# AN M/G/1 RETRIAL QUEUE WITH MULTIPLE WORKING VACATIONS AND LOCK DOWN TIMES

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***ABSTRACT:***
*We consider an M/G/1 retrial queue with general retrial times multiple working vacation and lock down times. During the lock down period, customers can be served at a lower rate. Both service times in a vacation period and in a service period are generally distributed random variables. Using supplementary variable method we obtain the probability generating function for the number of customers and the average number of customers in the orbit. Furthermore, wecarry out the waiting time distribution and some special cases of interest are discussed. Finally, some numerical results are presented.*

***MSC: 60K25,60K30***

***Key Words:*** Retrial queues; Working vacation; Supplementary variable method

## *1. Introduction*

*If the server is found to be busy, arriving , customers join a trial queue (called orbit),retry for service after some random amount of time. In telephone switching system we do have this type of applications and hence the last two decades retrial queue have been investigated extensively, Moreover, retrial queues are also used as mathematical models for several computer systems: packet switching networks, shared bus local area networks operating under the carriers-sense multiple access protocol and collision avoidance star local area networks etc. A large number of researchers working in various fields have analyzed retrial queues. For a detailed review of literature on retrial queues one may refer Falin and Templeton (1979),Gomez-corral(1999), Renganathan et al.(2002),and Kalyanaraman and Srinivasan(2003,2004).Recently, queueing system with vacations have been studied extensively, along with a comprehensive and excellent study on the vacation models ,including some applications such as production inventory system, communication systems, and computer systems As we known, there are mainly two vacation policies: classical vacation policy (also called ordinary vacation) and working vacation policy. The characteristic of a working vacation is that the server serves customers at a lower service rate during the vacation period, but in the case of classical vacation, the server stops the service completely during the vacation period.*

*In the literature of queueing systems with vacation has been discussed through a considerable amount of work in the recent past. Doshi (1990) has recorded prior work on vacation models and their applications in his survey paper. In recent year few authors who were concentrated on vacation queues are Madan and Gautam Choudhury (2005), Kalyanaraman and Pazhani Bala Murugan (2008) and Thangaraj and Vanitha(2010).*

*In this paper we study an Non-Markovian retrial queue with Multiple working vacation. The organization of the paper is as follows. In section 2 we describe the model. In section 3, we obtain the steady state probability generating function. Particular cases are discussed in section4. Some performance measures are obtained in section 5 and in section 6 Numerical study is presented.*

## 2. Model Description

*We consider M/G/1 queueing system where the primary customers arrive according to Poisson with arrival rate* $\lambda(>0)$,$S_k$ *be the probability that 'k' customers arrive in batch and X(Z) be its probability generating function. Let S(.),V(.),L(,) be the cumulative distributions of the service time, vacation time and Lockdown time, respectively. Let s(x),v(x),L(X) be the probability density functions of service time, vacation time and lockdown time, respectively.* $S^0(t)$ *denotes the remaining service time of batch in bulk service at an arbitrary time t.* $V^0(t)$ *and* $L^0(t)$ *denotethe remaining vacat5ion time of a server and lock down time of a server at an arbitrary time t, respectively. Let* $S^*$, $V^*$ *and* $L^*$ *denote the Laplace-Stiltjes transforms of S,V and L respectively.* $N_q(t)$ *and* $N_S(t)$ *are the number if customers in the queue and under service respectively, at time t.*

*We define the different states of the server at time 't':*

$$Y(t) = \begin{cases} 0, \text{ if the server is busy with bulk service} \\ 1, \text{ if the server is doing lockdown job} \\ 2, \text{ if server is on vacation} \end{cases}$$

*and define N(t) = j , if the server is on* $j^{th}$ *vacation starting from the idle period.*

*We define the following limiting probabilities:*

$P_{i\,j}(x,t)dt = P\{ N_s(t) = i,\ N_q(t) = j,\ x \le S^0(t) \le x+dt, Y(t)=0\},\ a \le i \le b, j \ge 0$

