

A Modern Understanding of the Origin of Mass

¹Apoorv Indrajit Belgundi

¹Greenwood High International School, Bangalore, India

Abstract : There are many firmly rooted misconceptions in the literature regarding the origin of the mass of subatomic particles. For example, even the official [1] Nobel Prize in physics states, "... for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles". As we will show in this paper, this statement is inaccurate or, at the very least, misleading. This error may not have been intentional (and might also be a misinterpretation on the reader's part). Nevertheless, it still fuels a widespread misconception further. This paper aims to adequately explain the meaning of mass in modern physics and provide a broad overview of our current understanding of this concept. Along the way, we hope to remove common misinterpretations surrounding mass among enthusiastic readers so they can truly understand the beauty of mass and the beauty of physics.

Index Terms : Mass, Higgs Mechanism, Spontaneous Symmetry Breaking, Quantum Chromodynamics, Seesaw Mechanism

I. INTRODUCTION

This paper first touches on what mass is and the two types of mass- Newtonian and Relativistic. Then we try to answer the origin of said mass. The Higgs mechanism beautifully explains the answer to the origin of the mass of elementary particles. However, we want to emphasize that the Higgs mechanism does not account for the mass of subatomic particles and neutrinos. This paper stresses how the Higgs mechanism gives mass to elementary particles through spontaneous symmetry breaking. It includes spontaneous symmetry breaking in the $U(1)$ model and proof of its validity using the Goldstone Theorem. This paper also discusses Quantum Chromodynamics and its surprisingly significant contribution to the nucleon's mass and other hadrons. In addition, we dive deep into the mystery behind the mass of neutrinos and examine possible methods like the Seesaw mechanism, explaining how the neutrino gets its mass. The discussion level is high enough to make the paper an engaging read for those with prior knowledge of quantum mechanics. However, readers new to quantum physics will not find themselves lost. The paper's first few pages are dedicated to acquainting them with the Standard Model, Symmetries, and fundamental quantum mechanical properties. "Mass" is a fundamental concept in physics and other sciences like chemistry (Mass is used in the form of "molar mass" in chemistry). However, little attention is given to understanding what mass is. Only a handful question its origin. The motivation of this paper is to show that the seemingly dull and straightforward property "mass" is complicated yet stunningly elegant and very interesting.

II. THE STANDARD MODEL OF PARTICLE PHYSICS

The Standard Model of particle physics is a model that describes three of the four fundamental forces of nature- the strong interaction, the weak interaction and electromagnetic and also classifies the elementary particles. The electromagnetic force is the interaction that binds the electrons to atoms and, as the name suggests, is also responsible for electricity and magnetism. The strong interaction binds the quarks to form protons and neutrons, constituting over 99% of an atom's mass [2]. The weak interaction force is responsible for particular radioactivity (beta emission) and chirality. Gravity and electromagnetism have a very long-range (their effect is calculable with classical mechanics).

In contrast, the strong and weak interaction has a range smaller than the nucleus of an atom [3]. The exchange of particles causes the effect of these forces. Gluons are the force carriers of strong interaction and act within the nucleus of atoms. Photons are the force carriers of the electromagnetic force, and W and Z bosons are the force carriers of the weak interaction. The graviton is a hypothetical particle and a hypothetical force carrier of gravity. There is no complete quantum field theory of gravitons, and they have not been discovered so far.

Quantum Field Theory (QFT)

A field is "a quantity or measurement assigned to all points in space". This means that there is a vector at every point in spacetime, giving the fields full representation (magnitude and direction). Quantum fields are the quantum theoretical generalizations of classical fields. QFT treats particles as excited states of their underlying quantum fields, which are more fundamental than the particles. The vibrations/excitations of quantum fields create and destroy particles. A field must have a state of the lowest energy. The vacuum is said to be the state of lowest energy. Most fields have a strength of zero in the vacuum, but there is no requirement in physics that the strength of a field has to be zero in the vacuum. For instance, the Higgs field has a positive strength in empty space. We explain later in this paper how its positive strength is crucial in providing intrinsic mass to elementary particles. Quantum fields are invariant under space and time translation - implying that their associated particles will have the same properties, independent of where and when they are observed [4]. Rather than addressing the mass of each object in the universe separately, we can focus on the properties of a few quantum fields whose excitations (quanta) are the building blocks of matter. For instance, if we understand one electron's properties (including mass), we understand all electrons' properties. More generally: If we understand the properties of the fields associated with the building blocks of matter, we should be able to deduce the properties - including mass! - of matter itself and those deductions will be valid universally." [4] The simplest example of a quantum field theory is quantum electromagnetism. Two fields exist in it. The electromagnetic field and the "electron field." These two fields continuously interact with each other, and energy and momentum are transferred. An electron absorbing a photon is a transfer of energy and momentum from the electromagnetic field to the electron field.

Spin Quantum Numbers

Spin is an intrinsic form of angular momentum carried by elementary particles, hadrons and atomic nuclei. Spin was initially thought of as the rotation of a particle around an axis. It is unclear whether particles rotate because, in reality, elementary particles are said to be point-like. However, physicists consider spin unrelated to any motion in space. Spin is one of the reasons why colour charge is essential in keeping protons and neutrons intact.

Spin quantum numbers are quantum numbers describing the 'magnitude' of the spin of a particle. Note that it does not tell us about the direction of spin. Spin quantum numbers have some fascinating properties:

1. Spin quantum numbers may have half-integer values
2. An elementary particle cannot be made to spin faster or slower.
3. However, the direction of the spin of a particle can be changed

The spin quantum number of a particle takes the form $s = n/2$, where n is any non-negative integer. Thus, the values of s can be $0, 1/2, 1, 3/2, 2$, and so on. The value of s depends on the type of the particle and cannot be changed.

A Brief Description of Particles

The broadest categorization of particles can be done in two categories- fermions and bosons.

