Convolution Theorem for Kushare Transform and Applications in Convolution Type Volterra Integral Equations of First Kind

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Abstract: **In this paper we state and prove convolution theorem for Kushare integral transform and solve convolution type of Volterra integral equation of first kind.**

Keywords: **Convolution theorem, Volterra integral equations of first kind, Integral transforms, Kushare transform.**

1. Introduction

Recently, Integral transforms are one of the mostly used simple mathematical technique to obtain the solutions of advance problems of space, science, technology, engineering, commerce and economics. The important feature of these integral transform is to provide exact solution of of problem without lengthy calculations.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, in September 2021, Kushare and Patil [1] introduced Kushare transform for facilitating the process of solving differential equations in time domain. Further in October 2021 Khakale and Patil [2] introduced Soham transform. As researchers are introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform to solve the problems on Newton's law of Cooling. In April 2022 D.P. Patil , etal [4] used Kushare transform for solving the problems on growth and decay . In October 2021, D.P. Patil [5] used Sawi transform in Bessel functions. Further, Patil [6] used Sawi transform of error functions to evaluate improper integrals. Laplace transform and Shehu transforms are used to Patil [7] in chemical sciences. Patil [8] solved wave equation by using Sawi transform and its convolution theorem using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9]. Solution of wave equation is obtained by using double Laplace and double Sumudu transforms by D .P. Patil [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Laplace, Elzaki and Mahgoub transforms are compared and used for solving system of first order and first degree by Kushare and Patil [12] . D.P.Patil [13] used Aboodh and Mahgoub transform for solving boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu , Aboodh , Elazki and Mahagoub transform and used it for solving boundary value problems. Patil et al [16] used Emad-Sara transform for solving Volterra Integral equations of first kind. Futher Patil with Tile and Shinde [17] used transform for solving Volterra integral equations for first kind. Vispute, Jadhav and Patil [18] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [19] used general integral transform of error function for evaluating improper integrals. Dinkar Patil, Prerana Thakare and Prajakta Patil used double general integral transform for the solution of parabolic boundary value problems [20]. Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [21]. D. P. Patil et al [22] used Soham transform in Newton's law of cooling. Dinkar Patil et al [23] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling, D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation.

In this paper, we use state and prove convolution theorem for Kushare transform and use it to solve convolution type Volterra integral equations of first kind. Paper is organized as follows, In second section we state preliminary concepts required to solve Volterra integral equations of first kind by using Kushare transform. Convolution theorem is proved in third section. Fourth section is reserved for main result. Fifth section is for application of Kushare transform in Volterra integral equations of first kind.

2. Preliminaries

In this section we state some preliminary concepts required to solve the problems in Chemical sciences by using Kushare transform.

Definition: An integral transform said to be KUSHARE transform changes the characterized for capacity of outstanding request to think about capacities in the set A characterized by [1]

$A=\{f(t)/ \exists M, T_1,T_2 >0, |f(t)<Me^{\frac{|t|}{Tj}}, \text{if } t\epsilon(1)^j\times [0,\infty)\}$……(1)

and is defined as,

$K[f(t)]=s(v)=v\int\int_0^\infty f(t)e^{-tv^\propto}dt, >t\geq 0, T_1\leq v\leq T_2$….(2)

For a given function in set A, the constant M must be finite number, T_1, T_2 may be finite or infinite .

2.1. **Kushare transform of derivatives**:[1]

1] $k[f'(t)]=v^\propto s(v)-vf(0)$

2] $k[f''(t)]=v^{2\propto}s(v)-v^{\propto+1} f(0)- v f'(0)$

3] $k[f^n(t)]=v^{n\propto}s(v)-\sum_{k=0}^{n-1} v^{\propto(n-k-1)}f^k(0)$

2.2. **Kushare transform of some preliminary functions:[1]**

Sr. No.	Function	Kushare transform of f(t)=k[f(t)]
1.	1	$\frac{1}{v^{\alpha-1}}$
2.	t^n	$\frac{\sqrt{n+1}}{v^{\alpha(n+1)-1}}$
3.	e^{at}	$\frac{v}{v^{\alpha}}$
4.	sinat	$\frac{av}{v^{2\alpha}+a^2}$
5.	cosat	$\frac{v^{\alpha+1}}{v^{2\alpha}+a^2}$