$L_n(x,t)dt = P\{ N_q(t) = n,\ x \le L^0(t) \le x+dt, Y(t)=1 \},\ n \ge 0$ *and*

$Q_{j\,n}(x,t)dt = P\{ N_q(t) = n,\ x \le V^0(t) \le x+dt, Y(t)=2, N(t)=j \},\ n \ge 0, j \ge 1$

*We define the Laplace Stieltjes Transform and the probability generating function as follows,*

$$P^*_{i\,n}(\theta) = \int_0^\infty e^{-\theta x} P_{i\,n}(x)dx \ ;$$

$$Q^*_{j\,n}(\theta) = \int_0^\infty e^{-\theta x} Q_{j\,n}(x)dx \ ;$$

$$L^*_n(\theta) = \int_0^\infty e^{-\theta x} L_n(x)dx \ ;$$

### 3. The Orbit Size Distribution for Multiple working vacation and Lock down:

$$-\frac{d}{dx}P_{i\,0}(x) = -\lambda P_{i\,o}(x) + \sum_{m=a}^{b} P_{mi}(0)s(x) + \sum_{1=1}^{\infty} Q_{1\,0}(0)S(x)$$

$$a \le i \le b \qquad (1)$$

$$-\frac{d}{dx}P_{i\,j}(x) = -\lambda P_{i\,j}(x) + \sum_{k=1}^{j} P_{i\,j-k}(x)\lambda s_k$$

$$a \le i \le b-1, j \ge 1 \qquad (2)$$

$$-\frac{d}{dx}P_{b\,j}(x) = -\lambda P_{b\,j}(x) + \sum_{m=a}^{b} P_{m\,bj}(0)s(x) + \sum_{k=1}^{\infty} P_{b\,j-k}(x)\lambda S_k + \sum_{1=1}^{\infty} Q_{1\,b+j}(0)S(x),$$

$$j \ge 1 \qquad (3)$$

$$-\frac{d}{dx}L_0(x) = -\lambda L_0(x) + \sum_{m=a}^{b} P_{m\,0}(0)L(x) \qquad (4)$$

$$-\frac{d}{dx}L_n(x) = -\lambda L_n(x) + \sum_{m=a}^{b} P_{m\,n}(0)L(x) + \sum_{l=1}^{\infty} L_{n-k}(x)\lambda S_k$$

$$1 \le n \le a-1 \qquad (5)$$

$$-\frac{d}{dx}L_n(x) = -\lambda L_n(x) + \sum_{k=1}^{n} L_{n-k}(x)\lambda s_k$$

$$n \ge a \qquad (6)$$

$$-\frac{d}{dx}Q_{1\,0}(x) = -\lambda Q_{1\,o}(x) + L_0(0)V(x) \qquad (7)$$

$$-\frac{d}{dx}Q_{1\,0}(x) = -\lambda Q_{1\,o}(x) + L_0(0)V(x) + \sum_{k=1}^{n} Q_{1\,n-k}(x)\lambda s_k$$

$$n \ge 1 \; (8)$$

$$-\frac{d}{dx}Q_{j\,0}(x) = -\lambda Q_{j\,o}(x) + Q_{j-1\,0}(0)V(x)$$

$$j \ge 2 \; (9)$$

$$-\frac{d}{dx}Q_{j\,n}(x) = -\lambda Q_{j\,n}(x) + Q_{j-1\,n}(0)V(x) + \sum_{k=1}^{n} Q_{j\,n-k}(x)\lambda s_k$$

$$j \geq 2\,, 1 \leq n \leq a-1 \quad (10)$$

$$-\frac{d}{dx} Q_{j\,n}(x) = -\lambda Q_{j\,n}(x) + \sum_{k=1}^{n} Q_{j\,n-k}(x)\lambda s_k$$

$$j \geq 2\,, n \geq a \quad (11)$$

Now ,taking *Laplace Stieltjes Transforms on both side of the equation(1) to (11) We get*

$$\theta P^*_{i\,0}(\theta) - P_{i\,o}(0) = \lambda P^*_{i\,0}(\theta) - \sum_{m=a}^{b} P_{m\,i}(0)\, S^*(\theta) - \sum_{1=1}^{\infty} Q_{1\,0}(0) S^*(\theta)$$

