# Electron beam velocity effects with kappa distribution function on kinetic Alfven waves in dusty plasma

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Abstract : The effect of electron beam on dispersion relation, and growth rate, perpendicular wave and marginal instability of the kinetic Alfven waves with kappa distribution function in a low  $\beta$  homogeneous plasma is discussed by investigating the trajectories of the charge particles. Particle aspect approach is adopted to investigate the trajectories of charged particles in the electromagnetic field of kinetic Alfven wave in the presence of kappa distribution function. Kinetic Alfven wave with kappa distribution function in the presence of beam effects are investigated. Expressions are found for dispersion relation, growth rate and growth length for dust particle of charge. The results are interpreted for the space plasma parameter appropriate to the auroral acceleration region of earth's magnetosphere. Kinetic effects of electrons and ions are included to study kinetic Alfven wave because both are important in the transition region.

Index terms: kinetic Alfven wave, Magneto-plasma, Auroral acceleration region, kappa distribution function, dusty plasma, beam effect.

## Introduction :

Kinetic Alfven waves play an important role in energy transport in driving field-aligned currents, particle acceleration and heating, inverted-V structures in magnetosphere- ionosphere coupling, solar flares and the solar wind<sup>1-7</sup>. They are also useful in explaining the ultra-low frequency emissions in the earth's magnetosphere. Field-aligned currents play a dominant role in the study of magnetized plasmas of magnetosphere-ionosphere coupling. The kinetic Alfven waves may be generated by the density in homogeneity of the plasma sheet and propagate to the ionosphere. Over the last decade it has been established that auroral luminosity is due to the impact of an accelerated electron beam coming towards the ionosphere and at the same event the up flowing ion beam has also been observed towards the magnetotal<sup>5-6</sup>. In the recent past the kinetic Alfven wave has been analyzed using particle aspect analysis in view of the auroral acceleration processes<sup>8</sup>.

They are perhaps of most importance in magnetospheric physics in the study of coupling between regions, where different dynamical conditions prevail but which are threaded by the same field<sup>9</sup>. it has been established that auroral luminosity is due to the impact of an accelerated electron beam coming towards the ionosphere and at the same event the upcoming ion beam has also been observed towards the magnetotail<sup>10,11</sup>. In the recent past particle aspect analysis was used to explain the auroral particle acceleration in the terms of Alfven waves and kinetic Alfven waves propagating parallel to or obliquely with respect to the ambient magnetic field<sup>12-13</sup>.One of the most important aspects of Alfven waves observed in the auroral region, and more recently in the high latitude magnetosphere.<sup>14</sup> is its ability to accelerate electrons to energies sufficient to cause visible aurora<sup>15</sup>.developed the idea that Alfven wave with perpendicular wavelength of the order of an ion acoustic gyro radius or less, carries a parallel electric field and can, thereby, accelerate electrons to form aurora<sup>16</sup>.

ion beam commonly believed to be produced by acceleration through a field aligned potential drop which also accelerates electrons downwards producing "inverted V" electron distributions<sup>29</sup>. the ion beam energy is closely related to the magnitude of the potential drop estimated from the enhancement of the electron loss cones. In order to examine up flowing ions only near the auroral acceleration region, data were used from only near perigee which was at an altitude of about 0.8 RE over the southern polar region.<sup>18,19</sup> have stated that the ion beam considered does not follow the kappa distribution, it is background plasma of the auroral acceleration region which may permit the kappa distribution due to converging magnetic field lines. The ion beam is supposed to follow the drifting kappa distribution function<sup>20</sup>. The ion beam in the direction of the wave motion may damp these waves if the ion beam velocity is smaller than the phase velocity of the wave; however, the ion beam opposite to the wave motion may excite the waves, as reported in this paper<sup>20</sup>.

Hasegawa (1976)<sup>21</sup> first suggested that small-scale kinetic Alfven waves possessed parallel electric fields and could be an efficient mechanism for accelerating particles on plasma sheet field lines. The kinetic Alfven waves Hasegawa had spatial scales perpendicular to B such that  $k_{\perp}\rho_s$  -1, where  $\rho_s$  is the ion acoustic gyro radius. This is the "kinetic" limit of kinetic Alfven waves. Subsequent theoretical investigations<sup>22</sup> focused on kinetic Alfven waves in the electron inertial limit for establishing parallel electric fields capable of accelerating auroral electrons. Considerable experimental evidence has accumulated from the Viking, Freja and FAST spacecraft, as well as low altitude sounding rockets, that kinetic Alfven waves in the inertial limit are an important feature of the altitude range between 1000 km and 2.5 RE<sup>23,24</sup>.have presented a model calculation of the properties of kinetic Alfven waves along a magnetic flux tube in which the altitude range encompasses both regimes.<sup>26</sup> examined the importance of ions and electrons kinetic effects in the acceleration of electrons in small scale Alfven waves above the auroral oval based on FAST satellite observations.<sup>26</sup>

