right)\right) \tag{19}$$

Integrating equations (18) and (19), we get

$$\int_0^1 F_2^p(x)dx = \sim \int_0^1 x\left(\frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x)]\right)dx = \frac{4}{9}f_u + \frac{1}{9}f_d \tag{20}$$

$$\int_0^1 F_2^n(x)dx = \int_0^1 x\left(\frac{4}{9}[d(x) + \bar{d}(x)] + \frac{1}{9}[u(x) + \bar{u}(x)]\right)dx = \frac{4}{9}f_d + \frac{1}{9}f_u \tag{21}$$

Note that $f_u = \int_0^1[xu(x) + x\bar{u}(x)]dx$ and f_u is the fraction of the nucleon momentum carried by the up and anti-up quarks. Experimentally in the proton, $f_u = 0.36$ and $f_d = 0.18$, so the up quarks carry twice the momentum of the down quarks. From simply adding f_u and f_d, we note that quarks carry around 50% of the proton's momentum [12, 16, 20]. The gluons and sea quarks carry the rest. If we take the initial hadron H to be a proton, the parton types must also yield a net proton [14, 21, 22], with $p\rangle = |uud\rangle$

$$\int_0^1 dx\, [f_u(x) - f_{\bar{u}}(x)] = 2 \tag{22}$$

$$\int_0^1 dx\, [f_d(x) - f_{\bar{d}}(x)] = 1 \tag{23}$$

$$\int_0^1 dx\, [f_s(x) - f_{\bar{s}}(x)] = 0 \tag{24}$$

Sea quarks are low-energy quark-antiquark ($q\bar{q}$) pairs present in nucleons which contribute to their energy, momentum and mass. The sea component arises from processes such as $g \rightarrow u\bar{u}$. Experimentally, at low values of x we expect the "sea" to dominate, whereas, at high values of x, the "sea" contribution is tiny [20].
Let us resolve the contributions made by quarks into valence and sea quarks [20, 22].

$$u(x) = u_v(x) + u_s(x) \tag{25}$$

$$d(x) = d_v(x) + d_s(x) \tag{26}$$

$$\bar{u}(x) = \overline{u_s}(x), \bar{d}(x) = \overline{d_s}(x) \tag{27}$$

With $m_u \approx m_d$, we can expect [22]

$$u_s(x) = d_s(x) = \overline{u_s}(x) = \overline{d_s}(x) = S(x) \tag{28}$$

Therefore, due to equation (28), equations (18) and (19) are transformed to

$$F_2^p(x) = x\left(\frac{4}{9}u_v(x) + \frac{1}{9}d_v(x) + \frac{10}{9}S(x)\right) \tag{29}$$

$$F_2^n(x) = x\left(\frac{4}{9}d_v(x) + \frac{1}{9}u_v(x) + \frac{10}{9}S(x)\right) \tag{30}$$

Thus, giving us a complete picture of the nucleon structure. Note that experimentally, at low values of x, we expect the "sea" to dominate, whereas, at high values of x, the "sea" contribution is tiny [20]. However, from equations (29) and (30), we see that the sea quarks significantly contribute to the momentum of nucleons.

VI. CONCLUSION

We have seen how momentum transfer in DIS can be used to find the internal structure of the nucleon. However, there is much more to understand about nucleons and their constituent quarks. As mentioned earlier, ongoing research tries to prove the existence of strange quarks in protons and neutrons. Their findings will surely give us a clearer idea about the structure of nucleons.

VII. REFERENCES

[1] B. Grinstein, 2003 CERN–CLAF School of High-Energy Physics , 27 (2006).
[2] F. Wilczek, Phys. Today 53, 22 (2000).
[3] T.-M. Yan and S. D. Drell, International Journal of Modern Physics A 29, 1430071 (2014).
[4] J. Blu¨mlein, arXiv preprint hep-ph/9512272 (1995).
[5] J. Pumplin, "Deep inelastic scattering: Measuring the parton structure of the proton using inclusive lepton-hadron collisions," (2014).
[6] J. Blu¨mlein, Progress in Particle and Nuclear Physics 69, 28 (2013).
[7] R. G. Roberts, The Structure of the proton: Deep inelastic scattering (Cambridge University Press, 1993).
[8] F. Huang, J. Xu, and X.-H. Yang, arXiv preprint arXiv:2103.07873 (2021).
[9] J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
[10] S. D. Drell, D. J. Levy, and T.-M. Yan, Physical Review D 1, 1035 (1970).
[11] R. Placakyte (H1and for the ZEUS), in 31st International Symposium on Physics In Collision (2011) arXiv:1111.5452 [hep-ph].
[12] A. Tapper, "Qcd and deep inelastic scattering," .
[13] M. Anselmino, A. Efremov, and E. Leader, Physics Reports 261, 1 (1995).
[14] A. D. Martin, arXiv preprint arXiv:0802.0161 (2008).
[15] G. Altarelli, R. K. Ellis, and G. Martinelli, Nuclear Physics B 143, 521 (1978).
[16] "Lecture 7 - the parton model," .
[17] A. Cafarella, QCD at hadron colliders and in Ultra High Energy Cosmic Rays, Other thesis (2006), arXiv:hep-ph/0605241.
[18] J. Dove, B. Kerns, R. McClellan, S. Miyasaka, D. Morton, K. Nagai, S. Prasad, F. Sanftl, M. Scott, A. Tadepalli, et al., Nature 590, 561 (2021).
[19] U. ki Yang, "Deep inelastic scattering (dis)," .
[20] M. Thomson, "Particle physics handout 6 : Deep inelastic scattering," (2011).
[21] F. Krauss, "Introduction to particle physics lecture 7," (2009).
[22] R. Vogt, Progress in Particle and Nuclear Physics 45, S105 (2000).