Analysis of improvements in soil quality with Mixing of Fly-Ash and stabilization of fragile sub-grades

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Abstract: **The characteristics of the pavement are quite sensitive to the characteristics of the subgrade soil. Therefore, the unstable sub-grade is strengthened by utilizing the most lucrative stabilizing procedure available to it. As a result of previous study, it was determined that stabilization with fly-ash at sub-grade was a feasible approach for enhancing soil quality. An attempt was made in this manner to observe the impact of fly-ash stabilization activity on the geotechnical features of poor subgrade soils through the use of a testing program. The stabilization of the soil beneath a pavement is one of the most significant and time-consuming processes in the construction of a pavement. For the purposes of this investigation, the influence of soil mixing with fly-ash on the stability and strength of subgrade pavements is being investigated. Different levels of fly ash were added to the soil to improve its fertility, including 9, 18, 27, and 36 percent. Some soil characteristics, such as consistency restrictions, CBR, compaction, and UCS testing, have been investigated on both modified and natural soils. Adding fly-ash admixture to a weak soil, according to the results of the studies, has a substantial impact on the soil's qualitative features. This percentage has been determined to be 18 percent, which has been shown to result in considerable increases in soil quality and strength, as well as a reduction in soil swelling and plasticity. This number is therefore considered to be the optimal percentage. With the percentage of Fly-Ash included in the soil sub-grade (F.A in percent) as the starting point for this study, the following parameters are determined: Plastic Limit (PL), Liquid Limit (LL), Optimum Moisture Content (OMC), Unconfined compressive strength (UCS), Maximum dry density (MDD), and Plasticity Index (PL). It is determined whether the soil has been stabilized by fly ash by performing CBR, compaction, specific gravity, sieve analysis, and a water absorption test on it. The parameters investigated include the Plastic Limit, the Liquid Limit, the California Bearing Ratio, and the Optimum Moisture Content. A realistic approach for stabilizing unstable sub-grades, according to the findings, is to incorporate fly-ash additive into the concrete mix.**

Keywords: **Soil Quality, Fly-Ash, stabilization of fragile sub-grades**

INTRODUCTION

Soil stabilization is the characteristic of the procedure of enhancing the engineering properties of the weak soil in order to stabilize. Soil stabilization is the predominant and useful method used to strengthen the qualitative properties of weak soil. Its basic objective is to lessen the compressibility and permeability of the frail soil and to enhance its shearing strength.

Soil Improvement Techniques

Various techniques that are required to improve the weak soil are as:-

- Surface compaction
- Vibration Method
- Dynamic Compaction
- Blast Densification
- Soil reinforcement
- Grouting
- Stabilization

Surface Compaction

This is the technique for enhancing the soil that is straight and generally utilized. Of example airfields, roads, or weak bases construction requires compacted base. It relies on the depth that is to be compacted. It is progressively viable when the depth is less. It is one of the most efficient techniques for the development of soil.

Vibration Method

This technique includes pushing a vibratory mechanism into the ground to improve the underground soil. There are two types of vibration methods. First is terra probe involving a vibratory pile hammer appended to a steel pipe. The vibration within the soil is carried out by the probe. The adjoining soil densifies because of this vibration and is sampled. Another one is called vibro float, which is commonly a developed probe containing of water jet and vibrator. For introducing the probe into the ground, the crane mechanism is helpful. When the vibrator creates the vibrations inside the ground then the densification soil occurs. The delivered water jet provides the means to add and extract the sample from the surface.. This system is otherwise known as vibro flotation. In-situ soil densification is included in the technique.

Dynamic Compaction
In-situ-densification is additional technique of soil. A special crane consists from mechanism competent for lifting weight of around 5 to 30 ton, from a particular height around 20-30 m and dribs the weight above the surface. The ground becomes compacted because a sequence continual lift and fall on similar place. It is known as grid pattern.

Blast Densification
In this method a sequence of holes are bored into the soil and then the explosive is placed into these bores. Number of shocking waves below the ground produced when the blast is completed from these explosives. Surrounding soil densifies from the result of waves. The technique is diverted out from the population in distant areas, because of safety factors. The best technique suited with sand.

