Pilot Allocation in Channel Estimation for MIMO-OFDM Systems Using Haar Wavelet Transform

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Abstract: In OFDM multiple carriers' square measure used and it provides higher level of spectral potency as compared to Frequency Division Multiplexing (FDM). In OFDM as a result of loss of orthogonality between the subcarriers there's repose carriers put downference: Inter carrier Interference (ICI) and inter symbol interference (ISI) and to beat this downside use of cyclic prefixing (CP) is needed, that uses two hundredth of obtainable information measure. Comparison between typical and traditional standard} FFT/DCT based mostly} OFDM systems with DWT based OFDM system are created in line with some conventional and non-conventional modulation ways over AWGN. The ripple families are used and compared with FFT/DCT {mostly based or based mostly primarily based mostly} OFDM system and located that DWT based OFDM system is best than FFT/DCT based OFDM system with regards to the bit error rate (BER) performance.

***Index terms* - MIMO - OFDM – Multi input and multi output orthogonal Frequency Division Multiplexing, AWGN – Adaptive White Gaussian Noise, CP – Cyclic prefixing, Channel Estimation, DWT- Discrete Wavelet Transform, BER – Bit Error Rate**

I. INTRODUCTION

Digital communications systems require each channel to operate at a specific frequency and with a specific bandwidth. In fact, communication systems have evolved so that the largest amount of data can be communicated through a finite frequency range. In this document we will focus on the recent evolution of communications systems into using various mechanisms for effectively using the frequency spectrum. More specifically, we will describe how frequency division multiplexing (FDM) and orthogonal frequency division multiplexing (OFDM) are able to effectively utilize the frequency spectrum. In addition, we will distinguish the two and describe why OFDM systems are currently being implemented in some of the newest and most advanced communications systems

OFDM is a subset of frequency division multiplexing in which a single channel utilizes multiple sub-carriers on adjacent frequencies. In addition the sub-carriers in an OFDM system are overlapping to maximize spectral efficiency. Ordinarily, overlapping adjacent channels can interfere with one another. However, sub-carriers in an OFDM system are precisely orthogonal to one another. Thus, they are able to overlap without interfering. As a result, OFDM systems are able to maximize spectral efficiency without causing adjacent channel interference. The frequency domain of an OFDM system is represented in the diagram below.

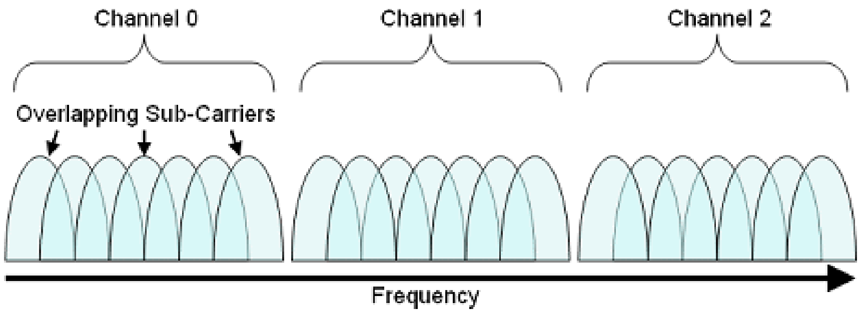

Figure 1 channel separation

Notice above that there are seven sub-carriers for each individual channel. Because the symbol rate increases as the channel bandwidth increases, this implementation allows for a greater data throughput than with an FDM system.

Each subcarrier carries one bit of information (N bits total) by its presence or absence in the output spectrum. The frequency of each subcarrier is selected to form an orthogonal signal set, and these frequencies are known at the receiver. Note that the output is updated at a periodic interval T that forms the symbol period as well as the time boundary for orthogonality. Figure 4 shows the resultant frequency spectrum. In the frequency domain, the resulting sin function side lobes produce overlapping spectra. The individual peaks of sub bands all line up with the zero crossings of the other sub bands. This overlap of spectral energy does not interfere with the system's ability to recover the original signal. The receiver multiplies (i.e., correlates) the incoming signal by the known set of sinusoids to produce the original set of bits sent. The digital implementation of an OFDM system will enhance these simple principles and permit more complex modulation.

