Effect of variable viscosity and thermal conductivity of free convective fluid along a vertical plate through porous medium

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***Abstract*: Effect of variable viscosity and thermal conductivity of free convective fluid along a vertical plate through porous medium has been studied. The dimensionless governing equations are solved using series solution technique. The influences of the various parameters on the flow field, Temperature field, Mass Concentration, Shearing Stress and rate of heat transfer are extensively discussed through graphs and tables.**

***Keywords*: Heat transfer, Mass transfer, MHD, Porous medium, free convection, variable thermal conductivity and variable viscosity.**

1.1 Introduction:

Magnetohydrodynamics plays a pivotal role in the present era for an alternative and more efficient means of power generation. This branch supersedes several other branches like Ocean dynamics, Chemical engineering, aerospace engineering, Plasma Jet, Pollution studies, Controlled Fusion Research, etc. Effects of a magnetic field on free convective flow past through porous medium bounded by an infinite vertical plate with constant heat flux was investigated by Raptis [1]. Senapati et. al. [2] have discussed the magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in the presence of chemical reaction. Senapati et. al. [3,4] have studied the effects of chemical reaction on MHD unsteady free convection flow through a porous medium bounded by a linearly accelerated vertical plate, and also effects of chemical reaction on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. Coupled heat and mass transfer by natural convection in fluid - saturated porous medium has drawn attention in last few years in the field of many important engineering and geophysics applications.

Investigations are made to know how the flow field, shear stress and heat flux vary within the boundary layer with thermal jump at the wall when the thermal conductivity is of temperature dependent. Hamad et. al. [5] analysed numerically using finite difference method jump effects on boundary layer flow of a Jeffery fluid near stagnation point on stretching / shrinking sheet with variable thermal conductivity. Idowu et. al. [6] have reported effects of heat and mass transfer on unsteady MHD oscillatory flow of Jeffery fluid in a horizontal channel with chemical reaction analytically using perturbation method. Uwanta and Omokhuale [7] have investigated effects of variable thermal conductivity on heat and mass transfer with Jeffery fluid numerically using implicit finite difference method. Okedoye and Asibor [8] have studied the effects of variable viscosity on magneto-hydrodynamic flow near a stagnation point in the presence of heat generation/absorption.

Keeping in view of the above analysis, the aim of the present investigation is to study the effects of variable viscosity and thermal conductivity of free convective fluid along a vertical plate through porous medium.

1.2 Formulation of problem: Let us consider a steady laminar electrically conducting incompressible MHD free convective fluid of variable viscosity and thermal conductivity flow along a vertical plate through porous medium. Let X' - axis is along the plate which moves with a constant velocity U_o. The magnetic field lines of strength H_o are supposed to be perpendicular to the plate along Y'- axis and magnetic permeability μ_e is constant throughout the field. It is assumed that the field has constant properties and variation in density is taken into account only in body force term. The temperature and mass concentration are respectively T'_∞ and C'_∞ throughout the field, whereas the temperature and mass concentration near the plate are respectively T'_w and C'_w . Then by using Boussineq's approximation the steady flow governed by the following equations:

$$\frac{\partial V'}{\partial y'} = 0 \implies v' = V_0 \qquad (1)$$

$$-V_0\frac{du'}{dy'} = \frac{1}{\rho}\frac{d}{dy'}\left(\mu\frac{du'}{dy'}\right) - \frac{\mu}{\rho}\frac{u'}{K'} - \frac{\sigma B_0^2 u'}{\rho} + g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty) \qquad (2)$$

$$-V_0\frac{dT'}{dy'} = \frac{1}{\rho C_p}\frac{d}{dy'}\left(k\frac{dT'}{dy'}\right) \qquad (3)$$

$$-V_0\frac{dC'}{dy'} = D\frac{d^2C'}{dy'^2} \qquad (4)$$

with boundary conditions

$$\left.\begin{aligned} &u' = U_0\ , T' = T'_w + \epsilon(T'_w - T'_\infty), C' = C'_w \text{ at } y' = 0\\ &u' = 0, T' = T'_\infty, C' = C'_\infty \ \text{ as } y' \to \infty \end{aligned}\right\} \qquad (5)$$