The transversely accelerated ions and their with kinetic Alfven waves in the auroral acceleration region have been recently reported by various workers.<sup>27-30</sup> in the analysis of the FAST satellite data. The equilibrium dipolar magnetic field of the Earth is curved in a meridonial plane and may introduce loss-cone effects in the particle distribution function.<sup>31-34</sup> In most of the theoretical work reported so far, the velocity distribution function has been assumed to be either ideally Max wellian or bi-Max-wellian, ignoring the steep kappa function feature. Plasma in mirror-like devices and in the auroral region with curved and converging field lines,

depart considerably from a Max-wellian distribution and have a steep kappa distribution.<sup>33,35</sup> In this paper we will discuss the effect of the kappa distribution function on the kinetic waves in the presence of ion beams and thermal anisotropy. The main objective of the present investigation is to examine the effect of ion beams on kinetic Alfven waves at different kappa indices, in view of the observations in the auroral acceleration region. The present analysis is based on.<sup>36</sup> theory of Landau damping, which was further extended.<sup>37,39,34</sup> The method adopted, known as particle analysis, has been widely used in the analysis of electrostatic and electromagnetic instabilities.<sup>32,33,35,37,38</sup> The relative importance of this approach over fluid and kinetic approach is also discussed.<sup>31,33,39</sup> The main advantages of this approach is to consider the energy transfer between waves and particles, along with the discussion of waves dispersion and growth/damping rate of the waves. The method may be suitable to deal with the auroral electrodynamics where particle acceleration is also important along with waves emissions. The results obtained by this approach are the same as those derived using a kinetic approach<sup>41</sup>.

The kinetic Alfven waves generated in the equatorial magneto- sphere travel towards the auroral ionosphere in the converging magnetic field; thus, it is assumed that the distribution may depart from ideally Max-wellian and allow for a kappa distribution function.<sup>34,35,38,</sup> In the past the mirror structure for the development of the quasi-static potential drop along auroral field lines has been adopted by various authors.<sup>40</sup> The various plasma instability processes may also lead to the kappa distribution.<sup>41</sup> The mirror geometry is the justification of their choice and the experimental evidence is not known, to our knowledge. The ion beam considered does not follow the kappa distribution; it is the background plasma of the auroral acceleration which may permit the kappa distribution, due to converging magnetic field lines. The electron beam may generate the kinetic Alfven waves, as mentioned in the Introduction. The ion beam in the direction of the wave motion may damp these waves, however, the ion beam opposite to the wave motion may excite the waves, as reported in this paper. In our present paper, we have considered the electrodynamics of the auroral ionospheric region by an kinetic Alfven wave study.<sup>28</sup> predicted some observational evidences through satellite data. Ion conics may enhance the growth rate of kinetic Alfven waves at the ionospheric region, which provides the potential of our theoretical model to study kinetic Alfven wave characteristics in the auroral acceleration region.<sup>34</sup> The kinetic Alfven turbulence plays an important role in the kappa relationship. It has been suggested that the kappa effects can enhance the anomalous resistivity for a given turbulence level. Since the steep kappa distribution in the presence of kinetic Alfven waves and the ion beam enhances the growth rate, the anomalous resistivity and transport resulting from this instability are likely to play a crucial role in the auroral acceleration region. The conversing magnetic field lines in the higher latitude auroral ionosphere may be considered suitable for the use of the generalized kappa distribution function. An upflowing ion beam, along with energetic particles may ex- cite kinetic Alfven waves. Both the ion and electrons are assumed to follow the kappa distribution function<sup>41</sup>.

#### **Basic trajectory :**

The kinetic Alfven wave is assumed to start at t=0 when the resonant particles are undisturbed. The main interest lies in the behaviour of kinetic Alfven waves, which satisfy the conditions.

$$V_{Tnd}, V_{Tni} << \frac{\omega}{K_{n}} << V_{Tne}; \ \omega << \Omega_{i}; \ \Omega_{e}, \Omega_{d}; K_{\perp}^{2} \rho_{e}^{2} << K_{\perp}^{2} \rho_{i}^{2}; K_{\perp}^{2} \rho_{d}^{2} < 1$$
(1)

Where  $V_{Tni}$ ,  $V_{Tne}$  and  $V_{Tnd}$  are the mean velocities of ions, electrons and dust particles along the magnetic field,  $\Omega_{i,e,d}$  are gyration cyclotron frequencies of the respective species.  $K_{\perp}$  and  $K_{\pi}$  are the components of real wave vector k perpendicular and parallel to the magnetic field  $B_{0}$ . Consider the two particles representation of electric field a kinetic Alfven wave of the from (A K Dwivedi 2015)

$$E_{\perp} = -\nabla_{\perp} \phi \text{ and } E_{\Pi} = -\nabla_{\Pi} \psi$$

$$\overline{E} = \overline{E}_{\perp} + \overline{E}_{\Pi}$$

$$\phi = \phi_1 \cos(k_{\perp} x + k_{\Pi} z - \omega t)$$

$$\psi = \psi_1 \cos(k_{\perp} x + k_{\Pi} z - \omega t) \qquad (2)$$

where  $\phi_1$  and  $\psi_1$  are assumed to be a slowly varying function of time t, and  $\omega$  is the wave frequency which is assumed as real. $u_x(\bar{r},t)u_y(\bar{r},t)$  and  $u_z(\bar{r},t)$  of the changed particles presence of KAW.