1. Fermions

Fermions, in general, have a half-integer spin (spin = $1/2$). Fermions are further divided into quarks and leptons. There are six quarks in total- up (u), down (d), strange (s), charm (c), top (t), and bottom (b). Likewise, there are six leptons – electrons (e), muons (μ), tau (τ), electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ). Besides common quantum numbers like spins and electric charges, quarks also have a colour charge. They participate in strong interactions and can also interact with W/Z bosons through weak interactions. Leptons do not have a colour charge and are affected by Electroweak interactions.

2. Bosons

Bosons are particles having an integer spin. There are two types of elementary bosons: gauge bosons -those with spin equal to 1 and mediate the strong, weak, and electromagnetic interactions- and scalar bosons with spin equal to 0. Only one scalar boson has been found to date – the Higgs boson.

3. Composite Particles/Hadrons

Hadrons are subatomic particles which have more than one quark. Hadrons are further divided into baryons and mesons. Ordinary baryons have three valence quarks, whereas ordinary mesons comprise a quark and an antiquark pair. Protons and neutrons - the particles that account for most of the mass of atoms are baryons. Therefore, the matter we see around us is called 'baryonic' matter.

Chirality

A chiral phenomenon is not identical to its mirror image. Chirality arises from the intrinsic quantum spin of particles. Chirality can either be clockwise or anticlockwise relative to the direction of motion. When the particle is spinning clockwise, its chirality is positive (right-handed), and the direction of its spin is the same as the direction of its motion. Chiral symmetry is the invariance under parity transformation [5]. The chirality is negative (left-handed) if the directions of spin and motion are opposite and the particle is spinning anticlockwise. For example, an electron having spin $1/2$ in the direction of its motion is said to have a chirality of $+1/2$ [5]. The chirality of a massive particle does not depend on the reference frame. Instead, it is a characteristic property of that particle. Most particles have left- and right-chiral states (except the neutrino, as we will see later).

III. SYMMETRY

The standard definition of the word symmetry tells us that if we take the mirror image along some line, a symmetric object looks the same or if we rotate a body by a certain amount, a symmetric object looks the same. For example, a snowflake is said to be symmetric under a specific rotation of 60 degrees. This definition can also be applied intuitively to some of the symmetries in physics [6]. Symmetry in physics is defined as "invariance under a specified group of transformations" [7]. For example, if we consider the temperature of a room to be homogeneous, then the temperature does not depend on the position of an observer within the room. So, we say that the temperature is invariant under a shift in an observer's position within the room. Such a definition of symmetry allows the concept of symmetry to be applied to abstract objects such as mathematical equations. [7] Symmetry can be broadly classified into global and local symmetry. A global symmetry affects all points of spacetime, whereas a local symmetry has a different symmetry transformation at different points of spacetime.

Discrete Symmetry

The Standard Model of particle physics states that one universe could be indistinguishable from another when a particular transformation occurs in one of the universes. C, P and T symmetries may be produced by these transformations.

1. C-Symmetry

It is an abbreviation for "charge symmetry" or "charge conjugation symmetry", wherein every particle in the universe is replaced by its antiparticle. For example, charge conjugation transforms an electron into a positron. Such a transformation is called a charge conjugation.

2. P-symmetry

In quantum mechanics, P-symmetry is an abbreviation for 'Parity symmetry'. In three dimensions, a parity transform refers to the simultaneous flip in the sign of all three spatial coordinates.

3. T-Symmetry

Time reversal is a transformation which flips the sign of the time variable $t \rightarrow -t$. This may or may not affect the physical quantities in classical physics. For example, time reversal does not affect:

- The position of a particle x in three-dimensional space
- The energy E of the particle
- The acceleration a of the particle

However, time reversal affects:

- The velocity v of a particle
- The linear momentum p and angular momentum l of a particle
- The magnetic field B

T-symmetry may exist in some arbitrary universe when the direction of time is reversed. However, according to the second law of thermodynamics- in our universe, entropy increases as time flows towards the future. Because of that, our universe does not show symmetry when subjected to time reversal (T-symmetry can be seen in local transformations but not in global ones like entropy).

Continuous Symmetries

Continuous symmetries are symmetries involving a transformation of space or time. The most common being time translation, spatial translation and spatial rotational symmetries. Another notable example is the Poincare transformation. They are continuous symmetries which preserve distances in Minkowski spacetime. They are studied primarily in special relativity.

Gauge Symmetry

Gauge symmetries exist where the Lagrangian (KE – PE) is invariant under local transformations. For example, take a ball falling from height h_2 with a constant acceleration. The velocity of the ball v at any point h_1 depends upon the change in altitude (Δh). The values of h_1 and h_2 are irrelevant as long as Δh remains constant. The velocity v will always remain the same as given by the formula: $v = \sqrt{2g(h_2 - h_1)}$. This is an elementary example of gauge symmetry, but these symmetries are an essential feature of our theories describing the universe. For example, even in quantum physics, Quantum electrodynamics is an abelian gauge theory with the symmetry group $U(1)$. Similarly, The Standard Model is also a non-abelian gauge theory with the symmetry group $U(1) \otimes SU(2) \otimes SU(3)$.

Symmetry Groups

Symmetries can be represented mathematically by a set (group) of unitary matrices $U(n)$ or by a set (group) of special unitary matrices - with unit determinants denoted by $SU(n)$.

Here, n is the dimension/size of the matrices.

1. $U(1)$ Group

The $U(1)$ group is mathematically a group of rotation around the unit circle, i.e., $e^{i\theta}$, where $\theta \in [0, 2\pi)$. Group $U(1)$ corresponds to the circle group- a circle group is the multiplicative group of all complex numbers with absolute value 1.

Geometrically, $U(1)$ group symmetry is the rotational symmetry of a circle rotated by an angle in 3-dimensional space. It is the symmetry group for electromagnetic interactions [8]—for instance, the interaction of photons and electrons.