As the viscosity and thermal conductivity as variable and linear function of temperature, so it is assumed as

$$\left.\begin{aligned}\mu &= \mu_0\left(1 - a\epsilon\left(\frac{T'-T'_\infty}{T'_w-T'_\infty}\right)\right)\\ k &= k_0\left(1 + b\epsilon\left(\frac{T'-T'_\infty}{T'_w-T'_\infty}\right)\right)\end{aligned}\right\} \tag{6}$$

Now let us introduce the following non-dimensional quantities

$$\left.\begin{aligned}&u = \frac{u'}{U_0}, y = \frac{y'V_0}{\nu_0}, \nu_0 = \frac{\mu_0}{\rho}, \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty}, C = \frac{C'-C'_\infty}{C'_w-C'_\infty}\\ &Gr = \frac{\nu_0 g\beta(T'-T'_\infty)}{V_0^2 U_0}, Gm = \frac{\nu_0 g\beta_c(C'-C'_\infty)}{V_0^2 U_0}, M = \frac{\sigma B_0^2 \nu_0}{\rho V_0^2}\\ &Pr = \frac{\rho\nu_0 C_p}{k_0}, Sc = \frac{\nu_0}{D}, \frac{1}{K_p} = \frac{\nu_0^2}{K' V_0^2}\end{aligned}\right\} \tag{7}$$

By substituting equations (6) and (7) in equations (2) to (5), we get

$$(1 - a\epsilon\theta)\frac{d^2u}{dy^2} - a\epsilon\left(\frac{d\theta}{dy}\right)\left(\frac{du}{dy}\right) + \frac{du}{dy} - \frac{1}{K}\left((1 - a\epsilon\theta)u\right) - Mu + Gr\theta + GmC = 0 \tag{8}$$

$$(1 + b\epsilon\theta)\frac{d^2\theta}{dy^2} + b\epsilon\left(\frac{d\theta}{dy}\right)^2 + Pr\left(\frac{d\theta}{dy}\right) = 0 \tag{9}$$

$$\frac{d^2C}{dy^2} + Sc\frac{dC}{dy} = 0 \tag{10}$$

with the boundary conditions

$$\left.\begin{aligned}&u = 1, \theta = 1 + \epsilon, C = 1 \text{ at } y = 0\\ &u = 0, \theta = 0, C = 0 \text{ as } y \to \infty\end{aligned}\right\} \tag{11}$$

By solving equation (10) using (11), we get

$$C = e^{-Scy} \tag{12}$$

1.3 Method of solution:

In order to solve equations (8) and (9), let us assumed

$$\left.\begin{aligned}u &= u_0 + \epsilon u_1 + O(\epsilon)\\ \theta &= \theta_0 + \epsilon\theta_1 + O(\epsilon)\end{aligned}\right\} \tag{13}$$

and substitute in equations (8) and (9) with boundary condition (11), we get the following by comparing constant and co-efficient of ϵ :

$$\frac{d^2u_0}{dy^2} + \frac{du_0}{dy} - \left(M + \frac{1}{K}\right)u_0 = -Gr\theta_0 - GmC \tag{14}$$

$$\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} - \left(M + \frac{1}{K}\right)u_1 = a\theta_0\frac{d^2u_0}{dy^2} - a\frac{du_0}{dy}\frac{d\theta_0}{dy} - \frac{a}{K}u_2\theta_0 - Gr\theta_1 \tag{15}$$

$$\frac{d^2\theta_0}{dy^2} + Pr\frac{d\theta_0}{dy} = 0 \tag{16}$$

$$\frac{d^2\theta_1}{dy^2} + Pr\frac{d\theta_1}{dy} = -b\theta_0\frac{d^2\theta_0}{dy^2} - b\left(\frac{d\theta_0}{dy}\right)^2 \tag{17}$$

with the boundary conditions

$$\left.\begin{aligned}&u_0 = 1, u_1 = 0, \theta_0 = 1, \theta_1 = 1 \text{ at } y = 0\\ &u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0 \text{ as } y \to \infty\end{aligned}\right\} \tag{18}$$