$$\begin{split} u_{x}(\vec{r}.t) &= -\frac{q}{m} \Big[ \phi_{1} K_{\perp} - \frac{V_{n} K_{n} K_{\perp}}{\omega} (\phi_{1} - \psi_{1}) \Big] \sum_{-\infty}^{+\infty} J_{n}(\alpha) \sum_{-\infty}^{+\infty} J_{l}(\alpha) \Big[ \frac{\Lambda_{n}}{a_{n}^{2}} \cos\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}} \cos(\xi_{nl} - \Lambda_{n+l}t) \\ &- \frac{\delta}{2\Lambda_{n-1}} \cos(\xi_{nl} - \Lambda_{n-l}t) \Big] \\ u_{y}(\vec{r}.t) &= -\frac{q}{m} \Big[ \phi_{1} K_{\perp} - \frac{V_{n} K_{n} K_{\perp}}{\omega} (\phi_{1} - \psi_{1}) \Big] \sum_{-\infty}^{+\infty} J_{n}(\alpha) \sum_{-\infty}^{+\infty} J_{l}(\alpha) \Big[ \frac{\Omega}{a_{n}^{2}} \sin\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}} \sin(\xi_{nl} - \Lambda_{n+l}t) \\ &- \frac{\delta}{2\Lambda_{n-1}} \sin(\xi_{nl} - \Lambda_{n-l}t) \Big] \\ u_{z}(\vec{r}.t) &= -\frac{q}{m} \Big[ \psi_{1} k_{n} - \frac{V_{\perp} K_{n} K_{\perp}}{\omega} (\phi_{1} - \psi_{1}) \frac{n}{\alpha} \Big] \sum_{-\infty}^{+\infty} J_{n}(\alpha) \sum_{-\infty}^{+\infty} J_{l}(\alpha) \frac{1}{\Lambda_{n}} [\cos\xi_{nl} - \delta\cos(\xi_{nl} - \Lambda_{n}t)] \end{split}$$

Where  $\delta=0$  for non-resonant particles and  $\delta=1$  for resonant particle and  $\Lambda_n = k_{\pi}v_{\pi} - \omega + n\Omega, \quad a_n^2 = \Lambda_n^2 - \Omega^2$ 

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(3)

$$\alpha = \frac{\kappa_{\perp} v_{\perp}}{\Omega},$$
  

$$\xi_{nl} = k_{\perp} x + k_{\Pi} z - \omega t + (l - n)(\theta - \Omega t)$$
(4)

 $\zeta_{nl} = \kappa_{\perp} x + \kappa_{n} z - \omega t + (t - n)(0 - 2t)$   $\theta$  is the initial phase of the velocity and  $\Omega = qB_0/mc$ ,  $u_x$  and  $u_y$  are the perturbed and velocities in the x and y direction respectively. The slowly varying quantities  $\phi_1$  and  $\psi_1$  are treated as a constant. Integration of eq. (3) gives the perturbed coordinates of particles x,y,z which in addition of trajectories of free gyration

Exhibits the true path of the particles. In the view of the approximations introduced in the beginning, the dominant contribution comes from the term n=0.  $J_s$  are Bessel's functions which arise from the different periodical variation of charged particles trajectories. The term represented by Bessel's functions show the reduction of the field intensities due to finite gyro radius effect. In order to find out the Density perturbation associated with the velocity perturbation,  $\vec{u}(\vec{r},t,\vec{v})$ , we consider the equation for nonresonant particles

## **Distribution function:**

The density perturbation associated with the velocity perturbation we consider the equation for non-resonant particles.

$$n_{1}(\vec{r}t) = F(\vec{v}) \sum_{-\infty}^{+\infty} J_{n}(\alpha) \sum_{-\infty}^{+\infty} J_{1}(\alpha) \frac{q}{m} \left[ \left\{ \phi_{1} \frac{v_{\Pi}k_{\Pi}}{\omega} (\phi_{1} - \psi_{1}) \right\} \left\{ \frac{k_{\perp}^{2}}{a_{n}^{2}} + \frac{\Omega^{2} v_{d}k_{\perp}m}{\Lambda_{n}a_{n}^{2}T_{\perp}} \right\} + \frac{k_{\Pi}^{2}}{\Lambda_{n}} \left\{ \psi_{1} - \frac{n}{\alpha} \frac{v_{\perp}k_{\perp}}{\omega} (\phi_{1} - \psi_{1}) \right\} \right] \cos\xi_{nl}$$

$$\tag{5}$$

The resonant particles we have.

