STATIC CONFORMALLY FLAT CHARGED FLUID SPHERES

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Abstract: **In this chapter we have presented some solutions of Einstein-Maxwell field equations for static conformally flate charged perfect fluid sphere by using a suitable form of mass density. We have also discussed the case4 of uniform charge density distribution or the case of uniform charge density sphere with the surface of charge spherical thin shell. Again the case of uniform mass density has been also considered. A conformally flat spherically symmetric non-static internal solution was obtained by Singh and Abdussattar. Letter on Ray and Rajbali found a general solution representing conformally flat perfect fluid distribution of spherical symmetry. They have also discussed various physical properties of the model.**

Keywords: **Einstein, Maxwell, mass density, conformally flate, spherical, symmetric, static uniform charge, hypergeometric, pressure.**

Introduction- Static conformally flat charged fluid spheres have much attracted the relatives in recent years. In 1968, De and Raychaudhuri [9] have shown that in relativistic unit a pressure-less charged dust distribution in equilibrium will have the absolute value of the charge to mass ratio as unity. Many workers have already studied the charged fluid distribution in equilibrium. While bailyn and Eimer and Nduka have presented some solution of static spherical distribution which are not free from singularity at the origin. Omote has given a solution where the charge to mass ratio of the spherical ball can be made arbitrary large. However, because of the repulsive action of the pressure, the charge to mass ratio of a spherical ball is expected to be less than unity in case of fluid matter as the ratio becomes unity from pressure less dust matter. This strange result was obtained by Omote possibly because of an un explained hypergeometric function appearing in his solution. Incidentally, Krori and Barua found some solutions for charge fluid distributions with spherical symmetry which are very regular. Chakravarti and De studied the problem of static charged fluid distributions in the form of spherical ball and presented some new solutions which are regular everywhere and of course, the solution could be matched with outside Reissner-Nord strong metric. in all there solution, the charge to mass ratio of the spherical ball Was as expected, less than unity. Singh and Yadav have also found some exact solution of charged fluid sphere in general relativity, some other workers in this line are Bonnor, Effinger , Kyle and Martin and Wilson who has considered the interior solution for a static charged sphere under different conditions.
A conformally flat spherically symmetric non-static internal solution was obtained by Singh and Abdussattar. Letter on Ray and Rajbali found a general solution representing conformally flat perfect fluid distribution of spherical symmetry. They have also discussed various physical properties of the model.

Gurses has shown that the only static distribution of the fluid with positive density and pressure which would generate a conformally flate metric through the an Einstein's equations without cosmological term is that described by the Schwarzschild interior solution. Burman discussed the motion of the particles in conformally flate space-time. Singh and Abdussattar has obtained a non static generalization of the Schwarzschild interior solution which is conformal to flate space-time. They have also shown that the model admits of distribution of discrete particles and disorder variation. Zalcev and Shikin have obtained conformally flate non-static solution in general relativity theory and scalar tensor-theories of gravitation. Collinson has shown that every conform ally flat ax symmetric stationary space-time is static, he has also proved that if the source is perfect fluid the space-time is the interior Schwarzschild field. Gupta has observed that if a conformally flat space-time describes a perfect fluid distribution of matter $\rho \neq 0$, then it is necessarily of embedding class one.

THE FIELD EQUATIONS

We use here the static spherically symmetric line element in the form

$$ds^2= e^3dt^2 - e^r dr^2 - r^2(d\theta^2+\sin^2\theta d\phi^2) \qquad (1)$$

Where α and β are function of r only.