2. $SU(2)$ and $SU(3)$ Groups

$SU(2)$ can be visualized as the rotational symmetry of a sphere. It consists of a set of 2×2 matrices with unit determinant. They model the weak nuclear interactions between pair of fermions (e.g. electron and neutrino) and a set of three bosons: Z^0 and W^\pm [9]. $SU(3)$ is for strong nuclear forces [9]. The equations govern interactions between the eight gluons and a set of three quarks (red, green and blue).

IV. SYMMETRY BREAKING

Symmetry breaking occurs when fluctuations acting on a system cross a critical point and decide the system's fate by determining which branch of a bifurcation is taken. For example, as shown in Fig. (1), the figure is symmetric when the ball is at position C. However, the ball is in a precarious equilibrium. If there is a slight disturbance in the ball's position by an external force, it will cause it to fall into one of the two depressions. The ball's new position will make the system asymmetric and thus break the symmetry.

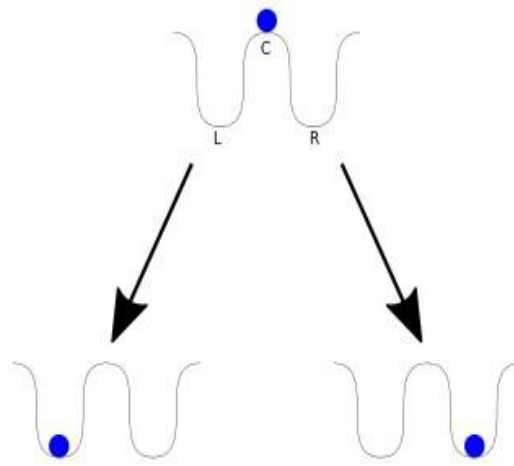


FIG. 1. Spontaneous Symmetry Breaking

V. WHAT IS MASS?

Newtonian Mass

According to Newtonian Mechanics, mass is matter's primary, conserved, and irreducible property [4]. Mass has two widely accepted definitions:

1. In everyday life, we define mass as the amount of substance in a body. More precisely, mass is defined "as a number attached to each particle or body obtained by comparison with a standard body whose mass is defined as unity".
2. Mass is also defined as the inertia of a body, i.e., the resistance it provides to acceleration, as given in the formula:

$$F = ma \quad (1)$$

and

$$p = mv \tag{2}$$

Where F stands for force, m for mass, a for acceleration, p for momentum and v for velocity. The equations Eq. (1) and Eq. (2) are the most fundamental formulas of Newtonian mechanics. Mass is also a part of Newton’s equation of gravitational force with which two bodies, m_1 and m_2 , at points r_1 and r_2 , attract each other.

$$F_g = \frac{-Gm_1m_2}{r^2} \tag{3}$$

Here $r = r_2 - r_1$, r is always positive, and $G = 6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ is Newton’s gravitational constant. Mass also constitutes the equations for kinetic energy and potential gravitational energy.

$$E_k = \frac{p^2}{2m} = \frac{mv^2}{2} \tag{4}$$

$$U_g = \frac{-Gm_1m_2}{r} \tag{5}$$

While the total energy in this case is:

$$E = Ek + Ug \tag{6}$$

Thus, the properties of mass according to Newton can be summarised as follows: [10]

- The mass of an isolated body is conserved. Therefore, it does not change with change in time.
- The body’s mass does not change from one reference frame to another.
- The mass of a body is a measure of its inertia
- Masses of bodies are sources of their gravitational attraction to each other

Relativistic Mass

Relativistic mass is commonly defined as “the mass of a body in motion”. However, we find it easiest to describe relativistic mass, m_R mathematically. It is simply the product of the Newtonian mass m (also known as the rest/intrinsic/proper mass in literature) and γ factor.

$$m_R = \gamma m \tag{7}$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

v is the relative velocity between initial reference frames, and c is the speed of light. Thus, from Eq. (8), we understand that γ is just a relation between the speed of light and the body’s speed. This understanding of γ will be essential in understanding the striking difference between relativistic and Newtonian mass.

Comparing Newtonian and Relativistic Mass

In modern physics, energy and momentum are the primary dynamical concepts, while mass is a parameter that appears in isolated bodies’ described energy and momentum [11]. We are comparing relativistic and Newtonian mass with the help of momentum first, as momentum is easier to grasp. Newtonian and Relativistic equations for momentum are contrasting.

$$p_N = mv \tag{9}$$

$$p_R = m_R v \tag{10}$$

Where p_N and p_R stand for Newtonian momentum and relativistic momentum, respectively. The addition of γ to p_R makes no difference at speeds we encounter in everyday life ($v \ll c$) as the numerical value of γ is very close to 1 and hence can be ignored. Thus, p_R is practically equal to p_N , and both give accurate results for the momentum of bodies at everyday speeds. However, when the magnitude of v is close to the speed of light ($v \sim c$), the value of γ factor increases drastically and needs to be accounted for (see Fig. 2). Newtonian mechanics does not yield accurate results when the speed of the body is near the speed of light. Thus, the relativistic formula of momentum Eq. (7) comes into play.

In popular scientific literature, the mass of the body increases as the velocity of a body approaches the speed of light. However, this is a common misconception that the value of relativistic mass (m_R) increases due to the γ factor, but the body’s Newtonian mass (m) remains unchanged. m_R increases infinitely because γ increases infinitely as the body approaches the speed of light (as shown in Fig. 2). Hence infinite energy is required for a massive body to be at the speed of light.