3. Convolution theorem

If $F(V)$ and $G(V)$ are Kushare Transform of $f(t)$ and $g(t)$ respectively. Then $\mathrm{K}[\mathrm{f(t)} * \mathrm{g(t)}] = \frac{1}{v} \mathrm{K}[\mathrm{f(t)}].\mathrm{K}[\mathrm{g(t)}]$.
Where * denotes convolution product.

Proof:

$$K[f(t)] = F(V)$$
$$k[G(T)] = G(V)$$

$$\mathrm{F(v).G(v)} = V\int_0^\infty e^{-tv^\alpha} f(t)dt.\mathrm{v}\int_0^\infty e^{-uv^\alpha} g(u)du$$
$$= v^2 \int_0^\infty \int_0^\infty e^{-(t+u)v^\alpha} f(t)g(u)dtdu$$

Let, $\mathrm{t} + \mathrm{u} = \mathrm{z}$

$\mathrm{dt} = \mathrm{dz}$

$$\mathrm{F(v).G(v)} = v^2 \int_0^\infty \int_0^\infty e^{-zv^\alpha} f(z-u)g(u)dz\,du$$

where, region of integration is R which is unbounded region. Changing the order of integration.

$$\mathrm{F(v).G(v)} = v^2 \int_0^\infty \int_0^t e^{-zv^\alpha} f(z-u)g(u)dudz$$
$$= v^2 \int_0^\infty e^{-zv^\alpha} (\int_0^t f(z-u)g(u)du)dz$$
$$\mathrm{F(v).G(v)} = \mathrm{v}\{\mathrm{v}\int_0^\infty e^{-zv^\alpha}(\int_0^t f(z-u)g(u)du)dz\}$$
$$= \mathrm{v.K}[\int_0^t f(z-u)g(u)du]$$
$$F(v).G(v) = V.K[f(t) * g(t)]$$
$$\frac{1}{v}F(v).G(v) = V.K[f(t) * g(t)]$$
$$K[f(t) * g(t)] = \frac{1}{v}K[f(t)].K[g(t)]$$

4. Main Result

Consider convolution type Volterra integral equation of first kind.

$$\mathrm{F(v)} = \int_0^x k(x-t)h(t)dt \qquad (1)$$

We apply Kushare transform on equation (1) and using convolution theorem to obtain,

$$F(v) = \frac{1}{v}K(v)H(v)$$
$$H(V) = \frac{vF(v)}{K(v)}$$

By taking inverse of kushare transform we obtain,

$$h(x) = \mathrm{K}^{-1}[\frac{vF(v)}{K(v)}]$$

similarly, consider convolution type of Volterra integral equation of second kind.

$$\mathrm{h(x)} = \mathrm{f(x)} + \lambda\int_0^x k(x-t)h(t)dt \qquad (2)$$

Again applying Kushare transform on above equation we obtain,

$$H(v) = F(v) + \lambda.\frac{1}{v}K(v).H(v)$$
$$H(v) - \lambda\frac{1}{v}\mathrm{K(v).H(v)} = \mathrm{F(v)}$$
$$H(v)[1 - \frac{\lambda}{v}K(v)] = \mathrm{F(v)}$$
$$H(v) = \frac{F(v)}{[1 - \frac{\lambda}{v}K(v)]}$$
$$H(v) = \frac{vF(v)}{v - \lambda K(v)}$$

Another problem of frequent interest in connection with (2) is the resolvent kernel that is, the determination of function $\Gamma(t)$ such that,

$$\mathrm{h(x)} = \mathrm{f(x)} + \lambda \int_0^x \Gamma(x-t) f(t) dt \qquad (3)$$

Now, above equation may be written as

$$\mathrm{H(v)} = F(v) + \frac{\lambda K(v)}{v-\lambda K(v)} \mathrm{F(v)}$$

So that Γ(t) is the inverse of Kushare transform of $\frac{\lambda K(v)}{v-\lambda K(v)}$.

