Approximate Solutions of Fractional order Mathematical Models Using Three Different Methods

[1]Pawar S. M., [2]Kale K. M.

[1,2]Research Scholar
P.G Department of Mathematics
N.E.S. Science College,
Nanded - 431602, (MH), India

Abstract: **In this paper, we using three different methods as FDTM (Fractional Differential Transfrom Method), FADM (Fractional Adomian Decomposition Method) and CFDTM (Conformable Fractional Differential Transfrom Method) are carried out for solving non-linear fractional Riccati differential equations. The fractional derivatives are described in the Caputo and Conformable sense. In these schemes, the solution takes the form of a convergent series with easily computable components. Furthermore, the fractional model solution generated by CFDTM is associated with the fractional model solution derived by FADM and FDTM for different fractional orders. Additionally, Python software is used to analyse the result numerically and graphically.**
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1. Introduction

During the last decades, the theory of fractional derivation has gained a lot of attention in the field of mathematics. There isn't a standard form for defining fractional derivative. However, the most generally used definitions found in [12] . Khalil et al. [9] came up with an innovative solution that extends the standard limit definition of the derivative of a function in order to solve some of these and other challenges. Using his novel definition of fractional derivative, he calculated several fractional derivative outcomes. In [4, 6, 7, 8] and [1], Khalil et al. provided a new concept of fractional derivative and demonstrated various findings utilising it. Almeida et al. developed the following limit definition of the derivative of a function in [2. Using his notion of fractional derivative, he also highlighted several key conclusions of fractional derivative. Katugampola [3] recently presented the concept of fractional derivative using his new definition. The major goal of this work is to present a limit definition of a function's derivative that obeys classical features such as linearity, product rule, quotient rule, and chain rule. [10, 11 for undefined and unexplained concepts and words. The differential transform method (DTM) is a numerical approach for solving differential equations. Zhou[17, firstly introduced the concept of DTM and by using this new DTM method he solved linear as well as nonlinear IVP in electrical science. Recently, a new analytical technique, named Fractional Differential Transform Method (FDTM), is developed to solve fractional differential equations (FDEs) which can be found in [18. FDTM forms fractional power series in the same way that DTM forms Taylor series. Many authors have done Studies by using Adomian decomposition method about solutions of different types of systems of fractional differential equations, which can be found in [14, 15, 27, 28.
In this paper, we present numerical and analytical solutions for the fractional Riccati differential equation

$$y^{(\alpha)} = A(t) + B(t)y + C(t)y^2,\ t > 0, n-1 < \alpha \le n$$

subject to the initial conditions
$y^k(0) = c_k,\ k = 0,1,\dots,n-1$, where $A(t), B(t)$ and $C(t)$ are given functions, $c_k, k = 0,1,\dots,n-1$, are arbitrary constants and a is a parameter describing the order of the fractional derivative. The general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses. In the case of $\alpha = 1$, the fractional equation reduces to the classical Riccati differential equation. The importance of this equation usually arises in the optimal control problems. The feed back gain of the linear quadratic optimal control depends on a solution of a Riccati differential equation which has to be found for the whole time horizon of the control process [14. The existing literature on fractional differential equations tends to focus on particular values for the order α. The value $\alpha = 1/2$ is especially popular. This is because in classical fractional calculus, many of the model equations developed used these particular orders of derivatives. In modern applications much more general values of the order a appear in the equations and therefore one needs to consider numerical and analytical methods to solve differential equations of arbitrary order.

2. Basic Ideas of the Fractional Differential Transform Mehtod (FDTM), Fractional Adomian Decomposition Mehtod (FADM) and Con CFDTM

In this part, we review several key conclusions from the FDTM and FADM, both of which are utilised to generate approximate analytical solutions for the in this work (1.1).