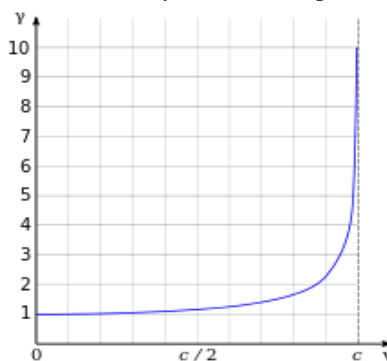


FIG. 2. Graph of the Lorentz factor shows the increase in the numerical value of γ with the increase in the body’s speed. The value of γ approaches infinity as the body’s velocity approaches the speed of light (c).

Now, we can begin comparing Newtonian and relativistic mass using energy. This is a fundamental equation of relativity [11]:

$$E^2 = p_R^2 c^2 + m^2 c^4 \tag{11}$$

From Eq. (10), when the body is at rest, $p_R = 0$. Hence, $E = E_0$ and then from Eq. (11), we get

$$E_0 = mc^2 \tag{12}$$

As in the case where v is nonzero but much smaller than the speed of light, $v \ll c$ [11]

$$E = E_0 + E_k = \sqrt{p_R^2 + m^2 c^4} = mc^2 + \frac{p_R^2}{2m} + \dots \tag{13}$$

and $E_k = \frac{p_R^2}{2m}$. So, we obtain the well-known Newtonian equation of momentum and kinetic energy. This means that m in Eq. (12) is the Newtonian mass (recall that Newtonian mass is constant regardless of the reference frame).

VI. THE HIGGS MECHANISM

The Higgs mechanism is responsible for providing intrinsic mass to most of the elementary particles through spontaneous symmetry breaking. The Higgs field vacuum expectation value is nonzero. This means that the Higgs field is present everywhere in the substrate of space. Particles interact with this field all the time, even in a vacuum. The field also impedes fermions from travelling at the speed of light.

History

In 1961, Sheldon Glashow presented the first $SU(2) \otimes U(1)$ gauge theoretic model for the electroweak interaction. By 1964, Peter Higgs, Brout, Englert, and Kibble extended the work of Goldstone on spontaneous symmetry breaking to gauge theories. However, they did not apply their framework to any phenomenologically relevant model. This was first done by Abdus Salam and Steven Weinberg in 1967 and 1968 for Glashow’s $SU(2) \otimes U(1)$ electroweak model. The key idea was that $SU(2) \otimes U(1)$ gauge symmetry is exact but hidden and that masses can be generated ‘dynamically’ by spontaneous symmetry breaking [12].

Importance and Properties of the Higgs Boson

The discovery of the Higgs boson in 2012 at the LHC at CERN was phenomenal. The Higgs boson proved (experimentally) the existence of the Higgs field and the validity of the Higgs Model. The Higgs boson is an elementary particle formed by the excitation of the Higgs field. The Higgs mechanism requires that a spin 0 particle with properties described by the Higgs mechanism exist. This particle was discovered in 2012 at the Large Hadron Collider (LHC) at CERN. The Higgs boson is popularly referred to as the God Particle or the particle that gives mass to other particles. This is wildly inaccurate and misleading. The Higgs boson can be thought of as a by-product of the Higgs mechanism, and it plays no direct role in giving intrinsic mass to elementary particles. The Higgs boson has no electric charge and is a massive particle with a mass of about $125.10 \pm 0.14 \text{ GeV}/c^2$ *1. It has a tiny mean lifetime of about 1.56×10^{-22} sec (predicted lifetime).

Working of a Field

As mentioned previously, if we excite a field, causing it to vibrate, the vibrations of the field are quanta (particles) of the field. The function of the position of a field can be viewed as a ball meant to oscillate. At the lowest possible energy of the field, the ball would sit at the bottom, as shown in Fig. (3). Exciting the field costs energy, making the ball oscillate to and fro. Suppose we add another dimension to the field. In that case, the particle can rotate in the internal space of the field, as shown in Fig. (4). This circular motion is very similar to angular momentum. This angular momentum is quantized- it comes in integer multiples of Planck’s constant. The oscillations of the ball correspond to quantum particles, and this angular momentum corresponds to an electric charge. Thus, a charged particle can be viewed as an excitation of a field in which the function of position (the ball) oscillates and spins simultaneously in the internal space of the field.

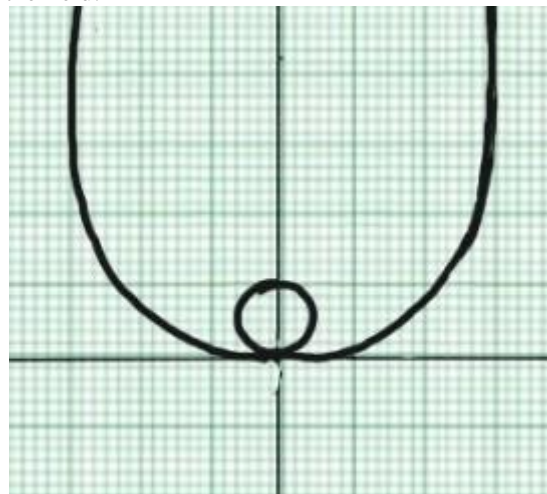


FIG. 3. Energy-Potential value graph

*1 In the following sections, we will set the speed of light to be $c = 1$. This convention allows us to express the mass as $125.10 \pm 0.14 \text{ GeV}$ for short.

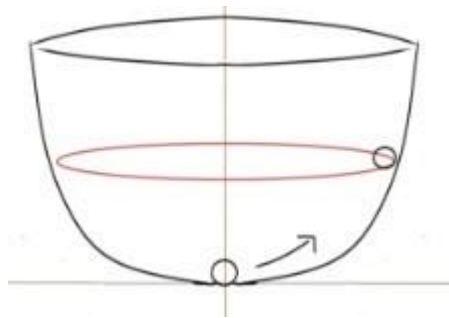


FIG. 4. Energy-Potential graph with a 3D Field

Higgs Model

Armed with a basic understanding of the internal working of fields, we can now go on to understand the working of the Higgs field. This field is responsible for providing mass to almost all elementary particles. This structure looks like a Mexican hat (A sombrero), which is why it is known as “The Mexican Hat” in physics.