$$a \leq i \leq b \quad (12)$$

$$\theta P^*_{i\,j}(\theta) - P_{i\,j}(0) = \lambda P^*_{i\,j}(\theta) - \lambda \sum_{k=1}^{j} P^*_{i\,j-k}(\theta) S_k$$

$$a \leq i \leq b-1,\ j \geq 1 \quad (13)$$

$$\theta P^*_{b\,j}(\theta) - P_{b\,j}(0)$$

$$= \lambda P^*_{b\,j}(\theta) - \left[\sum_{m=a}^{b} P_{m\,b+j}(0) + \sum_{1=1}^{\infty} Q_{1\,b+j}(0)\right] S^*(\theta) - \lambda \sum_{K=1}^{j} P^*_{i\,j-k}(\theta) S_k$$

$$j \geq 1 \quad (14)$$

$$\theta L^*_0(\theta) - L_0(0) = \lambda L^*_0(\theta) - \sum_{m=a}^{b} P_{m\,o}(0)\, L^*(\theta) \quad (15)$$

$$\theta L^*_n(\theta) - L_n(0) = \lambda L^*_n(\theta) - \sum_{m=a}^{b} P_{m\,n}(0)\, L^*(\theta) - \lambda \sum_{K=1}^{n} L^*_{n-k}(\theta)\, S_k \quad (16)$$

$$\theta L^*_n(\theta) - L_n(0) = \lambda L^*_n(\theta) - \lambda \sum_{K=1}^{n} L^*_{n-k}(\theta)\, S_k \quad (17)$$

$$\theta Q^*_{1\,0}(\theta) - Q_{1\,0}(0) = \lambda Q^*_{1\,0}(\theta) - \lambda \sum_{K=1}^{n} L^*_0(0) V^*(\theta) \quad (18)$$

$n$

$$\theta Q^{*}_{1\,n}(\theta) - Q_{1\,n}(0) = \lambda Q^{*}_{1\,n}(\theta) - L_n(0)V^{*}(\theta) - \lambda \sum_{k=1} Q^{*}_{1\,n-k}(\theta) s_k$$

$n \geq 1$

$$\theta Q^*_{j\,0}(\theta) - Q_{j\,0}(0) = \lambda Q^*_{j\,0}(\theta) - Q_{j-1\,0}(0)V^*(\theta) \quad (19)$$

$j \geq 2$ (20)

$$\theta Q^*_{j\,n}(\theta) - Q_{j\,n}(0) = \lambda Q^*_{j\,n}(\theta) - Q_{j-1\,n}(0)V^*(\theta) - \lambda \sum_{K=1}^{n} Q^*_{j\,n-k}(\theta)s_k$$

$j \geq 2, 1 \leq n \leq a-1$ (21)

$$\theta Q^*_{j\,n}(\theta) - Q_{j\,n}(0) = \lambda Q^*_{j\,n}(\theta) - \lambda \sum_{k=1}^{n} Q^*_{j\,n-k}(\theta)s_k$$

$j \geq 2, n \geq a$ (22)

$$P^*_i(z,\theta) = \sum_{n=0}^{\infty} P^*_{i\,n}(\theta)z^n; \;\; P_i(z,0) = \sum_{n=0}^{\infty} P_{i\,n}(0)z^n$$

$a \leq i \leq b$

$$Q^*_j(z,\theta) = \sum_{n=0}^{\infty} Q^*_{j\,n}(\theta)z^n; \;\; Q_j(z,\theta) = \sum_{n=0}^{\infty} Q_{j\,n}(0)z^n$$

$j \geq 2$

$$L^*(z,\theta) = \sum_{n=0}^{\infty} L^*_n(\theta)z^n; \;\; L(z,\theta) = \sum_{n=0}^{\infty} L_n(0)z^n$$

(23)

*Multiplying 18 by $z^0$ and equation 19 with $z^n$ summed over n from 0 to ∞*

*Using 23 we get,*

$$(\theta - \lambda + \lambda x(z))Q^*_1(z,\theta) = Q_1(z,0) - L(z,0)V^*(\theta) \quad (24)$$