2.1. Basic ideas of the FDTM

Let the fractional power series of an analytical and continuous function $\varphi(\zeta)$ in Riemann-Liouville sense is as follows [19]:

$$\varphi(\zeta) = \sum_{k=0}^{\infty} \Phi(k)(\zeta - \zeta_0)^{k/\alpha}$$

where α and $\Phi(k)$ are the order of fraction and FDT of $\varphi(\zeta)$ respectively. Let fractional IVP, in terms of the Caputo sense are as fallows.

$$\Phi(k) = \begin{cases} \text{If } k/\alpha \in Z^+, \dfrac{1}{(k/\alpha)!}\left[\dfrac{\mathrm{d}^{k/\alpha}\varphi(\zeta)}{\mathrm{d}\zeta^{k/\alpha}}\right]_{\zeta=\zeta_0} & \text{for } k = 0,1,2,\dots,(q\alpha-1) \\ \text{If } k/\alpha \notin Z^+ & 0, \end{cases}$$

where, q denotes the order of the fractional differential equation under consideration. Now we recall some important theorems of FDTM which can be used to find an analytical solution of model of dengue.
Theorem 2.1 If $\varphi(\zeta) = \psi(\zeta) \pm w(\zeta)$, then $\Phi(k) = \Psi(k) \pm \omega(k)$
Theorem 2.2 If $\varphi(\zeta) = \psi(\zeta)w(\zeta)$, then $\Phi(k) = \sum_{l=0}^{k}\Psi(l)\omega(k-l)$.
Theorem 2.3 If $\varphi(\zeta) = \psi_1(\zeta)\psi_2(\zeta)\dots\psi_{n-1}(\zeta)\psi_n(\zeta)$, then

$$\Phi(k) = \sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\cdots\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}\Psi_1(k_1)\Psi_2(k_2-k_1)\dots\Psi_{n-1}(k_{n-1}-k_{n-2})\Psi_n(k-k_{n-1})$$

Theorem 2.4 If $\varphi(\zeta) = (\zeta-\zeta_0)^r$, then $\Phi(k) = \delta(k-\alpha r)$ where,

$$\delta(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

Theorem 2.5 If $\varphi(\zeta) = D_{\zeta_0}^{q}[\psi(\zeta)]$, then $\Phi(k) = \frac{\Gamma(q+1+k/\alpha)}{\Gamma(1+k/\alpha)}\Psi(k+\alpha q)$.

2.2. Basic ideas of the FADM

Now have a look at the fractional differential equation [20]

$$D^\alpha y(\zeta) = A(\zeta) + B(\zeta)y + C(\zeta)y^2,\ \zeta > 0, n-1 < \alpha \le n$$

After applying I^α to the equation 2.2.1, we obtain,

$$y = \sum_{k=0}^{n-1} c_k\frac{\zeta^k}{k!} + I^\alpha[A(\zeta) + B(\zeta)y + C(\zeta)y^2],\ 1 \leqslant i \leqslant n$$

We adopt ADM to solve the equation 2.2.1. Let

$$y = \sum_{m=0}^{\infty} y_m(\zeta)$$

and

$$N(y) = \sum_{m=0}^{\infty} A_m,$$

where A_m are the Adomian polynomials. By using equations. 2.2.3 and 2.2.4 the equation 2.2.2, can be written as,

$$\sum_{m=0}^{\infty} y_m = \sum_{k=0}^{n-1} c_k\frac{\zeta^k}{k!} + I^\alpha\sum_{m=0}^{\infty}\left[A(\zeta) + B(\zeta)\sum_{m=0}^{\infty} y_m + C(\zeta)\sum_{m=0}^{\infty} A_m\right]$$

This can be expressed as

$$y_0(\zeta) = \sum_{k=0}^{n-1} c_k\frac{\zeta^k}{k!} + I^\alpha(A(\zeta))$$

$$y_{m+1}(\zeta) = I^\alpha(B(\zeta)y_n + C(\zeta)A_m), m \ge 0$$

The shortened series can be used to approximate the answer y_i.

$$\varphi_k = \sum_{m=0}^{k-1} y_m,\ \lim_{k\to\infty}\varphi_k = y_i(\zeta)$$

However, in many cases the exact solution in a closed form may be obtained. Moreover, the decomposition series solutions are generally converge very rapidly. The convergence of the decomposition series has investigated by several authors [25,26].