The Einstein-Maxwell equation for the charged perfect fluid distribution in general relativity are

$$R_{ij}-1/2Rg_{ij}=-8\pi T_{ij} \qquad (2)$$

$$[(-g)^{1/2}F^{ij}]_j=J^i(-8)^{1/2} \qquad (3)$$

$$F_{[ij;k]}=0 \qquad (4)$$

Where $T_{ij}$is the energy momentum tensor, J^i is the current four vector, R_{ij} is the Ricci tensor and R the curvature scalar. For the system under study the energy momentum tensor T^i_j splits up into two parts viz. M^i_j and $E^i_j$for matter and charges respectively i.e.

$$M^i_j = [(p+P)u^i u_j - p\delta^i_j] \quad \text{With} \quad u^i u_j = 1$$

The non-vanishing components of M^i_j are

$$M^1_1 = M^2_2 = M^3_3 = -\rho, \; M^4_4 = \rho \qquad (5)$$

Thus the Einstein-Maxwell field equation are

$$e^{-\alpha}\left(\frac{1}{r^2} + \frac{\alpha'}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E^2 \qquad (6)$$

$$\frac{1}{r^2} e^{-\alpha}\left(\frac{1}{r^2} + \frac{\beta'}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E^2 \qquad (7)$$

$$e^{-\alpha}\left[\frac{1}{4}\beta'\alpha' - \frac{1}{4}\beta'^2 - \frac{1}{2}\beta'' - \frac{1}{2}\left(\frac{\beta' - \alpha'}{r}\right)\right] = -8\pi\rho - E^2 \qquad (8)$$

Where p is the interior pressure and ρ is the pure gravitational mass density . To solve above equation we get

$$E^2 = \frac{Q^2(r)}{r^4} \qquad (9)$$

Where Q(r) represents the total charge contained within the sphere of radius r, we have

$$Q(r) = 4\pi \int \rho_e r^2 dr$$

Where ρ_e is the charge density.

SOLUTION OF THE FIELD EQUATIONS

We have five equation (1) – (4) and (6) in six variables $\alpha, \beta, Q(r), Ej, \rho$ and ρ. Hence the system is indeterminate. To make the system determinate we require one more relation. For this we choose uniform mass density in the form

$$\rho = A + Br^2, (B<0) \qquad (10)$$

Where A and B are constant. Now integrating equation (6) we get

$$e^{-\alpha} = 1 + \eta r^2 + \xi(r) \qquad (11)$$

Where η is integration constant and ξ(r) is

$$\xi(r) = 2r^2 \int \frac{e^2}{r} dr \qquad (12)$$

From (10), (11) and (12) we get

$$\xi'/r + \xi/r^2 = -8\pi(A + Br^2) - E^2 - 3\eta \qquad (13)$$

Now differentiating (2.3.4) we get

$$\frac{dE^2}{dr} + 2\frac{E^2}{r} = -\frac{16\pi}{3} Br \qquad (14)$$

Solution of (2.3.5) we get

$$E^2 = \frac{\bar{A} + b^2 r^2}{r^2} \qquad (15)$$

$$Q^2(r) = \bar{A} r^2 + b^2 r^6 \qquad (16)$$

Where $\bar{A}$ is the integration constant and

$$B^2 = \frac{-4\pi}{3} B, (B < 0)$$

Case 1 when $\bar{A} \neq 0$

Using (15) into (12) we find

$$\xi(r)=-\bar{A}+b^2r^2 \qquad (17)$$

From (17) and (11) we get

$$e^{-\alpha}=1-\bar{A}+b^2r^4+\eta r^2 \qquad (18)$$

Inserting (18) and (11) into (11) we can prove

$$\eta=-\frac{8\pi}{3}\,A=-\frac{1}{R^2} \qquad (a)$$

Where R^2 is different from R_0^2which is dependent on the pure gravitational mass density ρ; so that

$$e^{-\alpha}=C-\frac{r^2}{R^2}+b^2r^4 \qquad (19)$$

Where $C = 1-\bar{A}$

Using equation (15) and (19) into p eliminate of (a) and (19) we get

$$\beta''+\frac{1}{2}\beta'^2-\left(\frac{1}{2}-\frac{\frac{-r}{R^2}+2b^2r^3}{c-\frac{r^2}{R^2}+b^2r^4}\right)\beta=\frac{2(c-1)+2b^2r^4}{r^2\left(c-\frac{r^2}{R^2}+b^2r^2\right)} \qquad (20)$$