Soil Reinforcement
In this method the total performance of the soil improves. Reinforcement utilized with the end goal of various forms i.e. sheets, strips and bars. In order to lessen the tensile stress, strip bars are for the most part utilized. This is the most valuable technique for holding embankments, footings, structures and so on. It requires trained persons and it is also uneconomical, these are disadvantages of this method.

Grouting
The infusion of stabilizers into the soil includes directly in this method. Once under pressure the operation is done, extremely viscous reinforcements for permeable soil are utilized. Because of impermeable nature, this technique isn't applicable for clayey soil. Using this method the dams are well stabilized under the previous stratum. It is somewhat expensive procedure. Various materials exist utilized as grouts such as clay, bitumen, additives, polymers, cement and so forth.

Soil Stabilization
Soil stabilization is a procedure of altering or strengthening the soil's qualitative properties, collects it perfect for development purposes. The soil's load bearing capacity enhances while the soil's permeability and compressibility is diminished because of stabilization. New strategies to repair soil's geotechnical characteristics that is not stable are being shaped for designing purposes. Mainly three techniques are there for soil up-gradation are Soil stabilization, Evacuation of the undesirable materials and variable the ground water conditions.

Various Stabilization Techniques

Different techniques of stabilization of soil are listed below:-

1. Mechanical stabilization
2. Chemical stabilization.
3. Thermal stabilization
4. Cement stabilization
5. Electrical stabilization
6. Lime stabilization
7. Bituminous stabilization
8. Stabilization by geo-textiles

Mechanical Stabilization
In the process of Mechanical stabilization, the gradation of soil is altered to improve its properties. In order to achieve a good graded soil, the binders and aggregates are mixed suitably. Compaction effort is provided for improving the load bearing capacity of soil, so the mixed materials come to be precisely stable. The mechanical stabilization relies on mineral composition, gradation, and mechanical strength of compaction, aggregates and plasticity attributes. One of the easiest methods of soil stabilization is mechanical stabilization and is generally utilized for stabilizing sub bases, bases and road surfacing for an enormous scope.

Cement Stabilization
In this method, compaction efforts are provided to accomplish an upgraded material when the soil and cement is varied up with water. When the mixture of soil and cement is achieved it is known as soil-cement. During hydration, chemical response takes place among the cement and silica existent in soil because of which cementing act happens. The soil-cement improves into a harder material and strength is achieved due to result of this hydration. For weather situations, the mixture comes to be impervious. Cement rate utilized is 6% for Sandy soils and is 15% for clayey soils. For good results, suitable compaction of the mixture is necessary. In order to complete advanced results, a few admixtures are added to the mixture of soil-cement.

Lime Stabilization
Lime Stabilization generally suitable for clayey soils. When the lime is mixed with the soil Pozzolanic actions takes place and the nature of adsorbed layer changes. The MDD, OMC and PI is diminished, expanded and diminished correspondingly on adding lime. Thus by using lime stabilization, the durability and strength of soil gets increased. The road bases and sub-grade are usually

balanced out utilizing with lime.

Chemical Stabilization

For stabilization of soil various chemicals are used in chemical stabilization. The controlling of setting time and curing time is the main advantage of such process. For this purpose of stabilization various other chemicals are used.

- The reason of calcium chloride is to improve the moisture content in soil. Hence, calcium chloride is best for silty and clayey soils. Dry density is improved with the use of calcium chloride.
- Sodium chloride permits least moisture though another characteristic is like calcium chloride. Sodium chloride restricts the advancement of shrinking cracks.
- Sodium silicate on chemical reaction with soil enhances the power of soil. It is infused into the soil by creating its solution in water. Sodium chloride enhance thus by granting impenetrable nature to the soil and furthermore enhances its density.
- Certain kinds of coagulating chemicals, like calcium chloride and ferric chloride shows improvement in expanding the electrical affinity and furthermore enhances the penetrability by creating flocculated structures.

Bituminous Stabilization

The compound Bitumen is a hydrocarbon that has ability to dissolve in carbon disulphide and comes under category of non-aqueous systems of hydrocarbons. It is a kind of stabilization in which asphalt act as a binder. It is found that asphalt is very much viscous material that is why it cannot be applied legitimately and hence gasoline is supplanted as a dissolvable. It is discovered that by this type of stabilization, natural soil which has vicinity with asphalt can be enhanced. Asphalt has a property by which asphalt aides in expelling the voids from coarser grained soil therefore improving it from hard water, so makes it water proof which also enhance its strengthening capacity.