2.3. Basic ideas of the Conformable fractional differential transform method

Definition 2.6 [1] If $\phi: [0,\infty) \to \mathbb{R}$ be a function and $\forall\alpha \in (0,1)$, then the conformable fractional derivative of ϕ of order α is defined as

$$D^\alpha(\phi)(t) = \lim_{\mu\to 0}\frac{\phi(t+\mu t^{1-\alpha}) - \phi(t)}{\mu},\ t > 0$$

If $D_\alpha(\phi(t))$ and $\lim_{\mu\to 0^+}\phi^{(\alpha)}(t)$ is exist in $(0,c)$, where $c > 0$, then α-derivative is defined as $\phi^{(\alpha)}(0) = \lim_{t\to 0^+}\phi^{(\alpha)}(t)$
Definition 2.7 [1] Let $\phi: [0,\infty) \to \mathbb{R}$ and $\alpha \in (n, n+1]$ be an n-differentiable at t, where $t > 0$. Then conformable fractional derivative of ϕ is defined as

$$D^{n\alpha}(\phi)(t) = \lim_{\mu\to 0}\frac{\phi^{\lceil\alpha\rceil-1}\left(t+\mu t^{\lceil\alpha\rceil-\alpha}\right) - \phi^{\lceil\alpha\rceil-\alpha}(t)}{\mu},\ n-1 < \alpha \le n, t > 0$$

where $n \in \mathbb{N}$ and $\lceil\alpha\rceil$ is the smallest integer number greater than or equal to α. Provided $D^{n\alpha}(\phi)(0) = \lim_{\mu\to 0}D^{n\alpha}(\phi)(t), \phi(t)$ is n-differentiable and $D^{n\alpha}(\phi)(0) = \lim_{\mu\to 0}D^{\alpha}(\phi)(t), \phi(t)$ exists.

Definition 2.8 [1] Let , $0 \le \gamma \le t$ and ϕ be a function defined on $(\gamma, t]$, then New α-fractional integral is defined by

$$I^{\alpha}\phi(t) = \int_0^t \phi(t)t^{\alpha-1}dt,\ 0 < \alpha \le 1$$

provided integral exists.
Remark 1.4. 2] The most useful result is that

$$D^{n\alpha}(\phi)(t) = t^{\lceil\alpha\rceil-\alpha}\phi^{\lceil\alpha\rceil}(t).$$

where $\alpha \in (n, n+1]$ and ϕ is an $(n+1)$-differentiable function at $t > 0$. Definition 2.9 [11] Let $\phi(t)$ is infinitely α-differentiable function for some $\alpha \in (0,1]$. Conformable fractional differential transform of $\phi(t)$ is defined as

$$\Phi_{\alpha}(t) = \frac{1}{\alpha^k k!}\left[\left(T_{\alpha}^{t_0}\phi\right)^{(k)}(t)\right]_{t_0}$$

where $\left(T_{\alpha}^{t_0}\phi\right)^{(k)}(t)$ denotes the application of the fractional derivative k times.
Definition 2.10 [11] Let $\Phi_{\alpha}(k)$ be the conformable fractional differential transform of $\phi(t)$. Inverse conformable fractional differential transform of $\Phi(k)$ is defined as

$$\phi(t) = \sum_{k=0}^{\infty} \Phi_{\alpha}(k)t^{\alpha k}.$$

CFDT of initial conditions for integer order derivaties are defined as

$$\Phi_{\alpha}(k) = \begin{cases} \dfrac{1}{(\alpha k)!}\left[\dfrac{d^{(\alpha k)}y(t)}{dt^{(\alpha k)}}\right]_{t=t_0} & \text{for } \alpha k \in \mathbb{Z}^+ \\ 0 & \text{for } \alpha k \notin \mathbb{Z}^+ \end{cases}$$

where $\Phi_{\alpha}(k)$ is the fractional differential transform of $\phi(t)$.
Some basic properties of the CFDTM are presented in [11. Let $y(t), x(t)$, and $z(t)$ be functions of time t and $Y_{\alpha}(k), X_{\alpha}(k)$, and $Z_{\alpha}(k)$ are their corresponding fractional differential transforms with order of fraction α. Then for constants c and d the followings hold,
Theorem 2.11 If $y(t) = cx(t) \pm dz(t)$, then $Y_{\alpha}(k) = cX_{\alpha}(k) \pm dZ_{\alpha}(k)$.
Theorem 2.12 If $y(t) = x(t)z(t)$, then $Y_{\alpha}(k) = \sum_{r=0}^{k} X_{\alpha}(r)Z_{\alpha}(k-r)$.
Theorem 2.13 If $y(t) = t^p$ then $Y_{\alpha}(k) = \delta\left(k - \frac{p}{\alpha}\right)$ where $\delta(k) = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases}$
Theorem 2.14 if $\phi(t) = T_{\alpha}^{t_0}(y(t))$, then $\Phi_{\alpha}(k) = \alpha(k+1)Y_{\alpha}(k+1)$.
Theorem 2.15 if $\phi(t) = T_{\alpha}^{t_0}(y(t))$, for $m < \alpha \le m+1$, then