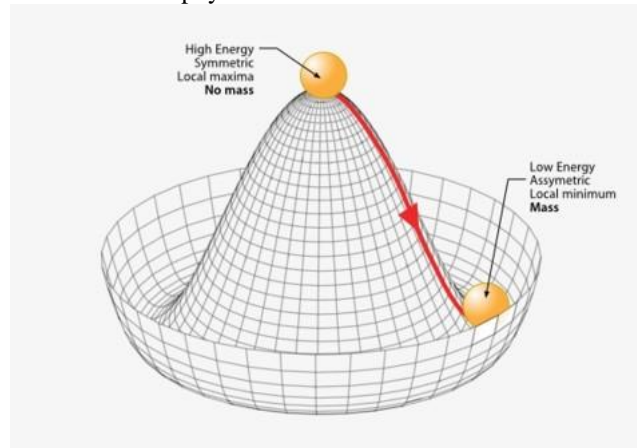


FIG. 5. Mexican Hat Potential

The ball on the top of the hat is highly unstable. The ball maintains symmetry at the top and is a function of the position of the field. Hence, the potential of the ball in the Higgs field is given by:

$$v(x) = \mu^2 \delta^2(x) + \lambda H_{min} \delta^3(x) + \frac{\lambda}{4} \delta^4(x) \tag{14}$$

where $\mu^2 \delta^2(x)$ refers to spin 1/2 particles (e.g., electrons), $H_{min} \delta^3(x) + \frac{\lambda}{4} \delta^4(x)$ refers to the energy present in the system and

$$H_{min} = \pm \sqrt{\frac{\mu}{2\lambda}} \tag{15}$$

where λ stands for the self-coupling interaction. The potential given in terms of the Higgs field reads:

$$v(H) = \frac{-1}{2} \mu^2 H H^\dagger + \frac{1}{2} \lambda (H^\dagger H)^2 \tag{16}$$

Where H is a two-component vector field given by:

$$H = \frac{H_{min} + \delta(x)}{\sqrt{2}} \times e^{-i\theta(x)\frac{\tau}{2}} \tag{17}$$

Where $e^{-i\theta(x)\frac{\tau}{2}}$ maintains the rotation of the particle and $\frac{H_{min} + \delta(x)}{\sqrt{2}}$ tells us about the oscillation of the ball. This ball oscillation breaks the symmetry, known as spontaneous symmetry breaking. The equations of these oscillations provide intrinsic mass to the particle! The existence of the Higgs field is essential for the universe we know to exist. Without the Higgs field, elementary particles will have no mass and travel at the speed of light, so composite particles like protons and neutrons will not exist.

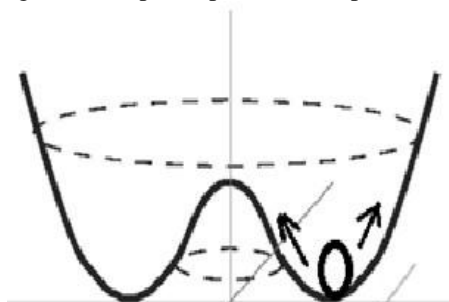


FIG. 6. Oscillation of the ball

Spontaneous Symmetry Breaking in the U(1) Model

The idea of symmetry breaking and the formation of mass can be easily understood mathematically by taking a closer look at the U(1) theory [12]. We start from a Lagrangian with a complex scalar field $\phi = -\frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ and a coupled gauge field $F^{\mu\nu}$ [12]

$$\mathcal{L}' = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(\partial_\mu - iqA_\mu)\phi|^2 - \mu^2|\phi|^2 - \lambda|\phi|^4 \tag{18}$$

The first two terms are kinetic, and the last two describe a potential $V(\phi) = \mu^2\phi*\phi + \lambda(\phi*\phi)^2$. We consider only the case where $\lambda > 0$, where the ϕ -field is self-interacting. From the two possibilities for the sign of μ^2 , the case $\mu^2 > 0$ leads to the theory of a massive scalar field. However, in the case $\mu^2 < 0$ shows two distinctive features. We get a negative mass term μ , and $V(\phi)$ possesses energy minima with $\frac{\partial V}{\partial \phi} = 0$ at $\phi = \pm v$ where $v = \sqrt{\frac{-\mu^2}{\lambda}}$. More precisely, $V(\phi_1, \phi_2)$ now has the form of the Mexican

Hat with $\phi_1^2 + \phi_2^2$ over the (ϕ_1, ϕ_2) -plane. Let us now see how the symmetry is broken in the energetically favoured state, the ground state, and the global and local U(1) [14]. If we rewrite ϕ as a field expansion of the vacuum state:

$$\phi = \frac{1}{\sqrt{2}}(v + \eta + i\varepsilon) \tag{19}$$

with real values for η and ε . This ansatz violates U(1) as after inserting Eq. (19) into Eq. (18), we get [12]

$$\mathcal{L}'' = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}q^2v^2A_\mu A^\mu + \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \varepsilon)^2 - v^2\lambda\eta^2 - qvA_\mu \partial^\mu \varepsilon + O(\eta^3) \tag{20}$$

where $O(\eta^3)$ means that the higher-order terms containing η^3 , η^4 and above are neglected. The particle content of \mathcal{L}'' is

- A massive real scalar field η
- A massless ε field
- A massive vector field A_μ .

The existence of a massive field is our goal, as it proves that the Higgs mechanism gives mass to elementary particles. Moreover, the existence of the massless particles proves spontaneous symmetry breaking due to the Goldstone theorem: For any generator of a symmetry that is broken in the ground state, there exists a massless scalar Goldstone boson [12].