Now we find solution of Abel`s integral equation using Kushare transform.

We write Abel`s integral equation in the form

$$\mathrm{F(t)} = \int_0^x \frac{g(x)}{(t-x)^{\alpha}} dx, 0 \le \alpha < 1 \qquad (4)$$

Above equation can be written as,

$$f(t) = \mathrm{g(t)} * t^{-\alpha}\mathrm{H(t)} \qquad (5)$$

where H(t) is Heaviside`s unit step function. Applying Kushare transform on equation (5) we obtain,

$$\mathrm{F(v)} = \frac{1}{\mathrm{v}} \mathrm{G(v)}\, \Gamma(1-\alpha) v^{-\alpha+2} \qquad (6)$$

$$G(v) = \frac{vF(v)}{\Gamma(1-\alpha)v^{-\alpha+2}}$$

$$G(v) = \frac{F(v)}{\Gamma(1-\alpha)} v.v^{\alpha-2}$$

$$G(V) = \frac{F(v)}{\Gamma(1-\alpha)} v^{\alpha-1}$$

$$G(v) = \frac{F(v)\Gamma(v)}{\Gamma(1-\alpha)} v^{\alpha-1}$$

$$v.G(v) = \frac{\sin(\pi\alpha)\Gamma(\alpha)}{\alpha} . \frac{1}{v} F(v) v^{\alpha-1}$$

$$v.G(v) = \frac{\sin(\pi\alpha)}{\alpha} \mathrm{K[f(t)} * t^{(\alpha-1)}]$$

$$K[\int_0^t g(x)dx] = \frac{sin(\pi\alpha)}{\alpha} K \int_0^t f(x)(t-x)^{\alpha-1} dx =$$

$$K[\int_0^t g(x)dx] = \frac{\sin(\pi\alpha)}{\alpha} \mathrm{K[w(t)]}$$

Where, $\mathrm{w(t)} = \int_0^t f(x)(t-x)^{(\alpha-1)} dx, \mathrm{w(0)} = 0$

We have,

$$\mathrm{K[w(t)]} == \frac{K(v)}{v}$$

Therefore,

$$vG(v) = \frac{\sin(\pi\alpha)}{\alpha} \mathrm{v.K[w(t)]}$$

$$K[g(t)] = \mathrm{K}[\frac{\sin(\pi\alpha)}{\alpha} \mathrm{w(t)}]$$

By uniqueness theorem, we obtain

$$g(t) = \frac{\sin(\pi\alpha)}{\alpha} \frac{d}{dt} [\int_0^t f(x)(t-x)^{(\alpha-1)} dx]$$

This is solution of Abel`s integral equation.

Application to Integral Equation:

In this section we illustrate some integral equations by using the kushare transform.

Ex.1) Solve the integral equation

$$\mathbf{t} = \int_0^t e^{t-x} g(x) dx$$

Solution: Apply Kushare transform we obtain

$$\frac{1}{v^{2\alpha-1}} = \frac{1}{v} \mathrm{K}\,[e^t].\mathrm{K}\,[\mathrm{g(t)}]$$

$$\mathrm{G(v)} = \frac{1}{v^{2\alpha-1}} \times \frac{v(v^{\alpha}-1)}{v} = \frac{v(v^{\alpha}-1)}{v(v^{2\alpha-1})} = \frac{v^{\alpha+1}-v}{v(v^{2\alpha-1})} = \frac{v^{\alpha+1}}{v(v^{2\alpha-1})} - \frac{v}{v(v^{2\alpha-1})}$$

$$= \frac{v^{\alpha}.v}{v(v^{2\alpha-1})} - \frac{1}{v^{2\alpha-1}} = \frac{v^{\alpha}}{v^{2\alpha}.v^{-1}} - \frac{1}{v^{2\alpha-1}} = \frac{1}{v^{\alpha-1}} - \frac{1}{v^{2\alpha-1}}$$