II. PILLOT ALLOCATION OF MIMO-OFDM SYSTEM FOR CHANNEL ESTIMATION USING WAVELET TRANSFORM

Algorithm

step 1: Mother function

Let $\phi(x)$ be some mother function. The $\phi(2x)$ is the same function compressed by a factor of 2. Binary compression can therefore be denoted as $\phi_j = \phi(2^j x)$.

step2: Wavelet functions

From the mother or scaling function and the coefficients we construct wavelet functions $\psi(x)$.

$$\psi(x) = \sum_k (-1)^k c_{M-k} \phi(2x - k)$$

step3: Multi Resolution Analysis (MRA)

Although we have quite general definitions for ϕ_{jk} and ψ_{jk} we need only use the j=0 level over and over again. This was a discovery by Mallet.

Here is the technique:

1) Multiply each a **pair** of input coefficients with the mother function coefficients on the top line and the wavelet coefficients in the bottom line.

Ex: For the non-reversible Haar transform this is

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \circ \begin{pmatrix} 3 \\ 2 \\ 4 \\ 1 \\ 4 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 5 \\ 3 \\ 6 \\ 2 \\ 4 \\ 2 \end{pmatrix} \quad \begin{pmatrix} m \\ w \\ m \\ w \\ m \\ w \\ m \\ w \end{pmatrix}$$

2) Now sort (an effective permutation) the above column matrix and bring all the mother generated coefficients to the top.

$$\begin{pmatrix} 5 \\ 1 \\ 5 \\ 3 \\ 6 \\ 4 \\ 4 \\ 2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 5 \\ 5 \\ 6 \\ 4 \\ 1 \\ 3 \\ 2 \\ 2 \end{pmatrix} \quad \begin{pmatrix} m \\ m \\ m \\ m \\ w \\ w \\ w \\ w \end{pmatrix}$$

3) Now repeat step 2 only on the coefficients labelled 'm'

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \circ \begin{pmatrix} 5 \\ 5 \\ 6 \\ 4 \\ 1 \\ 3 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 10 \\ 2 \\ 1 \\ 3 \\ 2 \\ 2 \end{pmatrix}$$

4) repeat step 2) and 3) until only the top coefficient has the 'm' label.

The Haar Transform

The Haar Function

The family of N Haar functions $h_k(t)$ are defined on the interval $0 \le t \le 1$ **Error! Reference source not found.**. The shape of the Haar function, of an index k, is determined by two parameters: p and q, where

$k = 2^p + q - 1$ and k is in a range of $k = 0, 1, 2, \cdots, N - 1$.

When $k = 0$, the Haar function is defined as a constant $h_0(t) = 1/\sqrt{N}$; when $k > 0$, the Haar function is defined as

$$h_k(t) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2} & (q-1)/2^p \le t < (q-0.5)/2^p \\ -2^{p/2} & (q-0.5)/2^p \le t < q/2^p \\ 0 & otherwise \end{cases}$$

From the above equation, one can see that p determines the amplitude and width of the non-zero part of the function, while q determines the position of the non-zero part of the Haar function.

The Haar Matrix

The discrete Haar functions formed the basis of the Haar matrix **H**

$$\mathbf{H}_{2N} = \begin{bmatrix} \mathbf{H}_N \otimes [1,1] \\ \mathbf{I}_N \otimes [1,-1] \end{bmatrix} \mathbf{H}(0) = 1$$

where

$$\mathbf{I}_N = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

The Kronecker product of $\boldsymbol{A} \otimes B$, where A is an m × n matrix and $\boldsymbol{B}$ is a p × q matrix, is expressed as

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}$$

When $\mathrm{N} = 2^n$

$$\mathbf{H}_\mathrm{N} = \begin{bmatrix} \phi \\ \mathrm{h}_{0,0} \\ \mathrm{h}_{1,0} \\ \mathrm{h}_{1,1} \\ \vdots \\ \mathrm{h}_{k-1,0} \\ \mathrm{h}_{k-1,1} \\ \vdots \\ \mathrm{h}_{k-1,2^{k-1}-1} \end{bmatrix}$$