$$\Phi_{\alpha}(k) = Y_{\alpha}\left(k + \frac{\alpha}{\beta}\right) = \frac{\Gamma(k\beta + \alpha + 1)}{\Gamma(k\beta + \alpha - m)}.$$

3. Applications
Example 3.1 We consider the fractional Riccati equation

$$y^{(\alpha)} = 1 - y^2$$

with the initial condition $y(0) = 0$. $y(t) = \frac{e^{2t^{\alpha}-1}}{e^{2\bar{\alpha}^{\alpha}+1}}$ is the exact solution of this equation.
To derive the solution of above FDE, we use the Adomian decomposition scheme:

$$y_0 = y(0) + I^{\alpha}(1) = \frac{1}{\Gamma(\alpha+1)}t^{\alpha}$$
$$y_{n+1} = -I^{\alpha}(A_n),\ n \ge 0$$

Using the above recursive relationship, the first few terms of the decomposition series are given by

$$y_0 = \frac{1}{\Gamma(\alpha+1)}t^{\alpha}$$
$$y_1 = I^{\alpha}(y_0^2) = -\frac{\Gamma(1+2\alpha)}{\alpha^2\Gamma(1+3\alpha)}t^{3\alpha}$$
$$y_2 = I^{\alpha}(2y_0y_1) = \frac{16\Gamma(2\alpha)\Gamma(4\alpha)}{\alpha\Gamma(1+3\alpha)\Gamma(1+5\alpha)}t^{5\alpha}$$
$$y_3 = I^{\alpha}(2y_0y_2 + y_1^2) = -\frac{(32\alpha^2\Gamma(2\alpha)\Gamma(4\alpha)\Gamma(1+3\alpha) + \Gamma(1+2\alpha)^2\Gamma(1+5\alpha))\Gamma(1+6\alpha)}{\alpha^4\Gamma(1+3\alpha)^2\Gamma(1+5\alpha)\Gamma(1+7\alpha)}t^{7\alpha}$$

The general form of the approximation $y(t)$ is given by

$$y(t) = \frac{1}{\Gamma(\alpha+1)}t^{\alpha} - \frac{\Gamma(1+2\alpha)}{\alpha^2\Gamma(1+3\alpha)}t^{3\alpha} + \frac{16\Gamma(2\alpha)\Gamma(4\alpha)}{\alpha\Gamma(1+3\alpha)\Gamma(1+5\alpha)}t^{5\alpha} \ldots$$

To derive the solution of above FDE, we use the Fractional Differential Transform Method scheme:
By using Theorems 2.4 and 2.5, Eq. (3.1) transforms to

$$Y(k+\alpha\theta) = \frac{\Gamma(1+k/\theta)}{\Gamma(\alpha+1+k/\theta)}\left[\delta(k) - \sum_{k_1}^{k} Y(k_1)Y(k-k_1)\right]$$

and using Eq. (2.1.2), initial conditions can be transformed as follows:

$$Y(k) = 0, \quad \text{for } k = 0,1,\dots,\alpha\theta - 1$$

Using Eqs. (3.2) and (3.3), Y(k) is obtained for different values of α and then using Eq. (2.1.2), y(x) is evaluated.

$$y(t) = \frac{1}{\Gamma(\alpha+1)}t^{\alpha} - \frac{\Gamma(1+2\alpha)}{\alpha^2\Gamma(1+3\alpha)}t^{3\alpha} + \frac{16\Gamma(2\alpha)\Gamma(4\alpha)}{\alpha\Gamma(1+3\alpha)\Gamma(1+5\alpha)}t^{5\alpha} \dots$$