$$n_{1}(\vec{r}t) = F(\vec{v}) \sum_{-\infty}^{+\infty} J_{n}(\alpha) \sum_{-\infty}^{+\infty} J_{1}(\alpha) \frac{q}{m} \Big[ \Big\{ \phi_{1} \frac{v_{\Pi}k_{\Pi}}{\omega} (\phi_{1} - \psi_{1}) \Big\} \Big\{ \frac{k_{\perp}^{2}}{a_{n}^{2}} + \frac{\Omega^{2} v_{d}k_{\perp}m}{\Lambda_{n}a_{n}^{2}T_{\perp}} \Big\} \cos\xi_{nl} + \frac{1}{2\Omega\Lambda_{n+1}} \cos(\xi_{nl} - \Lambda_{n+1}t) \Big( k_{\perp}^{2} - \frac{\Omega v_{d}k_{\perp}m}{\Lambda_{n}T_{\perp}} \Big) \frac{v_{d}k_{\perp}m}{\Lambda_{n}T_{\perp}} \cos(\xi_{nl} - \Lambda_{n}t) - \frac{1}{2\Omega\Lambda_{n+1}} \cos(\xi_{nl} - \Lambda_{n-1}t) \Big( k_{\perp}^{2} + \frac{\Omega v_{d}k_{\perp}m}{T_{\perp}} \Big) + \frac{k_{\Pi}^{2}}{\Lambda_{n}^{2}} \Big\{ \psi_{1} - \frac{n}{\alpha} \frac{v_{\perp}k_{\perp}}{\omega} (\phi_{1}\psi_{1}) \Big\} \{\cos\xi_{nl} + \Lambda_{n}t \sin(\xi_{nl} - \Lambda_{n}t) - \cos(\xi_{nl} - \Lambda_{n}t) \Big\} \Big]$$

$$\tag{6}$$

Where F(v) represent the kappa distribution function and  $V_d$  is the diamagnetic drift velocity which is defined by  $V_d = \frac{T_\perp}{m\Omega} \varepsilon_N; \varepsilon_n = \frac{1}{N} \frac{dN}{dy}$  inhomogeneous

To determine the dispersion relation and the growth rate, we use the by kappa distribution function with density perturbation . Kappa distribution: 

$$N(y,v) = N_0 \left[ 1 - \varepsilon \left( y + \frac{v_x}{\Omega} \right) \right] f_\perp(v_\perp) f_\Pi(v_\Pi)$$

$$f_\perp(v_\perp) = \left[ \frac{mv_\perp^2}{2\kappa_B} \right] \frac{2k_\perp}{k_\Pi - 1} \qquad f_\Pi(v_\Pi) = \left[ \frac{mv_\Pi^2}{2\kappa_B} \right] \frac{2k_\Pi}{2k_\Pi - 1}$$

$$T_{\Pi c} = T_\Pi \left( \frac{2\kappa}{2\kappa - 1} \right) \left[ 1 + i \frac{e_1 \cdot E_0 \cdot \overline{K}}{\kappa^2 V_{T\Pi}^2 \left( \frac{2\kappa}{2\kappa - 1} \right)} \right]$$
and
$$K = (k_\perp^2 + k_\Pi^2)^{1/2}$$
(7)
Where

And  $\varepsilon$  is a small parameter of the order of inverse of the density gradient scale length.

#### **Dispersion relation:**

To evaluated the dispersion relation, we calculate the integrated perturbed density for non-resonant particles as

$$n_{i,c,d} = \int_0^\infty 2\pi V_\perp dV_\perp \int_{-\infty}^\infty dV_\parallel n_i(r,t)$$
(8) With  
the help of eq.(5) and (7) use find the average densities for homogeneous plasma as  
$$\omega_{ri}^2 \left[ -K_\perp^2 \phi - K_\parallel^2 \right] + (1+2) (2k-1)$$

$$\bar{n}_{i} = \frac{\omega_{pi}}{4\pi e} \left[ \frac{-\kappa_{\perp} \psi}{\Omega_{i}^{2}} + \frac{\kappa_{\parallel}}{\omega_{i}^{2}} \psi \right] \left( 1 - \frac{1}{2} k_{\perp}^{2} \rho_{i}^{2} \right) \left( \frac{2k-1}{2k} \right)$$
(9)  
$$n_{e} = \frac{\omega_{pe}^{2}}{4\pi e V_{T\parallel e}^{2}} \psi$$
(10)  
$$\bar{n}_{d} = \frac{\omega_{pd}^{2}}{4\pi Z_{d} e} \left[ \frac{-k_{\perp}^{2} \phi}{\Omega_{d}^{2}} + \frac{k_{\parallel}^{2} \psi}{\omega^{2}} \right] \left( 1 - \frac{1}{2} k_{\perp}^{2} \rho_{d}^{2} \right) \left( \frac{2k-1}{2k} \right)$$
(11)

is observed that essential feature of the kinetic Alfven wave is retained even in this ideal case. For maxwell's equation we use the quasi-neutrality condition,