Electrical Stabilization

Electrical stabilization has been found for stabilization of clayey soil. Thus due to this stabilization of such soil, electro osmosis method has been made using direct current is made to cross through clayey soil that distracts the core water to cathode. It occurs as though accessibility of positive ions in water which are like cathode. Because of removal effective for the seepage reasons of cohesive soils.

Thermal Stabilization

Thermal stabilization improve the properties of soil to a great extent. The methodology included is heating and re-cooling of soil.

- Heating – The water content is reduced by this process. The procedure of electric repulsion lessens the clay particles and hence the bearing capacity of fine grained soil enhances. The absorbed water evaporates on ascent of temperature over 100^0C, and hence enhances the strength. The drawback of this technique is that, it is exceptionally broad.
- Freezing – The repulsion between the inter-particles improves in fine grained soil and the strength diminishes by the process of Cooling. In any case, it is found that when we lower the temperature up to 100 degree at that point freezing of pore water is initiated and in this way the fine grained soil arrives its stabilization point.

Merits and benefits

As soil properties changes to a great extent & construction of different structures will relies to a great extent upon the bearing capacity of soil, thus, needs to balance out the soil making it simple to imagine the load bearing capacity of the soil and even enhances the load bearing capacity. While working with soil, soil gradation is likewise an essential property which ought to be remembered. The soils might be of uniformly graded type which however sounds stable yet has more voids or well-graded type which is desirable as it has less number of voids. Along these lines, various sorts of soils are mixed together in order that strength properties of the soil are enhanced. Thus, stabilization of soil is to be done to search for these situations where it it turns out to be expensive to replace an inferior soil a superior one.

- ❖ The bearing capacity is increased because of increment in strength of soil by the stabilization of soil.
- ❖ Rather than going for deep or raft foundation, soil stabilization is increasingly affordable both as far as cost & energy so as to increase the bearing capacity of soil.
- ❖ It is likewise used to give greater stability to the soil in slopes or other such places.
- ❖ Soil water-proofing is additionally done by soil stabilization; which keeps water from inflowing into the soil and henceforth helps the soil from losing strength of soil.
- ❖ When temperature or moisture content gets changed, due to this volume of soil is decreased.
- ❖ Workability and the durability of soil are improved by soil stabilization.
- ❖ Occasionally soil stabilization is additionally used to avoid formation of dust or soil erosion, which is extremely useful particularly in dry and arid climate.

METHODOLOGY

Obtaining a uniform and thorough mixture is an important factor in soil stabilization. Two methodologies that are normally utilized in construction:

(1) Off-site mixing involving batch or continuous type mixing,

(2) On-site mixing.

Off-site mixing activities give more uniform mixtures because a greater control can be achieved with the amount of materials batched than with on- site mixing. Self-cementing fly-ash exhibits a moderately quick set (as meager as ten minutes), which diminishes strength with delayed compaction.

Application of Water

During the stabilization activity, the process of including and observing the mixing water is a critical step in the process of construction. When utilizing a mixing plant arrangement, general proposals for application of water are that it ought to be somewhere in the range of 80% and 110% of the optimum moisture content, , in view of the moisture-density relationship of the balanced out mixture, in order to acquire appropriate density at the time of compaction. Water can be included to the sub-grade soils before including fly-ash. However, this approach has a disadvantage that the sub-grade sometimes become unstable, which will then complicate the rest of the construction process. After the fly-ash has been fused into the soil, water can be included to the mixture, yet more goes of the mixing apparatus are normally needed and there is a loss in the strength of mixture that occurs because of the hydration of the fly-ash before conclusive compaction. Further adding water legitimately into the mixing drum of the pulvamixer is the best technique of controlling mixing water. The most uniform mixing is produced by this procedure and on the other hand minimal amount of delay is produced in the construction process.