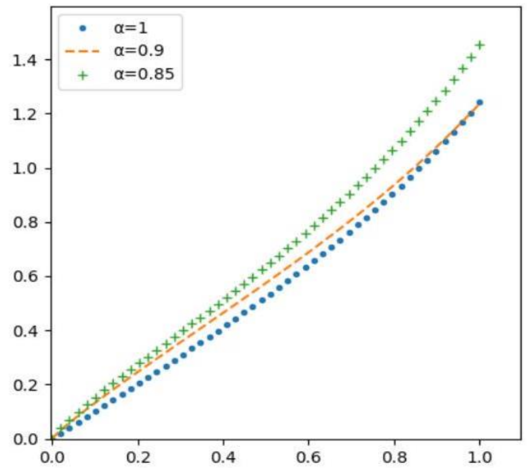
To derive the solution of above FDE, we use the Conformable Fractional Differential Transform Method scheme:
By the help of Theorem 2.12, Theorem 2.13 and Theorem 2.14, we can write

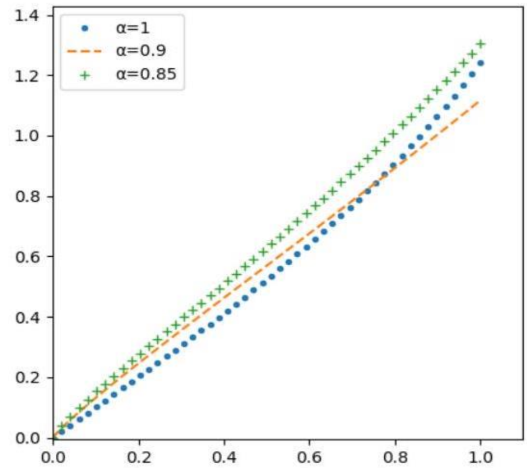
$$\alpha(k+1)Y_\alpha(k+1) = \delta(k) - \sum_{l=0}^{k} Y_\alpha(l)Y_\alpha(k-l)$$

Thereby, it is obtained that $Y_\alpha(k+1) = \frac{1}{\alpha(k+1)}\left(\delta(k) - \sum_{l=0}^{k} Y_\alpha(l)Y_\alpha(k-l)\right)$ For $k = 0,1.2, \dots$, the solution by means of CFDTM is found as

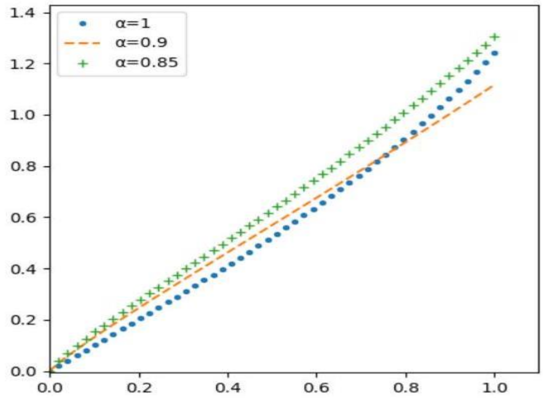
$$y(t) = \frac{t^{\alpha}}{\alpha} - \frac{t^{3\alpha}}{3\alpha^3} + \frac{2t^{5\alpha}}{15\alpha^5} - \frac{17t^{7\alpha}}{315\alpha^7} + \frac{62t^{9\alpha}}{2835\alpha^9} - \cdots$$

The obtained solution of (3.1) above is the fractional power series expansion of the exact solution for the first ten terms.





(a) Graph of solution of 3.1 for different (b) Graph of solution of 3.1 for different value of α by CFDTM. value of α by FDTM.



(c) Graph of solution of 3.1 for different value of α by FADM.

Figure 1: Comparision of the fourth iteration approximate solutions of CFDTM with the FDTM and FADM. Example 3.2 We consider the fractional Riccati equation

$$y^{(\alpha)} = 1 + 2y - y^2$$

with the initial condition $y(0) = 0$.