The Haar matrix is real and orthogonal, i.e.,

- $\mathbf{H} = \mathbf{H}^*$
- $\mathbf{H}^{-1} = \mathbf{H}^T$, i.e., $\mathbf{H}^T\mathbf{H} = \mathbf{I}$

An un-normalized 8-point Haar matrix $\mathbf{H_8}$ is shown below **Error! Reference source not found.**

$$\mathbf{H}[m,n] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

From the definition of the Haar matrix **H**, one can observe that, unlike the Fourier transform, **H** matrix has only real element (i.e., 1, -1 or 0) and is non-symmetric.

The first row of **H** matrix measures the average value, and the second row **H** matrix measures a low frequency component of the input vector. The next two rows are sensitive to the first and second half of the input vector respectively, which corresponds to moderate frequency components. The remaining four rows are sensitive to the four section of the input vector, which corresponds to high frequency components.The Haar function at each row of **H** matrix. Notice the width and location of the Haar function is changed. The Haar function with narrower width is responsible for analysing the higher frequency content of the input signal.

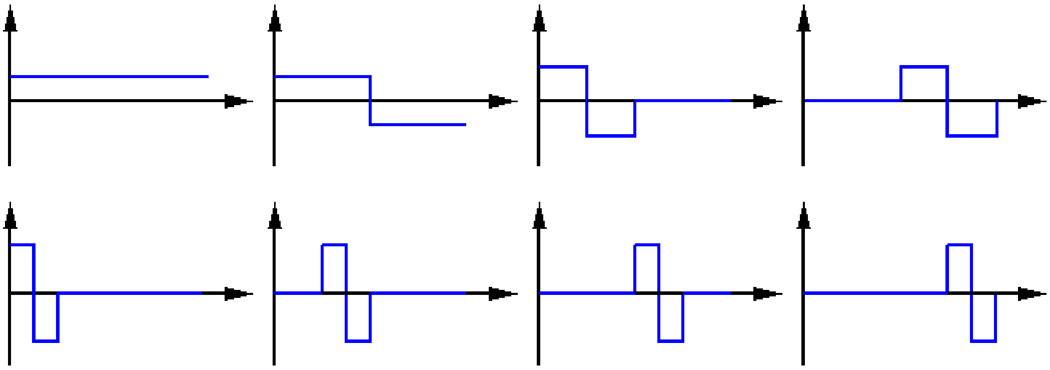

Figure 2. Haar functions for composing 8-point Haar transform matrix.

The inverse 2^k-point Haar matrix is described as$\mathbf{H}^{-1} = \mathbf{H}^{\mathrm{T}}\mathbf{D}$ **Error! Reference source not found.**

$$\mathbf{D}[m,n] = 0 \quad if\ m \neq n \mathbf{D}[0,0] = 2^{-k} \mathbf{D}[1,1] = 2^{-k} \mathbf{D}[\mathrm{n},\mathrm{n}] = 2^{-k+p} \quad if\ 2^p < n < 2^{p+1}$$

For k = 3, un-normalised inverse 8-points Haar transform.

$$\mathbf{D} = \begin{bmatrix} 1/8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$

The Haar Transform function

$\mathrm{HT}^n(f)$ of an N-input function $\mathrm{X}^n(f)$ is the 2^n element vector

$$\mathrm{HT}^n(f) = \mathbf{H}^n \mathrm{X}^n(f)$$

The Haar transform cross multiplies a function with Haar matrix that contains Haar functions with different width at different location.