By solving equations (14) to (17) using (18), we get

$$u = a_{14}e^{-b_1y} - a_{12}e^{-Pry} - a_{13}e^{-Scy}$$
$$+\epsilon\left(a_{20}e^{-b_1y} + a_{15}e^{-(b_1+Pr)y} + a_{16}e^{-2Pry} + a_{17}e^{-(Pr+Sc)y} + a_{18}e^{-Pry} + a_{19}ye^{-Pry}\right) \tag{19}$$

$$\theta = e^{-Pry} + \epsilon\left(a_{11}e^{-Pry} - \frac{b}{2}e^{-2Pry} - \frac{b}{Pr}ye^{-Pry}\right) \tag{20}$$

The non-dimensional skin friction

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = (-b_1a_{11} + a_{12}Pr + a_{13}Sc) - \epsilon\begin{pmatrix}a_{20}b_1 + a_{15}(b_1 + Pr) + 2Pra_{16}\\ +(Pr + Sc)a_{17} + Pra_{18} - a_{19}\end{pmatrix} \tag{21}$$

The non-dimensional Nusselt number

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = Pr + \epsilon\left(a_{11}Pr - bPr - \frac{b}{Pr}\right) \tag{22}$$

where

$$b_1 = \frac{1 + \sqrt{1 + 4\left(M + \frac{1}{K}\right)}}{2}, \quad a_{11} = 1 + \frac{b}{2}, a_{12} = \frac{Gr}{Pr^2 - Pr - \left(M + \frac{1}{K}\right)},$$

$$a_{13} = \frac{Gm}{Sc^2 - Sc - \left(M + \frac{1}{K}\right)}, \quad a_{14} = 1 + a_{12} + a_{13}$$

$$a_{15} = \frac{aa_{14}(ab_1^2 - b_1Pr - 1)}{(b_1 + Pr)^2 - (b_1 + Pr) - \left(M + \frac{1}{K}\right)}, \quad a_{16} = \frac{aa_{12} + Gr\frac{b}{2}}{4Pr^2 - 2Pr - \left(M + \frac{1}{K}\right)},$$

$$a_{17} = \frac{aa_{13}(1 + ScPr - Sc^2)}{(Sc + Pr)^2 - (Sc + Pr) - \left(M + \frac{1}{K}\right)}$$

$$a_{18} = \frac{Grb(2Pr + 1)}{Pr\left(Pr^2 - Pr - \left(M + \frac{1}{K}\right)\right)^2} - \frac{Gra_{11}}{Pr^2 - Pr - \left(M + \frac{1}{K}\right)},$$

$$a_{19} = \frac{Grb}{Pr\left(Pr^2 - Pr - \left(M + \frac{1}{K}\right)\right)}, \quad a_{20} = -(a_{18} + a_{15} + a_{16} + a_{17})$$

1.4 Result and Discussion:

In this paper, we have studied the effect of variable viscosity and thermal conductivity of free convective fluid along a vertical plate through porous medium. The effects of the parameters M, K, Gr, Gm, Sc, Pr, a and b on flow characteristics have been studied and shewn by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocities, heat and mass concentration are taken w.r.t. y and the values of shearing stress and Nusselt number are shewn in the tables for different values of flow parameters.

Velocity profiles: The velocity profiles are depicted in Figs. 1-4. Figure (1) shews the effects of the parameters M and K on velocity at any point of the fluid, when Gr = 2, Gm = 2, Sc = 2, Pr = 2, a = 0.2, b = 0.2, $\epsilon = 0.04$. It is noticed that the velocity increases with the increase of permeability parameter of porous medium (K), whereas decreases with the increase of magnetic parameter (M), physically the application of transverse magnetic field always results in resistive type force called Lorentz force which tends to resist the fluid motion, finally reducing the velocity. Also by increasing the permeability parameter, drag force decreases, which leads to velocity increases.

Figure (2) shows the effects of the parameters Gr and Gm on velocity at any point of the fluid, when M = 2, K = 2, Sc = 2, Pr = 2, a = 0.2 and b = 0.2, $\epsilon = 0.04$. It is noticed that the velocity increases with the increase of Grashof number (Gr) and modified Grashof number (Gm). Physically, convective amount due to cooling/heating at the plate decreases the concentration which enhances the magnitude of velocity.