Chiral Transformations due to the Higgs Field

The universe is not ambidextrous- it cares if a particle has a right- or left-handed chirality. When we use the example of electrons: - Left-handed electrons (and fermions in general) have an additional property called weak hypercharge. Weak hypercharge originates from the Higgs field. The Higgs field can be considered an infinite source of weak hypercharge as it simultaneously carries all possible values. Weak hypercharge is similar to electric charge, which enables electrons to “feel” the electromagnetic force. Likewise, weak hypercharge enables particles to “feel” the weak nuclear force. As mentioned before, only left-handed particles have weak hypercharge, and this asymmetry is part of a mystery known as Parity Violation. Physics is yet to explain why the universe treats right- and left-handed particles differently. The electron cannot switch from left to right-handedness without losing its weak hypercharge, and a right-handed electron cannot become left-handed without gaining weak hypercharge. The weak hypercharge gives inertia to the left-handed electron, thus making it “feel massive” because it can now couple with the Higgs field! However, the right-handed electrons should not be considered massless. When an electron travels, it constantly switches its handedness (by gaining and losing weak hypercharge) in incredibly short time scales. The electron is both right- and left-handed simultaneously because the interchange happens in time scales shorter than the Planck time. There are no “free” left- or right-handed electrons. This is explained perfectly by equation Eq. (23). We can separate the left and right chiral parts of an electron as follows:

$$e_L = \frac{1}{2}(1 - \gamma^5)e \tag{21}$$

and

$$e_R = \frac{1}{2}(1 + \gamma^5)e. \tag{22}$$

Therefore from Eq. (21) and Eq. (22), the electron field can be decomposed as

$$e = e_L + e_R \tag{23}$$

Equation Eq. (23) proves that the left and right chiral particles exist in a sort of superposition and form ordinary particles.

VII. QUANTUM CHROMODYNAMICS

Quantum chromodynamics (QCD) is the theory of the strong interaction. As the name suggests, the strong interaction is the strongest of all the four fundamental forces (about 1038 times as strong as gravitation). Its strength is essential as this force keeps the protons and neutrons bound in the nucleus of atoms, resisting the repulsive Coulomb force between protons [15]. QCD has three different types of charge (red, green and blue), labelled by “colour”. Quarks and gluons, the fundamental particles of the strong interaction, both have colour charges. These colour charges have nothing to do with physical colours; instead, they have properties analogous to electric charge (a part of Quantum Electrodynamics (QED)). It is important to note that only colour-singlet hadrons can exist. This means that to colours of constituent quarks must add up to “white”. For example, a pi-meson can be made up of an up quark (red) and an anti-up quark (anti-red). Another example is a proton consisting of an up quark (anti-red), an up quark (blue) and a down quark (green).

Significance of colour charge

Having three quarks in a particle should be impossible because at least two of them would have the same quantum properties (such as spin). According to Pauli's Exclusion Principle, two or more identical fermions cannot simultaneously occupy the same quantum state within a quantum system. However, delta baryons (baryons made of three up quarks or three down quarks) say otherwise. These delta baryons exist because of the difference in the colour charge of their constituent quarks! Therefore, two or three quarks with the same quantum properties such as spin, may still share the same quantum state, provided they are of different colours [15]. Thus, colour is a vital quantum property of the strong interactions and ensures the existence of some composite particles.

Inside the Nucleus

The nucleus of atoms is comprised of two baryons- protons and neutrons. Protons are comprised of two up quarks (each having a charge of $+2/3$) and one down quark (having a charge of $-1/3$). Thus, protons have a net charge of $+1$. Neutrons are comprised of two down quarks and one up quark. This makes their net charge equal to 0. The mass of the nucleus accounts for over 99% of the atom's mass, and the rest comes from the orbital electrons. So, it is safe to say that most of the mass of objects we see around us arises from the nucleus of atoms.

The mass of protons and neutrons does not come via the Higgs mechanism directly, as they are not elementary particles. The mass of a proton is about 938.27 MeV, and the mass of a neutron is about 939.56 MeV. The mass of an up quark (valence quark) is 2.2 MeV, and the mass of a down quark (valence quark) is 4.7 MeV. If we naively add the masses of their constituent quarks, the masses of protons and neutrons would be around 9.1 MeV and 11.6 MeV, respectively. So, the mass of the quarks accounts for only 0.969% of the proton's mass and 1.234% of the mass of a neutron.

Origin of the Missing Mass

Calculations for a three-quark particle like the nucleon are complicated due to the significant uncertainties that arise. Successful calculations, with all sources of uncertainty controlled, have been rare. In their work, Yi-Bo Yang and his collaborators have overcome some complications by using new computational methods they developed. These advances have enabled them to compute the contribution to the proton mass from four sources- the quark condensate (9%), the quark energy (32%), the gluonic field strength energy (37%), and the anomalous gluonic contribution (23%). The quark condensate is a mixture of the up and down quarks and a "sea" of virtual strange quarks [16] (more about this in the appendix). The other three terms are related to the dynamics of the quarks and gluons and their confinement within the proton. The quark energy and gluonic field strength equate to the kinetic energy of the confined quarks and gluons, respectively. The anomalous term is a quantum effect. It is associated with the QCD mass scale and consists of contributions from condensates of all quark flavours, including the strange, charm, bottom, and top quarks. This calculation shows that even if the up, down, and strange quark masses were all zero, the proton would still have more than 90% of its experimental mass [16]. This proves that nearly all the known mass in the universe comes from the dynamics of quarks and gluons. It is interesting to note how a similar version of the energy-mass equivalence principle exists even in quantum mechanics.