*Multiplying 20 by $z^0$ and equation 21 with $z^n$ summed over n from 0 to ∞ Using 23 we get,*

$$(\theta - \lambda + \lambda x(z))Q^*_j(z,\theta) = Q_j(z,0) - V^* \sum_{n=0}^{a-1} Q_{j-1\,n}(0)z^n$$

$j \geq 2$ *(25)*

*Multiplying 15 by* $z^0$ *and equation 16 with* $z^n$ *summed over n from 0 to* $\infty$

*Using 23 we get,*

$$L^*(z,\theta) = L(z,0) - L^*(\theta)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{mn}(0)\, z^n \tag{26}$$

*Multiplying 12 by $z^0$ and equation 13 with $z^j$ summed over n from 0 to ∞*

*Using 23 we get,*

$$(\theta - \lambda + \lambda x(z))P^*(z,\theta) = P_i(z,0) - S^*(\theta)\,[\sum_{m=a}^{b} P_{m\,i}(0) + \sum_{i=0}^{\infty} Q_{1\,i}\,(0)]$$

$$a \le i \le b-1 \tag{27}$$

*Multiplying 12 by $z^0$ and equation 14 with $z^n$ summed over n from 0 to ∞*

*Using 23 we get,*

$$Z^b(\theta - \lambda) + \lambda x(z))P_b^*(z,\theta)$$

$$= z^b P_b(z\,,0) - S^*(\theta)\,[\sum_{m=a}^{b} (P_m(z,0) - \sum_{j=0}^{b-1} P_{m\,j}(0)z^j)]$$

$$+ S^*(\theta)\,[\sum_{1=1}^{\infty} Q_1(z\,,0) - \sum_{j=0}^{b-1} Q_{1j}\,(0)\,z^j] \tag{28}$$

*By substituting $\theta = \lambda - \lambda X(z)$ in the equation 24 to 28 we get,*

$$Q_1(z\,,0) = V^*(\lambda - \lambda X(z))L(z\,,0) \tag{29}$$

$$Q_j(z\,,0) = V^*(\lambda - \lambda X(z))\sum_{n=0}^{a-1} Q_{j-1\,n}(0)\, z^n \quad j \ge 2 \tag{30}$$

$$L(z\,,0) = L^{*}(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m\,i}(0) z^{n}$$

*(31)*

$$P_{i}\ (\ z\,,0) = S^{*}(\lambda - \lambda X(z))\,[\sum_{m=a}^{b} P_{m\,i}(0) + \sum_{1=1}^{\infty} Q_{1\ i}(0)]$$

$$a \le i \le b-1 \qquad (32)$$

*And*

$$Z^{b}P_{b}(z\,,0) = S^{*}(\lambda - \lambda X(z))\,\{[\sum_{m=a}^{b-1} P_{m}(z\,,0) + P_{b}(z\,,0) - \sum_{j=0}^{b-1}\sum_{m=a}^{b} P_{m\,j}\,(0)z^{n}]$$

$$+ [\sum_{1=1}^{\infty} (Q_{1}(z\,,0) - \sum_{j=0}^{b-1} Q_{1\,j}\,(0)z^{j})]$$

$$Z^{b}P_{b}\,(z\,,0) = S^{*}(\lambda - \lambda X(z))\,\{\,[\sum_{m=a}^{b-1} P_{m}(z\,,0) + P_{b}(z\,,0) - \sum_{j=0}^{b-1}\sum_{m=a}^{b} P_{m\,j}\,(0)z^{n}]$$

$$+ [\sum_{1=1}^{\infty} (Q_{1}(z\,,0) - \sum_{j=0}^{b-1} Q_{1\,j}\,(0)z^{j})]\,\}$$

*(33)*

*Now Solving for $P_{b}(Z\,,0)$ in 33*

$$(Z^{b} - S^{*}(\lambda - \lambda X(z)))\,P_{b}(z\,,0)$$

$$= S^{*}(\lambda - \lambda X(z))\,\{[\sum_{m=a}^{b-1} P_{m}(z\,,0) - \sum_{m=a}^{b}\sum_{j=0}^{b-1} P_{m\,j}(0)z^{j}]$$

$$+ [\sum_{1=1}^{\infty} Q_{1}\,(z\,,0) - \sum_{1=1}^{\infty}\sum_{j=0}^{b-1} Q_{1\ j}\,(0)z^{j}]\} \qquad (34)$$