It

$$\bar{n}_{i} = \bar{n}_{e} + Z_{d}\bar{n}_{d}$$
We get relation between  $\psi$  and  $\phi$  as:  

$$\phi = \frac{\alpha_{d}^{2}}{k_{\perp}^{2}} \left[ \frac{\omega_{pc}^{2}}{\omega_{pd}^{2} v_{T\parallel}^{2} A_{2}} - \frac{k_{\parallel}^{2}}{\omega^{2}} \left( 1 + \frac{A_{1}B_{1}}{A_{2}} \right) B_{2}^{-1} \psi$$
(12)
Where

When

$$\begin{aligned} A_1 &= 1 - \frac{1}{2} k_\perp^2 \rho_i^2 \left[ \frac{2k_i - 1}{2k_i} \right], \qquad A_2 &= 1 - \frac{1}{2} k_\perp^2 \rho_d^2 \left[ \frac{2k - 1}{2k} \right] \\ B_1 &= \frac{N_0}{N_{d0}} \frac{m_d}{m_i} \frac{1}{z_d^2}, \qquad B_2 &= 1 - \frac{A_1 B_1}{A_2} \frac{\Omega_d^2}{\Omega_i^2} \end{aligned}$$

Using perturbed ion, electron and dust particle densities n<sub>i</sub>, n<sub>e</sub> and n<sub>d</sub> and Ampere's law in the parallel direction, we obtained the equation:

$$\frac{\partial}{\partial z} \nabla_{\perp}^{2}(\Phi - \Psi) = \frac{4\pi}{c^{2}} \frac{\partial}{\partial t} J_{z}$$
where
$$J_{z} = c \int_{0}^{\lambda} ds \int_{0}^{\infty} 2\pi V_{\perp} dV_{\perp} \int_{-\infty}^{\infty} dV_{\parallel} \frac{m_{j}}{2} [(N + n_{1})(V + u)^{2} - NV^{2}]_{j}$$
(13)

 $J_z$  is the current density which is contributed by first-order perturbations of density and velocity. we obtain the dispersion relation for the kinetic Alfven waves in homogeneous dusty plasma as:

$$\omega^{4} \left( \frac{\omega_{peB_{2}}^{2}}{k_{\Pi}^{4} \omega_{pd}^{2} V_{T\Pi e}^{4} V_{A}^{2} A_{2}} \right) - \omega^{2} \begin{cases} \frac{k_{1}^{2} B_{2}}{k_{\Pi}^{2} \alpha_{d}^{2}} \left( 1 + \frac{\omega_{pdA_{2}}^{2}}{c^{2}} \left( \frac{T_{\Pi cd}}{m_{d}} \right) \right) + \frac{\omega_{pi}^{2} A_{1}}{c^{2} k_{\Pi}^{2} \alpha_{d}} \left( 1 - \frac{\omega_{pi}^{2}}{\omega_{pd}^{2} V_{T\Pi e}^{2} A_{i}^{2}} \frac{T_{\Pi ci}}{m_{i}} - \frac{k_{1}^{2}}{\alpha_{i}^{2}} \frac{T_{\Pi ci}}{m_{i}} \right) + \\ \left( \frac{\omega_{pe}^{2}}{c^{2} \alpha_{d}^{2} V_{T\Pi e}^{2} k_{\Pi}^{2}} \frac{T_{\Pi cd}}{m_{d}} \right) + \left( \frac{B_{2}}{k_{\Pi}^{2} \omega_{pd}^{2}} - \frac{\omega_{pe}^{2}}{k_{\Pi}^{2} \omega_{pd}^{2} V_{T\Pi e}^{2} A_{2}} \right) \\ \frac{\omega_{pi}^{2}}{c^{2} \Omega_{i}} \frac{T_{\Pi ci}}{m_{i}} \left( 1 + \frac{A_{1}B_{1}}{A_{2}} \right) - \frac{\omega_{pdA_{2}}^{2} T_{\Pi cd}}{c^{2} \alpha_{d}^{2}} \frac{T_{\Pi cd}}{m_{d}} - \frac{A_{1}B_{1}}{A_{2}} + 1 = 0 \end{cases}$$
(14)  
Where,  $V_{A}^{2} = \frac{c^{2} \Omega_{i}^{2}}{\omega_{pd}^{2}}$  is the square of Alfven's speed.

The oscillatory motion of non-resonant electrons carriers the major part of energy. The wave energy density per unit wave length  $W_w$  is the sum of pure field energy and the changes in energy of the non-resonant particles  $W_{i,e,d}$ . it is observed that the wave energy is contained in the from of the oscillatory motion of the non-resonant electrons. **Growth rate:** 