For example:

$$\mathbf{I}_N \begin{bmatrix} 1.2 \\ 1.2 \\ 1.8 \\ 0.8 \\ 2 \\ 2 \\ 1.9 \\ 2.1 \end{bmatrix} = \begin{bmatrix} 13 \\ -3 \\ -0.2 \\ 0 \\ 0 \\ 1 \\ 0 \\ -0.2 \end{bmatrix}$$

The Haar transform is performed in levels. At each level, the Haar transform decomposes a discrete signal into two components with half of its length: an approximation (or trend) and a detail (or fluctuation) component. The first level of approximation $\boldsymbol{a}^1 = (a_1, a_2, \cdots, a_{N/2})$ is defined as

$$\mathrm{a_m} = \frac{\mathrm{X_{2m-1}} + \mathrm{X_{2m}}}{\sqrt{2}}$$

for $\mathrm{m} = 1,2,3, \cdots, \mathrm{N}/2$, where $\mathbf{X}$ is the input signal. The multiplication of $\sqrt{2}$ ensures that the Haar transform preserves the energy of the signal. The values of $\boldsymbol{a}^1$ represents the average of successive pairs of $\mathbf{X}$ value.

The first level detail $\boldsymbol{d}^1 = (d_1,\ d_2, \cdots,\ d_{N/2})$ is defined as

$$d_m = \frac{X_{2m-1} - X_{2m}}{\sqrt{2}}$$

for $\mathrm{m} = 1,2,3, \cdots, \mathrm{N}/2$. The values represents the difference of successive pairs of $\mathbf{X}$ value.

The first level Haar transform is achieved by

$$\mathrm{X} = \frac{\mathrm{a_1} + \mathrm{d_1}}{\sqrt{2}}, \frac{\mathrm{a_1} - \mathrm{d_1}}{\sqrt{2}}, \cdots, \frac{\mathrm{a_{N/2}} + \mathrm{d_{N/2}}}{\sqrt{2}}, \frac{\mathrm{a_{N/2}} - \mathrm{d_{N/2}}}{\sqrt{2}}$$

The successive level of Haar transform, the approximation and detail component are calculate in the same way, except that these two components are calculated from the previous approximation component only.

$$\boldsymbol{a}^1 = \sqrt{2}(5, 9, 11, 3)$$
$$\boldsymbol{d}^1 = \sqrt{2}(-1, -1, 2, 0)$$
$$\boldsymbol{a}^2 = (14, 14)$$
$$\boldsymbol{d}^1 = (-4, 8)$$

III. SIMULATION RESULTS

BERPERFORMANCE EVALUATION

By victimization MATLAB performance characteristic of DFT {based| based mostly| primarily based mostly} OFDM and riffle based OFDM area unit obtained for various modulations that area unit used for the LTE, as shown in figures. Modulations that might be used for LTE area unit QPSK, sixteen QAM and sixty four QAM (Uplink and downlink).