By use of transformation

$$r^2=\chi \quad \text{And} \quad \chi^{-1/2}e^{\beta/2}=Z$$

Equation (20) is transformed to

$$\frac{d^2z}{d\chi^2}+\left[\chi^{-1}\frac{1}{2}\left(\frac{1-2b^2R^2\chi}{cR^2-\chi+b^2R^2\chi 2}\right)\frac{dz}{d\chi}\right]=\frac{2(c-1)R^2}{4\chi 2(cR^2-\chi+b^2R^2\chi 2)^2} \qquad (21)$$

Again using the transformation

$$u=\frac{D}{R}\frac{1}{(c)^{\frac{1}{2}}}\ln\left[\frac{\left(c-\frac{\chi}{R^2}+b^2\chi 2\right)^{\frac{1}{2}}}{\chi}+\frac{c}{\chi}-\frac{1}{2\sqrt{c}}R^2\right] \qquad (22)$$

Where D is constant, equation (21) is changed into

$$D^2\frac{d^2z}{du^2}-\frac{(2c-1)R^2}{4}Z=0 \qquad (23)$$

The final solution of equation (20) may be written as

$$e^{\beta}=r^2\left(k_1\sinh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\left(\frac{c-\frac{r^2}{R^2}+b^2r^4}{r^2}\right)^{\frac{1}{2}}+\frac{c}{r^2}-\frac{1}{2\sqrt{cR}}\right]\right\}\right.$$
$$\left.+k_2\cosh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\left(\frac{c-\frac{r^2}{R^2}+h^2r^4}{r^2}\right)^{1/2}+\frac{c}{r^2}\frac{1}{2\sqrt{cR^2}}\right]\right\}\right)^2$$

where k_1 and k_2 are the integration constant. Now using equation (24) and (a) the pressure ρ is given by

$$8\pi\rho=4b^2r^2-\frac{3}{R^2}+\frac{2e}{r^2}-\left(\frac{e}{r^2}+\frac{1}{R^2}+r^2b^2\right)\beta' \qquad (25)$$

$$8\pi\rho=\frac{2c}{r^2}-\frac{3}{R^2}+4b^2r^2-\left(c-\frac{r^2}{R^2}+r^2b^4\right) \qquad (26)$$

$$X\frac{\frac{2}{r}-(2-\frac{1}{c})^{1/2}\left[\frac{\frac{r^2}{R^2}-2c}{r^2\left(1-\frac{r^2}{R^2}+b^2r^4\right)^{1/2}}-\frac{2c}{r^2}\right]}{\left(c_2-\frac{r^2}{R^2}+b^2r^4\right)+c-\frac{r^2}{2\sqrt{cR^2}}}$$

$$X\frac{k_1+k_2\tanh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\left(c-\frac{r^2}{R^2}+b^2r^4\right)^{1/2}+\frac{c}{r^2}-\frac{1}{2\sqrt{cR^3}}\right]\right\}}{k_2+k_1\tanh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\frac{\left(c-\frac{r^2}{R^2}+b^2r^4\right)^{1/2}}{r^2}+\frac{c}{r^2}-\frac{1}{2\sqrt{cR^2}}\right]\right\}}$$

CONCLUSION:

This model is physically reasonable and free from Singularity hence various physical parameters can be calculated.

The solution of Einstein-Maxwell field equation for Static conform ally flat that charged perfect fluid sphere by using a suitable form of mass density. The result gives uniform charge density and uniform mass density distribution also. Various physical parameters can be calculated by using different boundary conditions. In spherical symmetric metric, we have solved Einstein-Maxwell field equation by taking a suitable form of matter density and charge density hence various parameters can also be calculated by putting varies conditions.

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