Using the low of conservation of energy, calculate the growth rate of drift kinetic Alfven wave by

$$\frac{\mathrm{d}}{\mathrm{dt}}(W_w + W_r) = \tag{15}$$

With the help of we have found the growth rate of the drift kinetic Alfven wave with dusty plasma as:

$$\frac{\gamma}{\omega} = \frac{\pi^{1/2}\omega_{e}}{k_{\parallel} v_{T\parallel e} [1 + \frac{\omega_{pl}^{2} k_{\parallel}^{2} T_{\parallel}}{\omega_{e} \omega_{pe}^{2} m_{e}} \times (A_{x} + P_{x})} \frac{\Gamma(\kappa+1)}{\kappa^{2}} \left(1 + \frac{(\omega_{e})^{2}}{k_{\parallel}^{2} v_{T\parallel e}^{2}}\right)^{-(k+1)}$$
(16)

where  $\omega_e = \omega - k \pi V_{de}$  electron beam velocity

## Growth length:

$$\gamma = \frac{\Gamma(\kappa+1)}{\kappa^{2}_{2} \times \Gamma(\kappa-\frac{1}{2})} \times \left[ \frac{\sqrt{\pi} \times \sqrt{\omega_{e}}}{K_{\Pi}.VT_{\Pi e}.\left[1 + \frac{\omega_{p1}^{2}.VT_{\Pi e}^{2}}{\omega_{e}.\omega_{pe}^{2}}.(A_{x}+P_{x})\right]} \cdot \left[1 + \frac{\omega_{e}}{K_{\Pi}^{2}.VT_{\Pi e}^{2}}\right]^{-(\kappa+1)} \right]$$
(17)  
$$V_{p} = \left[ \frac{B}{\omega_{pd}^{2}.VT_{\Pi e}^{2}.V_{a}^{2}A_{2}} + \sqrt{B^{2} + 4} \cdot \frac{\omega_{pe}^{2}.B_{2}}{\omega_{pd}^{2}.VT_{\Pi e}^{2}.V_{a}^{2}A_{2}} \cdot C \cdot \frac{\omega_{pd}^{2}.VT_{\Pi e}^{2}.V_{a}^{2}A_{2}}{\omega_{pe}^{2}.B_{2}}\right]^{\frac{1}{2}}$$
(18)  
$$L_{g} = \frac{\left[ \frac{B}{\omega_{pd}^{2}.VT_{\Pi e}^{2}.V_{a}^{2}A_{2}} + \sqrt{B^{2} + 4} \cdot \frac{\omega_{pe}^{2}.B_{2}}{\omega_{pd}^{2}.VT_{\Pi e}^{2}.V_{a}^{2}A_{2}} \cdot C \cdot \frac{\omega_{pd}^{2}.VT_{\Pi e}^{2}.V_{a}^{2}A_{2}}{\omega_{pe}^{2}.B_{2}}\right]^{\frac{1}{2}}$$
(19)

 $\frac{\frac{1}{\kappa^{\frac{3}{2}} \times \Gamma\left(\kappa - \frac{1}{2}\right)}^{x} \left[ \frac{1}{K_{\Pi} \cdot VT_{\Pi e}} \left[ 1 + \frac{\omega_{pi}^{2} \cdot VT_{\Pi e}^{2}}{\omega_{e} \cdot \omega_{pe}^{2}} (A_{x} + P_{x}) \right]^{-1} \left[ \frac{1}{K_{\Pi}^{2} \cdot VT_{\Pi e}^{2}} \right] \right]$ Where B and C are define as, and L<sub>g</sub> is Growth length,

$$B = \frac{k_{\perp}^2 B_2}{k_{\Pi}^2 \Omega_d^2} \left( 1 + \frac{\omega_{pd}^2 A_2}{c^2} \left( \frac{T_{\Pi cd}}{m_d} \right) \right) + \frac{\omega_{pi}^2 A_1}{c^2 k_{\Pi}^2 \Omega_d} \left( 1 - \frac{\omega_{pi}^2}{\omega_{pd}^2 V_{\Pi e}^2 \Omega_i^2 A_2} \frac{T_{\Pi ci}}{m_i} - \frac{k_{\perp}^2}{\Omega_i^2} \frac{T_{\Pi ci}}{m_i} \right) + \left( \frac{\omega_{pe}^2}{c^2 \Omega_d^2 V_{T\Pi e}^2 k_{\Pi}^2} \frac{T_{\Pi cd}}{m_d} \right) + \left( \frac{B_2}{k_{\Pi}^2 V_A^2} - \frac{\omega_{pe}^2}{k_{\Pi}^2 \omega_{pd}^2 V_{T\Pi e}^2 A_2} \right)$$
$$C = \frac{\omega_{pi}^2}{c^2 \Omega_i} \frac{T_{\Pi ci}}{m_i} \left( 1 + \frac{A_1 B_1}{A_2} \right) - \frac{\omega_{pd}^2 A_2}{c^2 \Omega_d^2} \frac{T_{\Pi cd}}{m_d} - \frac{A_1 B_1}{A_2} + 1$$

### **Results and discussion:**

$$\begin{array}{ll} \Omega_i \!\!=\!\! 412 \; s^{\text{-1}} \;, & \Omega_d \!\!=\! 6.88^{\ast} 10^{\text{-10}} \;, & m_d \!\!=\! 10^{\text{-12}} g \;, & VT_{\parallel\!e} \!\!=\! 4^{\ast} 10^6 \text{ms}^{\text{-1}} \;, & \rho_i \!\!=\! 1.68^{\ast} 10^4 \;, \\ N_d \!\!=\! 1^{\ast} 10^3 \text{cm}^{\text{-3}} \;, & Z_d \!\!=\! 5.5 \; V_{de} \!\!=\! -1^{\ast} 10^7 \;, \end{array}$$