Here, $\mathrm{K[g(t)]} = \mathrm{G(v)}$

Applying inverse of Kushare transform we obtain

$$\mathrm{g(t)} = \mathrm{K}^{-1}(\mathrm{G(v)})$$

$$\mathrm{g(t)} = \mathrm{K}^{-1}\left(\frac{1}{v^{\alpha-1}}\right) - \mathrm{K}^{-1}\left(\frac{1}{v^{2\alpha-1}}\right)$$

$$\therefore \mathrm{g(t)} = 1 - \mathrm{t}$$

Ex.2) Solve the integral equation

$$\mathbf{Sin(2t)} = \int_0^t (\tau - t) h(t) d\tau$$

Solution: applying Kushare transform on above equation, we get

$$\frac{2v}{v^{2\alpha}+4} = \frac{1}{\mathrm{v}}\left(-\frac{1}{v^{2\alpha-1}}\right) H(v)$$

Here, $\mathrm{K[h(t)]} = \mathrm{H(v)}$

$$\mathrm{K[h(t)]} = -\frac{2v.v.v^{2\alpha-1}}{v^{2\alpha}+4} = \frac{-2v.v.v^{2\alpha}.v^{-1}}{v^{2\alpha}+4} = \frac{-2v^2 v^{2\alpha}}{v(v^{2\alpha}+4)} = \frac{-2v.v^{2\alpha}}{v^{2\alpha}+4}$$

$$= -2v[\frac{v^{2\alpha}+4-4}{v^{2\alpha}+4}] \quad = -2v\,[\frac{v^{2\alpha}+4}{v^{2\alpha}+4} - \frac{4}{v^{2\alpha}+4}] \quad = -2v\,[1 - \frac{4}{v^{2\alpha}+4}]$$

$$\mathrm{K}[\mathrm{h(t)}] = -2v + \frac{8v}{v^{2\alpha}+4} \qquad (7)$$

We know the property of Dirac-Delta Function

$$\mathrm{K}\,\{\delta(\mathrm{t}-\mathrm{a}).\mathrm{f(t)}\} = \mathrm{v}\int_0^\infty e^{-tv^\alpha} f(t)\delta(t-a)dt$$

$$\therefore \mathrm{v}\int_0^\infty f(t)\delta(t-a) = \mathrm{f(a)}$$

$$\mathrm{v}\int_0^\infty e^{-tv^\alpha}\delta(t-a)$$

$$v.\,[e^{-tv^\alpha}]_{t=a}$$

$$\mathrm{K}[\delta(\mathrm{t}-\mathrm{a})] = \mathrm{v}\,.\,[e^{-av^\alpha}]$$

But if we put a = 0 then,

$$\mathrm{K}[\delta(\mathrm{t})] = \mathrm{v}\;[e^0]$$

$$\therefore \mathrm{K}[\delta(\mathrm{t})] = \mathrm{v}$$

$$\therefore \delta(\mathrm{t}) = \mathrm{K}-1(\mathrm{v})$$

Applying inverse of Kushare transform on equation (7)

$$\mathrm{K}[\mathrm{h(t)}] = -2\,\mathrm{K}^{-1}(\mathrm{v}) = 4\,\mathrm{K}^{-1}\,[\frac{2v}{v^{2\alpha}+4}]$$

$$K[h(t)] = -2\delta(t) + 4\,sin2t$$

Ex.3) Find the function g(t) which satisfies the equation

$$\mathbf{G(t)} = \mathbf{t} + \int_0^t \boldsymbol{g(\tau)sin(t-\tau)d\tau}$$

Solution: Applying Kushare transform

$$\mathrm{G(v)} = \frac{1}{v^{2\alpha-1}} + \frac{1}{v}\frac{v}{v^{2\alpha}+1}G(v)$$

$$G(v) - \frac{v}{v(v^{2\alpha}+1)}G(v) = \frac{1}{v^{2\alpha-1}}$$

$$\mathrm{G(v)}\left[1 - \frac{v}{v(v^{2\alpha}+1)}\right] = \frac{1}{v^{2\alpha-1}}$$