Compaction of Fly-ash-Stabilized Soil

Based on the type of soil, various compaction equipments and techniques are available to expand the overall compaction of fly-ash soil ad-mixtures. Because of the self-cementing properties of fly-ash, it proved to be a successful stabilizer for fine grained and granular soils. Time factor due to delay in compaction ought to likewise be considered on the grounds that the balanced out material can lose strength as the fly-ash hydrates while being in an un-compacted state. For Class F fly-ash stabilization work, a most extreme compaction delay time of as long as four hours has been indicated.

Different stabilization techniques for soil

As there are numerous way to stabilize Natural soil because it is a complex and irregular material, Yet because of its broad presence all around the world and its minimum cost, it offers incredible pass for skillful use as an engineering material. Various types of stabilization techniques are:

- Mechanical stabilization
- Cement stabilization
- Lime stabilization
- Bitumen stabilization
- Chemical stabilization
- Thermal stabilization
- Electrical stabilization
- Stabilization by grouting
- Stabilization By geotextiles and fabrics

Fly-ash stabilization

Erdalcokca used fly-ash as a stabilizer with lime in 1998. In his research works lime is added to black cotton soils at 0 to 8% to set up datum esteems.. Next fly-ash is at 0 to 25% limit. The curing period is considered at 7 days age for his experiments. Fly-ash is the by-product material collected from the flue gases of a heater terminated with coal. Fly-ash comprises of hollow spheres of iron oxide, silicon aluminum and un-oxidized carbon. Thus, expansive soils are likely to be stabilized effectively by cation exchange using fly-ash. Addition of 18% fly-ash can decrease the selling capability considerably. There is slight diminishing in swelling potential from 18 - 23 % fly-ash expansion. Accordingly the ideal fly-ash content is close to 18% only. The activity, plasticity index, CBR, UCS, and swelling pressure etc. gave satisfactory results when black cotton soils are treated with fly-ash. Thus fly-ash is a good stabilizer now a day.

Hydration of fly-ash

Hydration is the evolution of cementitious material by the response of free lime (CaO) and pozzolans (AlO_3, SiO_2, Fe_2O_3) with addition of water. The hydrated calcium silicate gel or calcium aluminates gel (cementitious material) can tie static material together. For class C fly-ash, the calcium oxide (lime) of the fly-ash can respond with the siliceous and aluminous materials (pozzolans) of the fly-ash itself. class F fly-ash has generally low lime content and thus requires expansion of lime for hydration with the pozzolans of the fly-ash. For lime stabilization of soils, pozzolanic responses rely upon the siliceous and aluminous materials already present in the soil. Hydration of tricalcium aluminates in the ash is very important as it gives one of the essential cementitious items in numerous ashes. The quick hydration of the tricalcium aluminate results in the prompt setting of these materials, and on this basis, delay in compaction provide stabilized materials of low strength. The hydration chemistry of fly-ash is extremely complicated, so the stabilization procedure must focus primarily over the chemical properties of the fly-ash.

Weak sub-grade and treatment

Flexible pavement sub-grade is most prone to breakdown under the vehicle loads since the load from overlying layers is non-uniform and also the moisture content is very high. This layer is given least importance contrasted with different layers in pavement, even though the greater part of the pavement breakdown is because of the bearing capacity breakdown of the sub-grade layer. Various sub-grade soils show strength at low moisture content; such as clayey soils, however the strength and workability decrease with the expansion in water content past the optimum value. Such soils need to be either changed with better-quality fill material or given a reasonable treatment process (Prusinski and Bhattacharja, 1999). Substituting the sub-grade soil may not generally be the most ideal choice due to uneconomical hauling cost of the unearthed materials and the imported quality materials as well. In certain regions, the deficiency of the reasonable fill materials or the unviability of aggregate the makes replacing the frail sub-grade soil too costly. In such conditions, the engineering properties of the current frail sub-grade soil can be enhanced by the utilization of legitimate compaction techniques and by using some chemical stabilizers as well. Lime, Portland cement and fly-ash are the most widely recognized kinds of chemical stabilizers that is used to make the frail sub-grades stabilized; in this way providing an appropriate working stage or potentially sub-base layer for the construction of pavement.

EXPERIMENTAL RESULTS

Sub-grade soil results

The outcomes attained from the experimental analysis are appeared in table.