Fig.1 Variation of growth rate  $(\gamma/\omega)$  versus perpendicular wave number  $(K_{\perp} \rho_i)$  for different value of electron beam velocities  $V_{de}$ . The variation of growth rate  $(\gamma/\omega)$  with  $k_{\perp}$  at different values of electron beam velocities  $V_{de}$  at fixed values dust grain  $Z_{d}$ ,  $N_{d}$ , and kappa  $\kappa$ . Here it is noticed that the effect of electron beam velocities is to reduced the growth rate at higher values of electron beam velocities with. Thus, the  $V_{de}$  controls the wave growth in the dusty magneto-plasma and transfers the energy to the particles by inverse Landau damping.



Fig. 2 The variation of growth length  $(\gamma_L)$  versus perpendicular wave vector  $(K \perp \rho i)$  cm<sup>-1</sup> for different values of electron beam velocities  $V_{De}$  at kappa  $\kappa$ .

the relation between Growth length  $(L_g)$  versus perpendicular wave number  $k_{\perp}$  for different values of electron beam velocities  $V_{de}$ , at the fixed values of kappa  $\kappa$  and dust grain  $Z_d$ . It is observed that the electron beam velocities  $V_{de}$ , enhanced the frequency .it is clear that the growth length $(L_g)$  increases with increase of applied electron beam velocities. the increase of growth length $(L_g)$  with increasing  $k_{\perp}$ pi.



Fig.3 Variation of growth rate  $(\gamma/\omega)$  with perpendicular wave number  $(K_{\perp}\rho_I)$  for different values of electron beam velocities at kappa ( $\kappa$ ).

The variation of growth rate  $(\gamma/\omega)$  with  $k_{\perp}$  at different values of electron beam velocities  $V_{de}$  at fixed values dust grain  $Z_d$ ,  $N_d$ , and kappa  $\kappa = 4$ . Here it is noticed that the effect of electron beam velocities is to reduced the growth rate at higher values of electron beam velocities with. Thus, the  $V_{de}$  controls the wave growth in the dusty magneto-plasma and transfers the energy to the particles by inverse Landau damping.



Fig. 4 The variation of growth length  $(\gamma_L)$  versus perpendicular wave vector  $(K \perp \rho i)$  cm<sup>-1</sup> for different values of electron beam velocities  $V_{De}$  at kappa  $\kappa$ .

the relation between Growth length  $(L_g)$  versus perpendicular wave number  $k_{\perp}$  for different values of electron beam velocities  $V_{de}$ , at the fixed values of kappa  $\kappa$ =4 and dust grain  $Z_d$ . It is observed that the electron beam velocities  $V_{de}$ , enhanced the frequency .it is clear that the growth length $(L_g)$  increases with increase of applied electron beam velocities. the increase of growth length $(L_g)$  with increasing  $k_{\perp}\rho i$ .



Fig.5 The variation of real frequency ( $\omega$ ) sec<sup>-1</sup> versus perpendicular wave vector ( $K_{\perp} \mathbf{\rho}_I$ ) cm<sup>-1</sup> for different values of N<sub>d</sub> at kappa distribution function ( $\kappa$ ).

shows the relation between wave frequency  $\omega$  versus perpendicular wave number  $k_{\perp}$  for different values of  $N_d$ , at the fixed values of electron beam velocities  $V_{de}$ , dust grain  $Z_d$  and kappa ( $\kappa$ ). It is observed that the  $N_d$ , enhanced the frequency .it is clear that the wave frequency  $\omega$  increases with increase of applied  $N_d$ . the increase of  $\omega$  with increasing  $k_{\perp}\rho i$ .



Fig. 6 The variation growth rate  $(\gamma/\omega)$  versus perpendicular wave vector  $(K_{\perp}\boldsymbol{\rho}_{I})$  cm<sup>-1</sup> for different values of N<sub>d</sub> at kappa distribution function ( $\kappa$ ).

The variation of growth rate  $(\gamma/\omega)$  with  $k_{\perp}$  at different values of  $N_d$  at fixed values electron beam velocities  $V_{de}$ , dust grain  $Z_d$  and kappa  $\kappa$ . Here it is noticed that the effect of  $N_d$  is to reduced the growth rate at higher values of  $N_d$  with. Thus, the  $N_d$  controls the wave growth in the dusty magneto-plasma and transfers the energy to the particles by inverse Landau damping.



Fig.7 The variation of growth length ( $\gamma_L$ ) versus perpendicular wave vector (K $\perp \rho i$ ) cm<sup>-1</sup> for different values of N<sub>d</sub> at kappa distribution function ( $\kappa$ ).

the relation between Growth length  $(L_g)$  versus perpendicular wave number  $k_{\perp}$  for different values of  $N_d$ , at the fixed values of electron beam velocities  $V_{de}$ , dust grain  $Z_d$  and kappa ( $\kappa = 3$ ). It is observed that the  $N_d$ , enhanced the frequency .it is clear that the growth length( $L_g$ ) increases with increase of applied  $N_d$ . the increase of growth length( $L_g$ ) with decreasing  $k_{\perp}\rho$ i.