*From the equation 34 we get*

$$P_{b}(Z\,,0) = \frac{S^{*}((\lambda - \lambda X(Z))f(Z))}{(Z^{b} - S^{*}((\lambda - \lambda X(Z))))} \qquad (35)$$

*Where*

$$f(z) = \sum_{m=a}^{b-1} P_m(z,0) - \sum_{j=0}^{b-1} \sum_{m=a}^{b} P_{m\,j}(0) z^j + \sum_{1=1}^{\infty} \left(Q_1(z,0) - \sum_{j=0}^{b-1} Q_{1,1}(0) z^j\right)$$

*Substituting the expression for* $P_m(z,0), (a \le m \le b-1)$ *from 32*

$Q_1(z,0)$ *from 29 and* $Q_j(z,0), j \ge 2$ *from 30 in* $f(Z)$

*We get,*

$$f(Z) = S^*(\lambda - \lambda X(z)) \sum_{i=a}^{b-1} \left[\sum_{m=a}^{b} P_{m\,i}(0) + \sum_{1=1}^{\infty} Q_{1,i}(0)\right]$$

$$- \sum_{j=0}^{b-1} \sum_{m=a}^{b} P_{m\,j}(0) z^j + [V^*(\lambda - \lambda X(z)) L^*(\lambda - \lambda X(z))] \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m\,n}(0) z^n$$

$$+ V^*(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{j=2}^{\infty} Q_{j-1\ n}(0) z^n$$

*From the equation 24 and 29 we get,*

$$Q_1^*(z,\theta) = \frac{V^*(\lambda - \lambda X(z)) - V^*(\theta) L^*((\lambda - \lambda X(z))) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m\,n}(0) z^n}{(\theta - \lambda + \lambda X(z))} \qquad (36)$$

*Similarly from the equations 25 and 30 we get,*

$$Q_j^*(z,\theta) = \frac{(V^*(\lambda - \lambda X(z) -) V^*(\theta) \sum_{n=0}^{a-1} Q_{j-1\ n}(0)\ z^n}{(\theta - \lambda + \lambda X(z))}$$

$$j \ge 2 \qquad (37)$$

*Similarly from the equations 26 and 31 we get*

$$L^*(z,\theta) = \frac{(L^*(\lambda - \lambda X(z)) - L^*(\theta)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}(0) z^n}{(\theta - \lambda + \lambda X(z))}$$

$$a \le i \le b-1 \qquad (38)$$

*from the equations 27 and 32 we get*

$$P_i^*(z,\theta) = \frac{(S^*(\lambda - \lambda X(z))) - S^*(\theta)(\sum_{m=a}^{b} P_{mi}(0) + \sum_{1=1}^{\infty} Q_{1i}(0))}{(\theta - \lambda + \lambda X(z))}$$

$$a \le i \le b-1 \qquad (39)$$

*From the equation 28 and 35, We get*

$$P_b^*(z,0) = \frac{(S^*(\lambda - \lambda X(z)) - S^{*(\theta)})f(z)}{(\theta - \lambda - \lambda X(z))\,(Z^b - S^*(\lambda - \lambda X(z)))} \qquad (40)$$

*Let $P(Z)$be the probability generating function of the queue size at an arbitrary time epoch,*

*Then,*

$$P(Z) = \sum_{m=a}^{b-1} P_m^*(z,0) + P_b^*(Z,0) + +L^*(z,0) + \sum_{1=1}^{\infty} Q^*_{1(Z,0)} \qquad (41)$$