Exact solution of this equation is $y(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)$, Following the analysis presented above gives the recurrence relation

$$y_0 = y(0) + I^{\alpha}(1) = \frac{1}{\Gamma(\alpha+1)}t^{\alpha}$$

$$y_{n+1} = I^{\alpha}(2y_n - A_n),\ n \geq 0,$$

where A_n are Adomian polynomials for the nonlinear term $F(y) = y^2$. Using the above recursive relationship and Mathematica, the first few terms of the decomposition series are given by

$$
\begin{aligned}
y_0 &= \frac{1}{\Gamma(\alpha+1)} t^{\alpha} \\
y_1 &= I^{\alpha}(2y_0 - y_0^2) = \frac{2^{1-2\alpha}\cos(\pi\alpha)\Gamma(1/2-\alpha)}{\sqrt{\pi}\alpha\Gamma(\alpha)} t^{2\alpha} - \frac{2\Gamma(2\alpha)}{\Gamma(\alpha)\Gamma(1+\alpha)\Gamma(1+3\alpha)} t^{3\alpha} \\
y_2 &= I^{\alpha}(2y_1 - 2y_0y_1) = \frac{3^{3-2\alpha}\cos(\pi\alpha)\Gamma(1/2-\alpha)}{\sqrt{\pi}\Gamma(\alpha)\Gamma(1+3\alpha)} t^{3\alpha} - \frac{2\Gamma(2\alpha)}{\Gamma(\alpha)\Gamma(1+\alpha)\Gamma(1+4\alpha)} t^{4\alpha} \\
&+ \frac{4\Gamma(2\alpha)\Gamma(1+4\alpha)}{\Gamma(\alpha)\Gamma(1+\alpha)^{\alpha}\Gamma(1+3\alpha)\Gamma(1+5\alpha)} t^{5\alpha} + \frac{12\Gamma(-2\alpha)\Gamma(3\alpha)\sin(2\pi\alpha)}{\pi\Gamma(\alpha)\Gamma(1+4\alpha)} t^{4\alpha}
\end{aligned}
$$

and so on. The first eleven terms of the decomposition series are give by

$$
y(t) = \frac{1}{\Gamma(\alpha+1)} t^{\alpha} + \frac{2^{1-2\alpha}\cos(\pi\alpha)\Gamma(1/2-\alpha)}{\sqrt{\pi}\alpha\Gamma(\alpha)} t^{2\alpha} - \frac{2\Gamma(2\alpha)}{\Gamma(\alpha)\Gamma(1+\alpha)\Gamma(1+3\alpha)} t^{3\alpha} \cdots
$$

To derive the solution of above FDE, we use the Fractional Differential Transform Method scheme:
By using Theorems 2.4 and 2.5, Eq. (1) transforms to

$$
Y(k+\alpha\theta) = \frac{\Gamma(1+k/\theta)}{\Gamma(\alpha+1+k/\theta)} \left[\delta(k) + 2Y(k) - \sum_{k_1}^{k} Y(k_1)Y(k-k_1) \right]
$$

and using Eq. (2.1.2), initial conditions can be transformed as follows:

$$
Y(k) = 0, \quad \text{for } k = 0,1,\dots,\alpha\theta - 1
$$

Using Eqs. (3.5) and (3.6), Y(k) is obtained for values of $\alpha = 1/2$ and $\theta = 2$, then using Eq. (2.1.2), y(x) is evaluated.

$$
y(t) = \frac{2}{\sqrt{\pi}} t^{1/2} + 2t + \frac{16(\pi-1)}{3\pi^{3/2}} t^{3/2} + \frac{\pi-4}{\pi} t^2 - \frac{32(3\pi^2+44\pi-32)}{45\pi^{5/2}} t^{5/2} + \cdots
$$

To derive the solution of above FDE, we use the Conformable Fractional Differential Transform Method scheme:
By the help of Theorem 2.12, Theorem 2.13 and Theorem 2.14, we can write

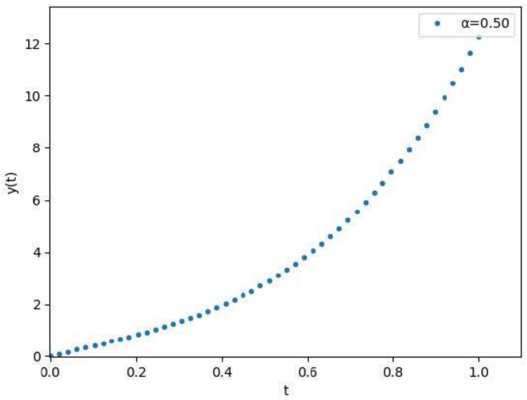
$$
\alpha(k+1)Y_{\alpha}(k+1) = \delta(k) + 2Y_{\alpha}(k) - \sum_{l=0}^{k} Y_{\alpha}(l)Y_{\alpha}(k-l)
$$