Fig. 8 The variation of real frequency ( $\omega$ ) sec<sup>-1</sup> versus perpendicular wave vector ( $K_{\perp} \mathbf{\rho}_i$ ) cm<sup>-1</sup> for different values of dust grain  $Z_d$  at kappa distribution function ( $\kappa$ ).

shows the relation between wave frequency  $\omega$  versus perpendicular wave number  $k_{\perp}$  for different values of dust grain  $Z_d$ , at the fixed values of electron beam effect  $V_{de}$ ,  $N_d = 1*10^3$  cm<sup>-3</sup> and kappa ( $\kappa$ ). It is observed that the dust grain  $Z_d$ , enhanced the frequency. it is clear that the wave frequency  $\omega$  increases with increase of applied dust grain  $Z_d$ . the increase of  $\omega$  with increasing  $k_{\perp}\rho$ i.



Fig. 9 The variation growth rate  $(\gamma/\omega)$  versus perpendicular wave vector  $(K_{\perp}\rho_I)$  cm<sup>-1</sup> for different values of dust grain  $Z_d$  at kappa distribution function ( $\kappa$ ).

The variation of growth rate  $(\gamma/\omega)$  with  $k_{\perp}\rho_i$  at different values of dust grain  $Z_d$  at fixed values electron beam velocities  $V_{de}$ ,  $N_d$ , and kappa  $\kappa$ . Here it is noticed that the effect of electric field is to reduced the growth rate at higher values of electric field with. Thus, dust grain  $Z_d$  controls the wave growth in the dusty magneto-plasma and transfers the energy to the particles by inverse Landau damping.



Fig. 10 The variation of growth length ( $\gamma_L$ ) versus perpendicular wave vector (K $\perp \rho i$ ) cm<sup>-1</sup> for different values of dust grain Z<sub>d</sub> at kappa distribution function ( $\kappa$ ).

the relation between Growth length  $(L_g)$  versus perpendicular wave number  $k_{\perp}$  for different values of dust grain  $Z_d$ , at the fixed values of electron beam velocities  $V_{de}$ ,  $N_d$ , and kappa ( $\kappa = 3$ ). It is observed that the dust grain  $Z_d$ , enhanced the frequency .it is clear that the growth length  $(L_g)$  increases with increase of applied dust grain  $Z_d$ . the increase of growth length  $(L_g)$  with decreasing  $k_{\perp}\rho i$ .



Fig. 11 The variation of frequency ( $\omega$ ) sec<sup>-1</sup> versus perpendicular wave vector ( $K_{\perp} \mathbf{\rho}_{I}$ ) cm<sup>-1</sup> for different values of kappa distribution function ( $\kappa$ ) and fix value  $Z_{D}$ ,  $N_{d}$ , and  $V_{de}$ .

shows the relation between wave frequency  $\omega$  (rad/sec) versus perpendicular wave number  $k_{\perp}$  for different values of kappa distribution function  $\kappa$ , at the fixed values of dust grain  $Z_d$  and electron beam velocities  $V_{de}$ , and  $N_d$ . It is observed that the kappa distribution function  $\kappa$ , enhanced the frequency .it is clear that the wave frequency  $\omega$  increases with increase of applied kappa function  $\kappa$ . the increase of  $\omega$  with increasing  $k_{\perp}\rho i$ .



Fig. 12 The variation growth rate  $(\gamma/\omega)$  versus perpendicular wave vector  $(K_{\perp}\rho_1)$  cm<sup>-1</sup> for different values of kappa distribution function ( $\kappa$ ) and fix value  $Z_d$ ,  $N_d$ , and  $V_{de}$ .

The variation of growth rate  $(\gamma/\omega)$  with  $k_{\perp}$  at different values of kappa distribution function  $\kappa$  at fixed values electron beam velocities  $V_{de}$ ,  $N_{d}$ , and dust grain  $Z_d$ . Here it is noticed that the effect of kappa function is to reduced the growth rate at higher values of kappa  $\kappa$  with. The growth rate slows down as we increase the value of the kappa.



Fig. 13 The variation of growth length ( $\gamma_L$ ) versus perpendicular wave vector (K $\perp \rho i$ ) cm<sup>-1</sup> for different values of kappa distribution function ( $\kappa$ ) and fix value Z<sub>d</sub>, N<sub>d</sub>, and V<sub>de</sub>.

the relation between Growth length  $(L_g)$  versus perpendicular wave number  $k_{\perp}$  for different values of kappa function  $\kappa$ , at the fixed values of electron beam velocities  $V_{de}$ ,  $N_d$  and dust grain  $Z_d$ . It is observed that the kappa function  $\kappa$ , enhanced the frequency .it is clear that the growth length( $L_g$ ) increases with increase of applied kappa function  $\kappa$ . the increase of growth length( $L_g$ ) with increasing  $k_{\perp}\rho i$ .

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