*Using the equation 36 to 40 in $P(Z)$ with We get,*

$$P(Z) = \frac{(S^*(\lambda - \lambda X(z)) - 1)\sum_{j=a}^{b-1}(\sum_{m=a}^{b} P_{mi}(0) + \sum_{1=1}^{\infty} Q_{1i}(0))}{(-\lambda + \lambda X(z))}$$
$$+ \frac{[S^*(-\lambda + \lambda X(z)) - 1]f(z)}{(-\lambda + \lambda X(z))\,(Z^b - S^*(-\lambda + \lambda X(z)))}$$
$$+ \frac{[L^*((\lambda - \lambda X(z)) - 1]\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{mn}(0)z^n}{(-\lambda + \lambda X(z))}$$
$$+ \frac{(V^*(\lambda - \lambda X(z) - 1)L^*(\lambda - \lambda X(z)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{mn}(0)z^n}{(-\lambda + \lambda X(z))}$$
$$+ \frac{(V^*(\lambda - \lambda X(z) - 1)\sum_{n=0}^{a-1}\sum_{j=1}^{\infty} Q_{jn}(0)z^n}{(-\lambda + \lambda X(z))} \qquad (42)$$

*Let ,*

$$P_i = \sum_{m=a}^{b} P_{mi}(0), \quad q_i = \sum_{1=1}^{\infty} Q_{1i}(0), \text{ and } L_i = P_i + q_i \qquad (43)$$

*Simplifying the equation 42 by using 43*

*We obtain ,*

$$P(Z) = \frac{\begin{array}{c}(S^*(\lambda - \lambda X(z)) - 1)\sum_{j=a}^{b-1}(z - z^i)\, L_i \\ +(z^b - 1)(V^*(\lambda - \lambda X(z)) - 1)\sum_{i=0}^{q} q_i z^i \\ +(S^*(\lambda - \lambda X(z)) - 1)\,(1 - L^*(\lambda - \lambda X(z))) \\ +V^*(\lambda - \lambda X(z))\sum_{i=0}^{a-1} p_i\, z^i \\ (Z^b - 1)\,(V^*(\lambda - \lambda X(z)))\, L^*(\lambda - \lambda X(z) - 1)\sum_{i=0}^{a-1} p_i\, z_i\end{array}}{(-\lambda + \lambda X(z))\,(z^b - S^*(-\lambda + \lambda X(z)))} \qquad (44)$$

*The Probability generating function* $P(z)$ *has to Satisfy the condition* $P(1) = 1$*,*

*In order to satisfy the condition applying L'Hospital's rule and Evaluating* $\lim_{Z\to 1} P(z)$

*And equating the expression to 1,*

*We have to satisfy*

$$E(S)\sum_{m=a}^{b-1}(b - i)P_i + b(E(L) + E(V))\sum_{i=0}^{a-1} P_i + bE(V)\sum_{i=0}^{a-1} q_i$$

$$-\lambda(E(L)E(X)E(S))\sum_{i=0}^{a-1} P_i = b - \lambda E\,(X)E(S) \qquad (45)$$

*Since* $p_i$ *and* $q_i$ *are probabilities of 'i' customers being in the queue at service completion epoch and vacation completion epoch, respectively, it follows that the left hand side of the above expression must be positive.*

*Thus* $P(1) = 1$ *is satisfied if and only if* $b - \lambda E(X)E(S) > 0$

$$define\ \rho\ as\ \frac{-\lambda E(X)E(S)}{b},$$

*Thus* $\rho < 1$*Is the condition to be satisfied for the existence of steady state model for the under consideration.*

### *4.1 Expected Length of Busy Period:*

*Let B be the busy Period random variable,*

*We define the another random variable 'M' as*

$$M = \begin{cases} \text{if the server finds } < 'a' \text{ customers after the first service} \\ \text{if the server finds } 'a' \text{ customers after the first service} \end{cases}$$

*Now, expected length of busy period $E(B)$is given by*

$$E(B) = E\left(\frac{B}{M} = 0\right) P(M = 0) + E\left(\frac{B}{M} = 1\right) P(M = 1)$$

$$= E(S)P(M = 0) + [E(S) + E(B)]P(M = 1),$$

*Where $E(S)$ is the expected service time.*

*Solving for*

$$E(B) = \frac{E(S)}{P(M = 0)} = \frac{E(S)}{\sum_{I=0}^{a-1} P_i} \qquad (46)$$

### 4.2 Expected Length of Idle Period:

*If I be idle period the random variable, then the expected length of idle period is given by,*