$$\mathrm{G(v)} = \frac{(\frac{1}{v^{2\alpha-1}})}{(1-\frac{v}{v(v^{2\alpha}+1)})} \quad = \frac{(\frac{1}{v^{2\alpha-1}})}{(\frac{v(v^{2\alpha}+1)-v}{v(v^{2\alpha}+1)})} \quad = \frac{(\frac{1}{v^{2\alpha-1}})}{(\frac{v[v^{2\alpha+1-1}]}{v(v^{2\alpha}+1)})} \quad = \frac{(\frac{1}{v^{2\alpha-1}})}{(\frac{v^{2\alpha}}{v^{2\alpha}+1})}$$

$$= \frac{1}{v^{2\alpha-1}} \times \frac{v^{2\alpha}+1}{v^{2\alpha}} \quad = \frac{1}{v^{2\alpha-1}}\,[\frac{v^{2\alpha}}{v^{2\alpha}} + \frac{1}{v^{2\alpha}}] \quad = \frac{1}{v^{2\alpha-1}}[1 + \frac{1}{v^{2\alpha}}]$$

$$\mathrm{G(v)} = \frac{1}{v^{2\alpha-1}} + \frac{1}{v^{4\alpha-1}} \qquad (8)$$

We know, $K[t^n] = \frac{\Gamma(n+1)}{v^{\alpha n+\alpha-1}}$

$$\mathrm{K}\,[t^3] = \frac{\Gamma(3+1)}{v^{3\alpha+\alpha-1}}$$

$$\mathrm{K}\,[t^3] = \frac{\Gamma(4)}{v^{4\alpha-1}}$$

But we know,

$$\Gamma(\mathrm{n}) = (\mathrm{n}-1)\,!$$

$$\Gamma(4) = (4-1)! = 3!$$

$$\mathrm{K}[t^3] = \frac{3!}{v^{4\alpha-1}}$$

$$\mathrm{K}^{-1}\left(\frac{1}{v^{4\alpha-1}}\right) = \frac{t^3}{3!}$$

Put in equation (8)

$$\mathrm{G(v)} = \frac{1}{v^{2\alpha-1}} + \frac{1}{v^{4\alpha-1}}$$

Applying inverse of Kushare transform,

$$\mathrm{g(t} = K^{-1}(\frac{1}{v^{2\alpha-1}}) + K^{-1}(\frac{1}{v^{4\alpha-1}})$$

$$\mathrm{g(t)} = t + \frac{t^3}{3!}$$

Ex.4) Find the resolvent kernel Γ(t) of the following equation.

$$\boldsymbol{g(t) = f(t) + \lambda\int_0^t e^{(t-\tau)}\,g(\tau)d\tau}$$

Solution: let, $\Omega(v)$be the kushare transform of Γ(t) then,

$$\Omega(v) = \frac{\lambda}{v-\lambda.K[e^t]} \quad = \frac{(\lambda.\frac{v}{v^\alpha-1})}{(v-\lambda.\frac{v}{v^\alpha-1})} \quad = \frac{(\frac{\lambda v}{v^\alpha-1})}{(\frac{v(v^\alpha-1)-\lambda v}{v^\alpha-1})} \quad = \lambda.\frac{v}{v(v^\alpha-1)-\lambda v} \quad = \lambda.\frac{v}{v.v^\alpha-v-\lambda v} =$$

$$\lambda.\frac{v}{v(v^\alpha-(1+\lambda))} \quad = \frac{\lambda}{v^\alpha-(1+\lambda)}$$

Applying inverse of Kushare transform we obtain

$$\Gamma(t) = \lambda.e^{-(1+\lambda)t}$$

Given equation leads in this case

$$g(t) = f(t) + \lambda \int_0^t e^{-(t-\tau)(1+\lambda)} f(\tau)d\tau$$

Conclusion: We have applied Kushare transform for obtain the solution of the Volterra integral equations of first kind.

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