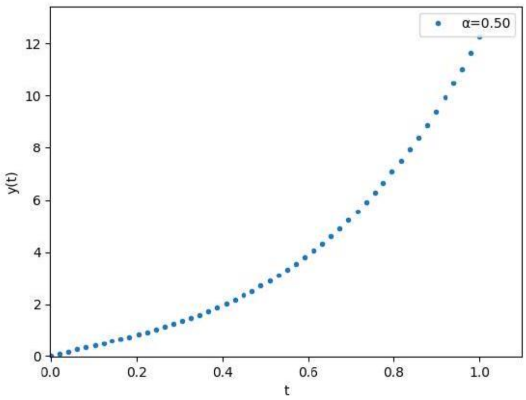
Thereby, it is obtained that

$$
Y_{\alpha}(k+1) = \frac{1}{\alpha(k+1)} \left(\delta(k) + 2Y_{\alpha}(k) - \sum_{l=0}^{k} Y_{\alpha}(l)Y_{\alpha}(k-l) \right)
$$

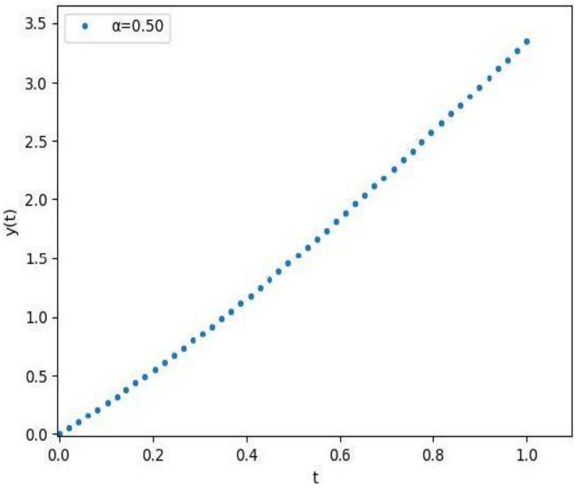
For $k = 0,1.2, \dots$, the solution by means of CFDTM is found as

$$
y(t) = \frac{t^{\alpha}}{\alpha} + \frac{t^{2\alpha}}{\alpha^2} + \left(\frac{2}{3\alpha^3} - \frac{1}{\alpha^2}\right) t^{3\alpha} + \left(\frac{4}{3\alpha^4} - \frac{3}{\alpha^3}\right) t^{4\alpha} + \cdots
$$





(a) Graph of solution of 3.4 for different (b) Graph of solution of 3.4 for different value of α by CFDTM. value of α by FDTM.



(c) Graph of solution of 3.4 for different value of α by FADM.

Figure 2: Comparision of the fourth iteration approximate solutions of CFDTM with the FDTM and FADM. Example 3.3 Consider the modified alpha fractional differential equation

$$y^{\alpha}+y=0, y(0)=1, \text{ for } \alpha\in(1,2].$$

Now we will find the solution of this equation by usnig CFDTM. Here Exact solution of 3.7 is $y(\zeta)=e^{-\frac{1}{\alpha}\zeta^{\alpha}}$. By using the Theorem 2.12, Theorem 2.13 and Theorem 2.14, we can write the above equation as follow.

$$\alpha(k+1)Y_{\alpha}(k+1)+Y_{\alpha}(k)=0,\ Y_{\alpha}(0)=1$$

As a result, the recurrence relation is as follows:

$$Y_{\alpha}(k+1)=-\frac{1}{\alpha(k+1)}Y_{\alpha}(k),\ Y_{\alpha}(0)=1$$

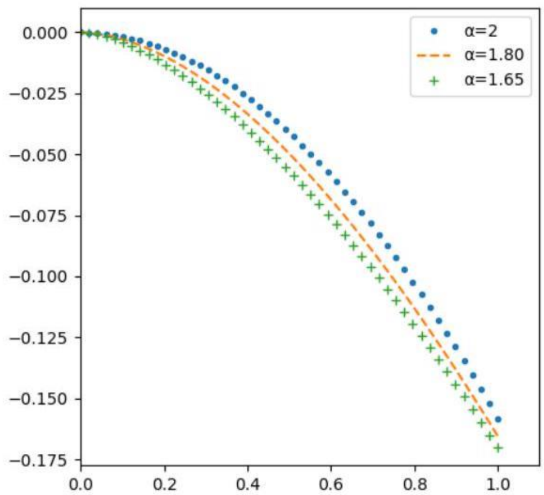
Take $k=0,1,2,\dots,n$

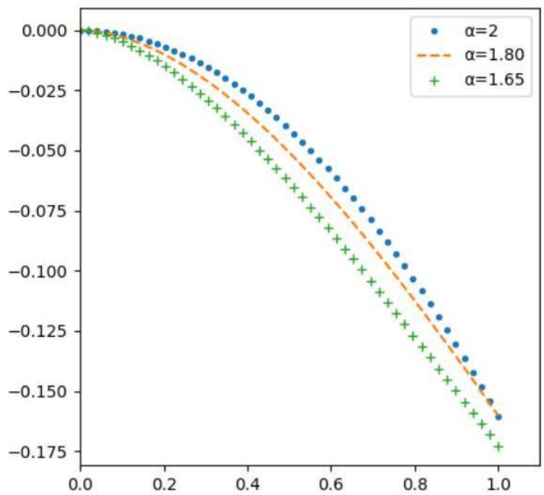
$$Y_{\alpha}(1)=-\frac{1}{\alpha}Y_{\alpha}(0)=-\frac{1}{\alpha}$$
$$Y_{\alpha}(2)=-\frac{1}{2\alpha}Y_{\alpha}(1)=\frac{1}{2!\,\alpha^{2}}$$
$$Y_{\alpha}(3)=-\frac{1}{3\alpha}Y_{\alpha}(2)=-\frac{1}{3!\,\alpha^{3}}$$
$$\vdots$$
$$Y_{\alpha}(n)=\frac{(-1)^{n}}{n!\,\alpha^{n}}.$$

Hence the solution of 3.7 by using MFDTM can be written as,

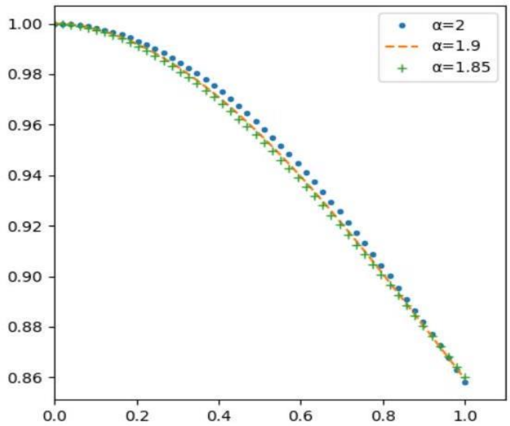
$$y(\zeta)=\sum_{n=0}^{\infty}\frac{(-1)^{n}}{n!\,\alpha^{n}}t^{n\alpha}=e^{-\frac{1}{\alpha}t^{\alpha}}$$

Now graphical nature of the solution of the equation by using FADM ,CFDTM and FDTM are shown in the figure.





(a) Graph of solution of 3.7 for different (b) Graph of solution of 3.7 for different value of α by CFDTM. value of α by FDTM.



(c) Graph of solution of 3.7 for different value of α by FADM.

Figure 3: Comparision of the fourth iteration approximate solutions of CFDTM with the FDTM and FADM.

4. Conclusion

This work uses FDTM, CFDTM and FADM to solve a non-linear fractional order mathematical model on dengue. Furthermore, the fractional model solution produced by CFDTM is associated with the solution of the same model estimated by FADM and FDTM for different fractional orders. Two alternative strategies FADM, FDTM and CFDTM have been used to solve and analyse a non-linear fractional order mathematical model. In terms of infinite series for various orders and by specifying fixed components with various time intervals, an approximate solution to the specified model is established. The Python programme is used to analyse the solution numerically and visually. The outcomes of these numerical simulations have been positive.

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