VIII. NEUTRINOS: A GRAND MYSTERY

Neutrinos are strange and peculiar particles. Neutrinos are extremely weakly interacting, and trillions of neutrinos pass harmlessly through our bodies each second [17]. They rarely interact with matter at all. Neutrinos only interact with gravity and weak interaction [18]. A neutrino is electrically neutral (as its name suggests) and incredibly light- they have a mass of about a millionth of an electron. Weirdly, although neutrinos are elementary particles, the Higgs mechanism does not give the particle its mass. The Standard Model predicted neutrinos had no mass to add to their strangeness! However, physicists at the Super-Kamiokande Observatory in Japan collected evidence and showed that neutrinos have a mass that was very close to zero. There are three flavours of neutrinos- the electron neutrino (ν_e), tau neutrino (ν_τ) and muon neutrino (ν_μ). The peculiarity of neutrinos does not end here. The neutrinos that we have observed to date are only left-handed. Why we have not detected right-handed electrons yet is an open question.

Potential Sources of the Mass of Neutrinos

1. The Higgs Mechanism

The Higgs mechanism does not accurately describe the mass of neutrinos. If we try to calculate the mass of an electron neutrino using the Higgs model, we get a mass similar to that of an electron. This calculated mass is enormous compared to what has been observed. One line of thought is that the neutrino does not "fit into" the Higgs model (or couple with the Higgs field) because of its incredibly light mass (The Higgs boson weighs about 125 GeV and is a very heavy particle).

2. Neutrinos as Majorana Fermions

Each particle in the universe has its opposite- an antiparticle. As their name suggests, antiparticles have some mirror properties to ordinary particles. For example, antiparticles have an opposite charge. However, the anti-neutrino is also neutral as the neutrino is neutrally charged. Such similarities in properties have led many to claim that the neutrino is its antiparticle! This would mean that right-handed neutrinos are, in fact, anti-neutrinos. Thus, neutrinos can be classified as Majorana particles [19] (a fermion that is its antiparticle). Suppose the neutrinos are classified as Majorana particles. In that case, their extremely lightness can be explained by the seesaw mechanism, where a heavy (currently hypothetical) neutrino pairs up with a lighter neutrino to give it mass.

IX. CONCLUSION

In conclusion, we can say that physics has answered a lot of the questions on the origin of mass. We have seen how elementary particles have attained an intrinsic mass through spontaneous symmetry breaking, the Higgs mechanism, and the Goldstone theorem. The bad (or maybe good?) news is that there is much to learn about the mass of neutrinos and nucleons. We are yet to find a complete, effective theory describing these particles accurately. We, in the 21st century, are now left with the "darkest" puzzles in physics. The "ordinary matter" that this paper is about constitutes only a tiny percentage of the universe's total mass. The rest is made of dark matter, which is almost a total mystery to modern physics. We do not even know what particle dark matter is made of, let alone its origin. It would be interesting if the dark matter particle is some supersymmetric particle, as the origin of its mass will be tied up with the larger question of supersymmetry breaking.

X. APPENDIX

Failure of the Higgs Mechanism in Describing the Mass of Neutrinos

As we have seen before, when the electroweak symmetry $SU(2) \otimes U(1)$ is broken by the vacuum expectation of the Higgs doublet $\langle H^0 \rangle = v_{wk} \approx 246$ GeV, the field gives mass to the gauge bosons and the fermions (all fermions except the neutrino). Neutrino mass is based on the $SU(3) \otimes SU(2) \otimes U(1)$ group under which quarks and leptons transform as follows

- Quarks $Q_L^T = (\mu_L, d_L) \left(3, 2, \frac{1}{3}\right)$, $\mu_R \left(3, 1, \frac{4}{3}\right)$, and $d_R \left(3, 1, -\frac{2}{3}\right)$
- Leptons $L^T = (v_L, e_L)(1, 2, -1)$ and $e_R(1, 1, -2)$
- Higgs Boson $H(1, 2, 1)$

As discussed previously, a popular way to introduce neutrino masses is to introduce a righthanded neutrino (say N_R) (one per flavour) into the Standard Model. The Standard Model Lagrangian will allow a ‘‘Yukawa coupling’’ [20] of the form $h_{\nu} L H N_R$, which after symmetry breaking, leads to a form $h_{\nu} v_{wk}$. However, h_{ν} is expected to be in the same order as the charged fermion couplings in the model (recall that $v_{wk} \approx 246$ GeV). This mass is too huge to describe the neutrinos' mass.

The Seesaw Mechanism

(This part will be best understood after reading the previous subsection) It is interesting to note that N_R s are singlets (a particle whose spin quantum number is equal to 0) and are thus allowed to have Majorana masses. We denote them by $M_R N_R^T C^{-1} N_R$ (note that C is the Dirac charge conjugation matrix). The masses M_R are not limited by gauge symmetry and can be arbitrarily large ($M_R \sim h_{\nu} v_{wk}$). This, along with the mass induced by Yukawa couplings (Dirac mass M_D), leads to the mass matrix for the neutrinos (M_{ν}) (left and right-handed neutrinos together), which has the form [20]:

$$\begin{pmatrix} \omega & M_D \\ M_D^T & M_R \end{pmatrix} \quad (24)$$

where M_D and M_R are 3×3 matrices. After diagonalizing the M_{ν} matrix, we get the mass matrix for the light neutrino masses to be as follows:

$$M_{\nu} = -M_D^T M_R^{-1} M_D. \quad (25)$$

M_R can be much larger than M_D , which is of order $h_{\nu} v_{wk}$. Because of this, we find that $m_{\nu} \ll m_{e, u, d}$ very naturally. This is known as the seesaw mechanism and provides a natural explanation of why neutrino masses are small [21].