*$E(I) = E(I_i) + E(L)$ where $I_1$ is the random variable denoting the 'Idle period due to multiple vacation process',$E(L)$ is the expected lockdown time.*

### 5.1 Expected Queue Length:

*The expected queue length $E(Q)$ at arbitrary time epoch is obtained by differentiating $P(z)$ at $z = 1$ and is given by*

$$E(Q) = \sum_{n=0}^{\infty} np_n = P'(1)$$

### 5.2 Expected Waiting Time :

*The expected waiting time is obtained by using the Little's formula*

$$E(W) = \frac{E(Q)}{\lambda E(X)}$$

*Where $E(Q)$ is expected queue length*

### 6.Numerical Example:

*A numerical model is analysed with the following assumptions:*

*(i) Service time distribution is k-Erlang with k=2 and service rate $\mu$*

(ii) *Batch size distribution of the arrival is geometric with mean = 2*
(iii) *Vacation time is exponential with parameter* $\alpha = 10$
(iv) *Lock down time is exponential with parameter* $\beta = 7$
(v) *Minimum service capacity* $a = 3$
(vi) *Maximum service capacity* $b = 10$
(vii) *Traffic intensity*

$$\boldsymbol{\rho} = \frac{\mathbf{2\lambda k}}{\boldsymbol{b\mu}}$$

*Since k=2 and b=10* $z^b - S^*(\lambda - \lambda X(z))$ *will become polynomial of degree twelve and it will have 9 roots outside and one on the unit circle* $|z| = 1$. *The zeros of the function* $z^b - S^*(\lambda - \lambda X(z))$*are found by using MATLAB and using the same, the simultaneously equations are solved.*

*The expected queue length* $E(Q)$,*expected length of idle period* $E(I)$ *and expected length of busy period* $E(B)$ *and expected waiting times* $E(W)$ *are computed and tabulated as detailed below.*

*In table (a),the results of performance measures of the queueing system are presented for the service rate 2.0 and the arrival rate ranging from 2.0 to 4.5*

*The following observations are made:*

(i) *Expected queue length increases, as arrival rate increases,*
(ii) *Expected queue length decreases ,as service rate increases for a particular arrival rate(considering all the tables together).*

*Table (a):*

*Arrival Rate versus Performance Measures for* $\mu = 2.0$

| $\lambda$ | $\rho$ | $E(Q)$ | $E(B)$ | $E(I)$ | $E(W)$ |
|---|---|---|---|---|---|
| 2.0 | 0.4 | 2.5032 | 2.9640 | 0.1364 | 0.6258 |
| 2.5 | 0.5 | 3.7087 | 3.3116 | 0.1288 | 0.7417 |
| 3.0 | 0.6 | 5.5215 | 3.9815 | 0.1210 | 0.9202 |
| 3.5 | 0.7 | 8.5602 | 5.2090 | 01142 | 1.2229 |
| 4.0 | 0.8 | 14.6847 | 7.7676 | 0.1084 | 1.8356 |
| 4.5 | 0.9 | 33.1938 | 15.5832 | 0.1038 | 3.6882 |

*In table (b),the results of performance measures of the queueing system are presented for the service rate 2.5 and the arrival rate ranging from 2.0 to 6.0*

*The following observations are made:*

(i) *Expected queue length increases, as arrival rate increases,*

(ii) *Expected queue length decreases ,as service rate increases for a particular arrival rate(considering all the tables together).*