Table 5 Results of unmodified sub-grade soil

S. No.	Property	Sub-grade soil(S.S)
1	Liquid limit	36.05
2	Plastic limit	22.55
3	Plasticity index	13.85
4	Shrinkage limit	26.05
5	Average Grain Size D_{50} (mm)	0.14
6	Coefficient of Uniformity C_u	2.81
7	Coefficien of Curvature C_c	1.51
8	Maximum dry density	19.75
9	O.M.C.	10.75
10	U.C.S.	58.75
11	Classification According to Indian Standard Typical Soil Classification	SM Silty Sand

According to Indian standard soil classification system, the silty sand soil (SM) is chosen for investigation as shown in Table 5. To strengthen the soil, numerous extends of Fly-ash are utilized. Various qualitative properties of fly-ash are referenced in Table 2.

91%Sub-grade soil (S.S) + 9%Fly-ash (F.A)

These are the results formed when 9% of the soil Sub-grade is replaced with the Fly-ash and it's observed that liquid limit reduces significantly while unconfined compressive strength (UCS) increases a little bit. MDD and OMC also decreases and increases respectively with the expansion of fly-ash within sub-grade soil.

Table 6 Results of soil sub-grade with 9% of fly-ash

S. NO.	PROPERTY	9% S. S. + 91% F. A.
1	Liquid limit	33.75
2	Plastic limit	24.75
3	Plasticity index	14.45
4	Shrinkage limit	21.95
5	Maximum dry density	17.95
6	O.M.C.	13.77
7	U.C.S.	60.85

82%Sub-grade soil (S.S) + 18% Fly-ash (F.A)

Table 7 Results of soil sub-grade with 18% of fly-ash

S. NO.	PROPERTY	82% S. S. + 18% F. A.
1	Liquid limit	31.85
2	Plastic limit	23.95
3	Plasticity index	11.75
4	Shrinkage limit	18.85
5	Maximum dry density	18.05
6	O.M.C.	13.99
7	U.C.S	90.85

On increasing the Percentage of Fly-ash by 9% more i: e total of 18% it's found that there is A noticeable change in the UCS as Well as Liquid limit. The UCS shoots up to 90.85 KN/m^2 which is quite good, whereas OMC and MDD increases a little bit.

73% Sub-grade soil (S.S) + 27% Fly-ash (F.A)

In this case, the UCS Start reducing which clearly means further addition of fly-ash will result in lower Strength but OMC increases with the further adding up of fly-ash in the Soil. When the Percentage of Fly-ash increases, the Plasticity Index lessens too.

Table 8 Results of soil sub-grade with 27% of fly-ash

S. NO.	PROPERTY	73% S. S. +27% F. A.
1	Liquid limit	32.75
2	Plastic limit	25.25
3	Plasticity index	11.63
4	Shrinkage limit	24.85
5	Maximum dry density	17.85
6	O.M.C.	14.4
7	U.C.S.	88.85

64% Sub-grade soil (S.S) + 36% Fly-ash (F.A)

Table 9 Results of soil sub-grade with 36% of fly-ash

S. NO.	PROPERTY	64% S. S. +36 % F. A.
1	Liquid limit	35.75
2	Plastic limit	27.45
3	Plasticity index	9.54
4	Shrinkage limit	26.05
5	Maximum dry density	16.95
6	O.M.C.	15.75
7	U.C.S.	86.35

On further checking of the Effect of increasing the Fly-ash content it is noted that UCS further drop down to 86.35kN/m^2 which means if we further replace the soil with fly-ash there will be a dropdown in the Strength of soil which is not the motto of our Project and hence we stop increasing the fly-ash percentage.