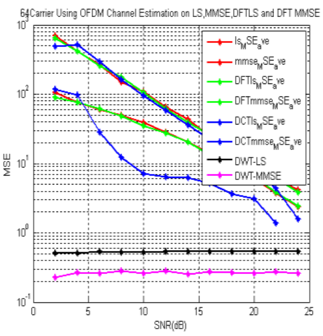

Figure 3 Comparison Analysis of DFT Vs DCT Vs. DWT Process Using 64QAM

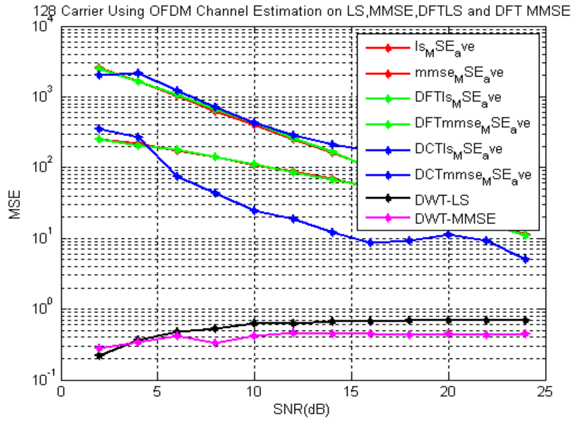

Figure 4. Comparison Analysis of DFT Vs DCT Vs. DWT Process Using 128QAM

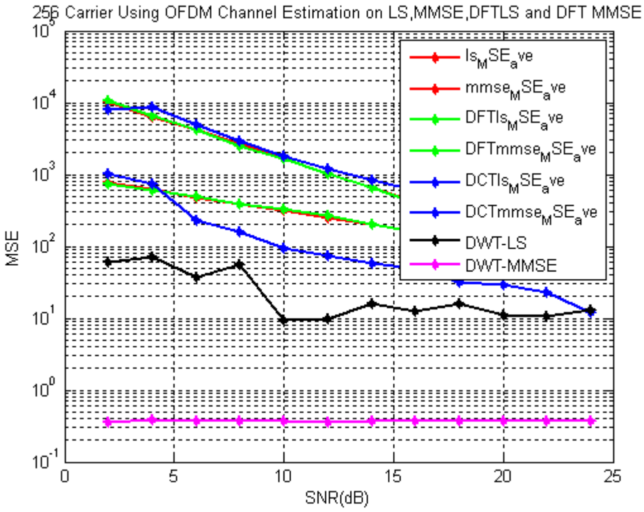

Figure 5. Comparison Analysis of DFT Vs DCT Vs. DWT Process Using 256QAM

IV. CONCLUSION

A tendency to analyzed the performance of rippling based mostly OFDM system and compared it with the performance of DFT based mostly OFDM system. From the performance curve wave got determined that the BER curves obtained from rippling {based mostly primarily based mostly} OFDM are higher than that of DFT based OFDM. We have a tendency to used 3 modulation techniques for implementation that are QPSK, 16QAM and 64 QAM, that are employed in LTE. In rippling based mostly OFDM differing types of filters may be used with the assistance of various wavelets out there.

V. REFEENCES

1. G.L. Tuber, J.R. Barry, S.W. McLaughlin, Ye Li, M.A. Ingram and T.G. Pratt, "Broadband MIMO-OFDM wireless communications," *Proceedings of the IEEE, vol. 92, No. 2, pp. 271-294*, February. 2004.
2. Jan-Jaap van de Beek, O.Edfors, and M.Sandell, "On channel estimation in OFDM systems," Presented at in *proceedings of Vehicular Technology, Chicago, pp. 815-819*, 1995.
3. B.Song, L.Gui, and W.Zhang, "Comb type pilot aided channel estimation in OFDM systems with transmit diversity," *IEEE Trans. Broadcast., vol. 52, pp. 50-57*, March. 2006.
4. Noh. M., Lee. Y. , and Park. H. "A low complexity LMMSE channel estimation for OFDM," *IEE Proc. Commum., vol. 153, No. 5, pp. 645- 650*, 2006.
5. Oppenheim, Schafer with Buck, Discrete-Time Signal Processing, *PEARSON Education, 2nd edition*, 2005.
6. Upena Dalal, "Wireless Communication", *Oxford University press, july 2009, pp.365 – 408*
7. Manish J. Manglani and Amy E. Bell, "Wavelet Modulation Performance in Gaussian and Rayleigh Fading Channels", *Electrical and Computer Engineering Department, Virginia*, 2001.
8. B.G.Negash and H.Nikookar, "Wavelet Based OFDM for Wireless Channels", *International Research Centre for Telecommunication Transmission and Radar, Faculty of Information Technology and Systems, Delft University of Technology*, 2001.
9. S. Adhikari, S. L. Jansen, M. Kuschnerov, B. Inan, and W. Rosenkranz, "*Analysis of spectrally shaped DFT-OFDM for fiber nonlinearity mitigation," Opt. Express*, to be published.
10. S. Nakajima, "Effects of spectral shaping on OFDM transmission performance in nonlinear channels," *in Proc. 16th ISTMWC 2007*, Budapest,
Hungary, Jul., pp. 1–5.

11. O. Gaete, L. Coelho, B. Spinnler, and N. Hanik, “Pulse shaping using the discrete Fourier transform for direct detection optical systems,” in *Proc. ICTON, Stockholm, Sweden, Jun. 2011, pp. 1–4, paper We.A1.2.*
12. C. Xia and D. van den Borne, “Impact of the channel count on the nonlinear tolerance in coherently-detected POLMUX-QPSK modulation,” in *Proc. OFC 2011, Los Angeles, CA, Mar. 2011, pp. 1–3, paper OWO1.1*.