*Table (b):*

*Arrival Rate versus Performance Measures for* $\mu = 2.5$

| $\lambda$ | $\rho$ | $E(Q)$ | $E(B)$ | $E(I)$ | $E(W)$ |
|---|---|---|---|---|---|
| 2.0 | 0.32 | 1.7407 | 1.8425 | 0.1531 | 0.4352 |
| 2.5 | 0.40 | 2.4089 | 1.8869 | 0.1464 | 0.4818 |
| 3.0 | 0.48 | 3.2834 | 2.0399 | 0.1378 | 0.5472 |
| 3.5 | 0.56 | 4.4844 | 2.3118 | 0.1293 | 0.6406 |
| 4.0 | 0.64 | 6.2391 | 2.7524 | 0.1217 | 0.7799 |
| 4.5 | 0.72 | 9.0346 | 3.4863 | 0.1152 | 1.0038 |
| 5.0 | 0.80 | 14.1367 | 4.8481 | 0.1098 | 1.4137 |
| 5.5 | 0.88 | 26.1829 | 8.0770 | 0.1053 | 2.3803 |
| 6.0 | 0.96 | 86.9028 | 24.3414 | 0.1016 | 7.2419 |

*The numerical results are also Presented.*

*Table in (c),(d)and (e) from the following parameters:*

(i) *Vacation rate* $\alpha = 10$
(ii) *Arrival rate* $\lambda = 2$
(iii) *Minimum Service capacity a= 3*
(iv) *Maximum service capacity b=10*

*Lockdown rate ranging from 2 to 10*

*From these tables, the following observations are made:*

(i) *Expected queue length increases, as lockdown rate increases*
(ii) *Expected queue length decreases, When the service rate increases for a particular lockdown rate .*

*Table (C):*

*Lockdown Rate versus Performance Measures for* $\mu = 1.5$

| β | E(Q) | E(B) | E(I) | E(W) |
|---|---|---|---|---|
| 2 | 3.0388 | 5.3609 | 0.1169 | 0.7597 |
| 3 | 3.7096 | 5.7111 | 0.1183 | 0.9274 |
| 4 | 4.0262 | 5.9443 | 0.1190 | 1.0066 |
| 5 | 4.2090 | 6.1072 | 0.1193 | 1.0523 |
| 6 | 4.3276 | 6.2266 | 0.1196 | 1.0819 |
| 7 | 4.4106 | 6.3177 | 0.1198 | 1.1027 |
| 8 | 4.4719 | 6.3893 | 0.1199 | 1.1180 |
| 9 | 4.5191 | 6.4471 | 0.1200 | 1.1298 |
| 10 | 4.5564 | 6.4947 | 0.1201 | 1.1391 |

*Table (d):*

*Lockdown Rate versus Performance Measures for* $\mu = 2.0$

| β | E(Q) | E(B) | E(I) | E(W) |
|---|---|---|---|---|
| 2 | 1.5581 | 2.7625 | 0.1285 | 0.3895 |
| 3 | 1.9857 | 2.7670 | 0.1321 | 0.4964 |
| 4 | 2.2097 | 2.8395 | 0.1340 | 0.5524 |
| 5 | 2.3461 | 2.8928 | 0.1352 | 0.5865 |
| 6 | 2.4376 | 2.9329 | 0.1359 | 0.6094 |
| 7 | 2.5032 | 2.9640 | 0.1364 | 0.6258 |
| 8 | 2.5524 | 2.9888 | 0.1368 | 0.6381 |
| 9 | 2.5097 | 3.0089 | 0.1371 | 0.6477 |
| 10 | 2.6213 | 3.0255 | 0.1374 | 0.6553 |

*Table (e):*

*Lockdown Rate versus Performance Measures for* $\mu = 2.5$

| β | E(Q) | E(B) | E(I) | E(W) |
|---|---|---|---|---|
| 2 | 1.0897 | 1.7364 | 0.1384 | 0.2724 |
| 3 | 1.3531 | 1.7618 | 0.1449 | 0.3383 |
| 4 | 1.5120 | 1.7892 | 0.1484 | 0.3780 |
| 5 | 1.6155 | 1.8113 | 0.1505 | 0.4039 |
| 6 | 1.6877 | 1.8287 | 0.1520 | 0.4219 |
| 7 | 1.7407 | 1.8425 | 0.1531 | 0.4352 |
| 8 | 1.7813 | 1.8536 | 0.1539 | 0.4453 |
| 9 | 1.8134 | 1.8628 | 0.1545 | 0.4533 |
| 10 | 1.8393 | 1.8704 | 0.1550 | 0.4598 |

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