COMPARISON AND DISCUSSION

Comparison of results of various samples (LL, PL, PI & SI)

Table 10 Comparison of sub-grade soil and stabilized sub-grade soil (LL, PL, PI & SI)

S. No	Property	Sub-grade Soil	91% S.S + 09 % F.A	82 % S.S + 18 % F.A	73 % S.S + 27 % F.A	64 % S.S + 36 % F.A
1	Liquid limit (%)	36.05	33.75	31.85	32.75	35.75
2	Plastic limit (%)	22.55	24.75	23.95	25.25	27.45
3	Plasticity Index (%)	13.85	14.45	11.75	11.63	9.54
4	Shrinkage limit (%)	26.05	21.95	18.85	24.85	26.05

6.2 Comparison of results of various samples (MDD, OMC & UCS)

Table 11 Comparison of sub-grade soil and stabilized sub-grade soil (MDD, OMC & UCS)

S. No	Property	Sub-grade Soil	91% S.S + 09 % F.A	82 % S.S + 18 % F.A	73 % S.S + 27 % F.A	64 % S.S + 36 % F.A
1	Maximum dry Density (KN/m^3)	19.75	17.95	18.05	17.85	16.95
2	O.M.C. (%)	10.75	13.77	13.99	14.4	15.75
3	U.C.S.(KN/m^2)	58.75	60.85	90.85	88.85	86.35

In the above table the values represents the increases or decreases over the unmodified sub-grade soil property result. Analysis of test data in all the cases of sub-grade soil + fly-ash, the 82% S.S + 18% F.A. set gives optimized results than other as the value of UCS comes out to be Maximum In this case only and Also the Liquid Limit Is least in this case only. The Shrinkage Limit is also Low when the percentage of fly-ash is 18%. Three sets nearly 91% S.S + 09% F.A., 73% S.S. + 27% F.A &64% S.S. + 36% F.A. set. By observing the above results when 82% S.S + 18% F.A. are kept constant the most optimum results are obtained as shown in table 10 and table 11.

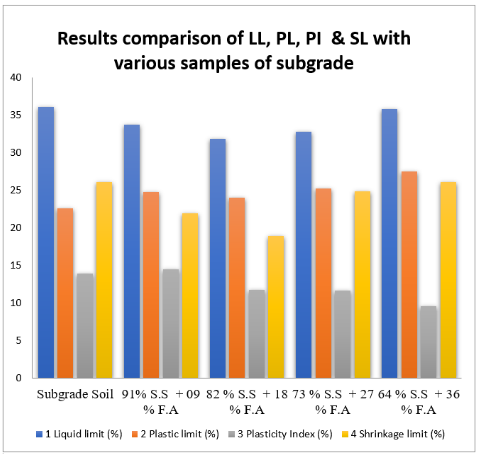

Fig. 7: Graphical comparison of soil sub-grade to the stabilized soil sub-grade (LL, PL, PI & SL)

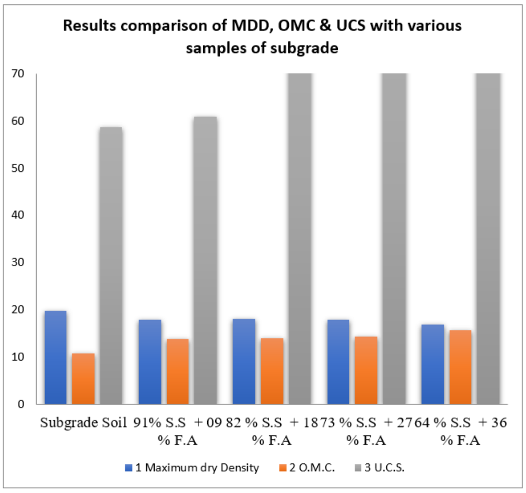

Fig. 8: Graphical comparison of soil sub-grade to the stabilized soil sub-grade (MDD, OMC & UCS)

Conclusion and future work

- With increased levels of fly-ash, the OMC increases while the MDD decreases. In addition, the optimal estimate of the acquired fly-ash blend was around 18%.
- It is also observed that UCS increases by roughly 27 percent of the fly-ash blend but thereafter decreases.
- This experiment shows that the derivative fly-ash is also an acceptable balancing out compound.
- The recommended proportions for the sub-grade soil + by item combination are 82 percent S.S + 218 percent F.A.
- The liquid limit increased as the level of fly ash increased, but the plastic limit decreased.
- The plasticity index of the soil is also reduced when more than 18 percent fly ash is added.
- When compared to the sub-grade soil, the unconfined compressive strength (UCS) of the balanced-out sub-grade soil is increased by adding 18% fly-ash.
- When 18 percent fly-ash is added, the shrinkage limit of stabilized sub-grade soil is reduced in comparison to sub-grade soil.

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