Supersymmetry

The Standard Model gives us very accurate results for low-energy phenomena. However, it is a work in progress and must be extended to describe high-energy physics accurately. For example, the standard model does not explain the cause of the weakness of gravity compared to the other three fundamental forces. This is popularly known as the ‘‘Hierarchy Problem’’. Supersymmetry (also known as SUSY) provides a compelling reason behind the hierarchy problem. SUSY brings another symmetry between fermions and bosons [22, 23]. This additional symmetry is a crucial feature of grand unification and modern string theories. Supersymmetry predicts that every Standard Model particle has a supersymmetric partner (super partner) of the opposite type. For example, a superpartner of a fermion would be a boson. Super partners are expected to be much more massive than their counterparts. They are predicted to have a mass around the electroweak energy- $100 \sim 1000$ GeV.

To date, no evidence of supersymmetry has been found. Recent experiments at the LHC (Large Hadron Collider) have ruled out some of the simplest supersymmetric theories.

Strange Quarks in Nucleon

Studies such as those of Yang [16] and his collaborators have shown that the contribution of the mass of valence quarks is around 9% to 11% of the total mass of the nucleon. This seems rather odd, given that the total mass of up and down quarks is not near this number and is around 1% of the nucleon's mass. To deal with this, physicists speculate that another quark contributes to the combined mass of the valence quarks. Top quarks are insanely massive particles with a mass of about 173.34 ± 0.76 GeV [24]. Similarly, bottom and charm quarks are massive particles with masses of about 4.532 GeV [25] and 1.15 to 1.35 GeV [26], respectively. The masses of these quarks far exceed the mass of a nucleon, so they have been ruled out.

Interestingly, the strange quarks' mass equals 101 MeV [27]. This mass falls in the correct range. Suppose one strange quark is included in the nucleon. In that case, numerically, the total mass of the valence quarks will be around 8.6% of the proton's mass and around 8.4% of the mass of a neutron. This calculated percentage is very close to Yang's predictions [16]. This is part of ongoing research, and the findings might change our understanding of protons and neutrons!

Finding the Mass of the Higgs Boson

While confirming the Higgs boson's existence, numerous particle collisions were observed at the LHC. In this process, scientists had to distinguish the signal (Signal is the process we are interested in) from the observation of the background noise (Background is all of the events which end up in the sample but are not a result of the signal. ‘‘Noise’’ is particle ‘‘debris’’ that arises from the collision of particles at very high energies) [28]. A typical noise source is a random combination of particles that look like the signal. We must note that we can never distinguish between the signal and the noise with 100% accuracy. Our best chance is to apply some selection criteria to remove the background events and retain most of the signal. Note that in particle physics, researchers are rarely concerned with analyzing single events. Instead, we look at distributions of the same measurements made many times in the data.

As mentioned earlier, Higgs bosons are highly unstable and decay into other particles almost instantly after they are produced. Because of this, we can only see the products of their decay. Each possible way the Higgs boson can decay is known as a “decay channel” [29]. The most common decay channels among those that we are capable of seeing are:

$$\begin{aligned}
 H &\rightarrow b + \bar{b} \\
 H &\rightarrow \tau + \bar{\tau} \\
 H &\rightarrow \gamma + \gamma \\
 H &\rightarrow W^+ + W^- \\
 H &\rightarrow Z^0 + Z^0
 \end{aligned}$$

Where b and \bar{b} stand for a bottom quark and its antiquark, τ and $\bar{\tau}$ stand for a tau lepton and its antiparticle, γ stands for photons, W^+ and W^- stand for the W boson and its antiparticle, and Z^0 stands for Z bosons [29]. $b\bar{b}$ is the most common channel, but many other processes can produce these particles. Therefore, it is hard to tell whether they came from the Higgs boson or something else. The best channels to look for the Higgs decay are the $\gamma\gamma$ and the two Z^0 boson channels. Taking the example of searching for the Higgs boson decaying to a pair of photons ($H \rightarrow \gamma\gamma$), we start with some samples of recorded events that contain two reconstructed photons. We must then apply some selection criteria to filter out the background events that look least likely to be products of the Higgs boson decay. After plotting the invariant mass distribution of photon pairs, we end up with Fig. (7). The signal usually appears as a bump in the distribution. Here, it is centred at the mass of the Higgs boson (125 GeV) [28]. Hence, we find the mass of the Higgs boson. From Fig. (7), we can also calculate how many Higgs boson decays $H \rightarrow \gamma\gamma$ have occurred.

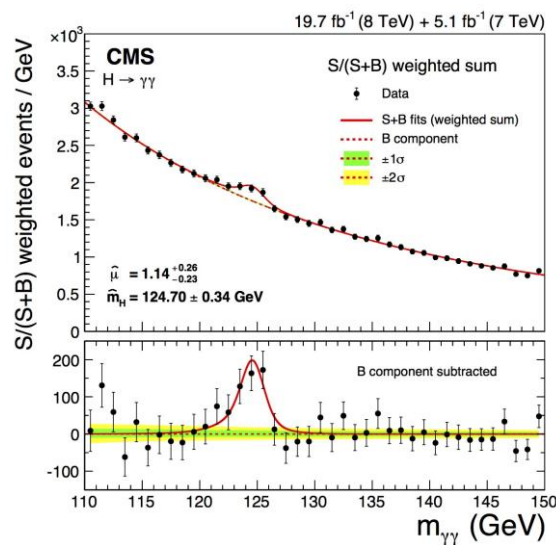


FIG. 7. The distribution of Photon pairs' invariant mass is measured by the CMS collaboration at the Large Hadron Collider [30, 31].

Spin Direction

In classical mechanics, angular momentum possesses both a magnitude and a direction. Likewise, in quantum mechanics, spin has a magnitude and a direction. The component of angular momentum for spin- s particle measured along any direction can only take on the value:

$$S_i = \hbar s_i \tag{26}$$

where $\hbar = \frac{h}{2\pi}$ (it is Plank’s reduced constant) and

$$s_i \in \{-s, -(s - 1), \dots, (s - 1), s\}$$

where s_i is the spin component along the i -axis (either $x, y,$ or z), s_i is the spin projection (direction) quantum number along the i -axis, and s is the principal spin quantum number.

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