Figure-(3) shows the effects of parameters Sc and Pr on velocity at any point of the fluid, when M = 2, K = 2, Sc = 2, Pr = 2, a = 0.2 and b = 0.2, $\epsilon = 0.04$. It is noticed that the velocity decreases with the increase of Prandtl number (Pr) and Schmidt number (Sc). Physically, kinematic viscosity is dominated by both mass and thermal diffusion.

Figure-(4) shows the effect of the parameters a and b on velocity at any point of the fluid, when M = 2, K = 2, Sc = 2, Pr = 2, $\epsilon = 0.04$, Gr = 2 and Gm = 2. It is noticed that the velocity increases with the increase of variable viscosity parameter (a) and variable thermal conductivity parameter (b).

Temperature Profile: The temperature Profile is depicted in Fig-5-6. Figure – (5) shows the effect of ϵ and Pr, when all other parameters are absents except b = 0.2. It is noticed that temperature falls by the increase of ϵ and Prandtl number (Pr).

Figure-(6) shows the effects of ϵ and b, when all other parameters are absent except Pr = 2. It is noticed that temperature rises for $0 < y < 0.8$ and falls for $0.8 < y$ by the increase of variable thermal conductivity parameter (b) and ϵ.

Mass concentration Profile: The mass concentration profile is depicted in figure-7. Figure- (7) shows the effect of ϵ and Sc on Mass concentration profile in the absence of all other parameters. It is noticed that the mass concentration decreases with the increase of ϵ and Schmidt number (Sc).

Table-(1) shows the effects of different parameters on Shearing stress. It is noticed that shearing stress increases in the increase of variable viscosity parameter (a), (ϵ), Prandtl number (Pr) and Magnetic parameter (M), whereas decreases with the increase of permeability parameter (K) of porous medium, Schmidt number (Sc), thermal conductivity parameter (b), Grashof number (Gr) and modified Grashof number (Gm).

Table-(2) shows the effects of different parameters on Nusselt Number. It is noticed that the Nusselt number increases with the increase of Prandtl Number (Pr) and (ϵ), whereas decreases with the increase of thermal conductivity parameter (b).

Table-1: Effect of Shearing Stress/Skin friction near the plate

S.No.	M	K	Gr	Gm	Pr	Sc	ϵ	a	b	τ_0
1	1	0.5	2	2	1	0.71	0.02	0.2	0.2	-1.31
2	1	0.5	2	2	1	0.71	0.02	0.2	5	-18.56
3	1	0.5	2	2	1	0.71	0.02	0.2	10	-61.03
4	1	0.5	2	2	1	0.71	0.02	4	0.2	-1.18
5	1	0.5	2	2	1	0.71	0.02	5	0.2	-1.11
6	1	0.5	2	2	1	0.71	0.2	0.2	0.2	-1.11
7	1	0.5	2	2	1	0.71	0.4	0.2	0.2	-0.89
8	1	0.5	2	2	2	0.71	0.02	0.2	0.2	-4.65
9	1	0.5	2	2	3	0.71	0.02	0.2	0.2	1.36

10	1	0.5	2	2	1	1	0.02	0.2	0.2	-1.53
11	1	0.5	2	2	1	2	0.02	0.2	0.2	-4.87
12	1	0.5	2	3	1	0.71	0.02	0.2	0.2	-1.64
13	1	0.5	2	-1	1	0.71	0.02	0.2	0.2	-0.31
14	1	0.5	3	2	1	0.71	0.02	0.2	0.2	-1.52
15	1	0.5	1	2	1	0.71	0.02	0.2	0.2	-1.1
16	1	1.5	2	2	1	0.71	0.02	0.2	0.2	-2.15
17	1	2.5	2	2	1	0.71	0.02	0.2	0.2	-2.5
18	2	0.5	2	2	1	0.71	0.02	0.2	0.2	-1.04
19	3	0.5	2	2	1	0.71	0.02	0.2	0.2	-0.88

Table-2: Effect of Nusselt Number near the plate

S.No	Pr	ϵ	b	Nu
1	0.71	0.02	0.2	0.72
2	0.71	0.02	5	0.55
3	0.71	0.02	10	0.37
4	0.71	0.2	0.2	0.78
5	0.71	0.4	0.2	0.85
6	1	0.02	0.2	1.01
7	2	0.02	0.2	2.03

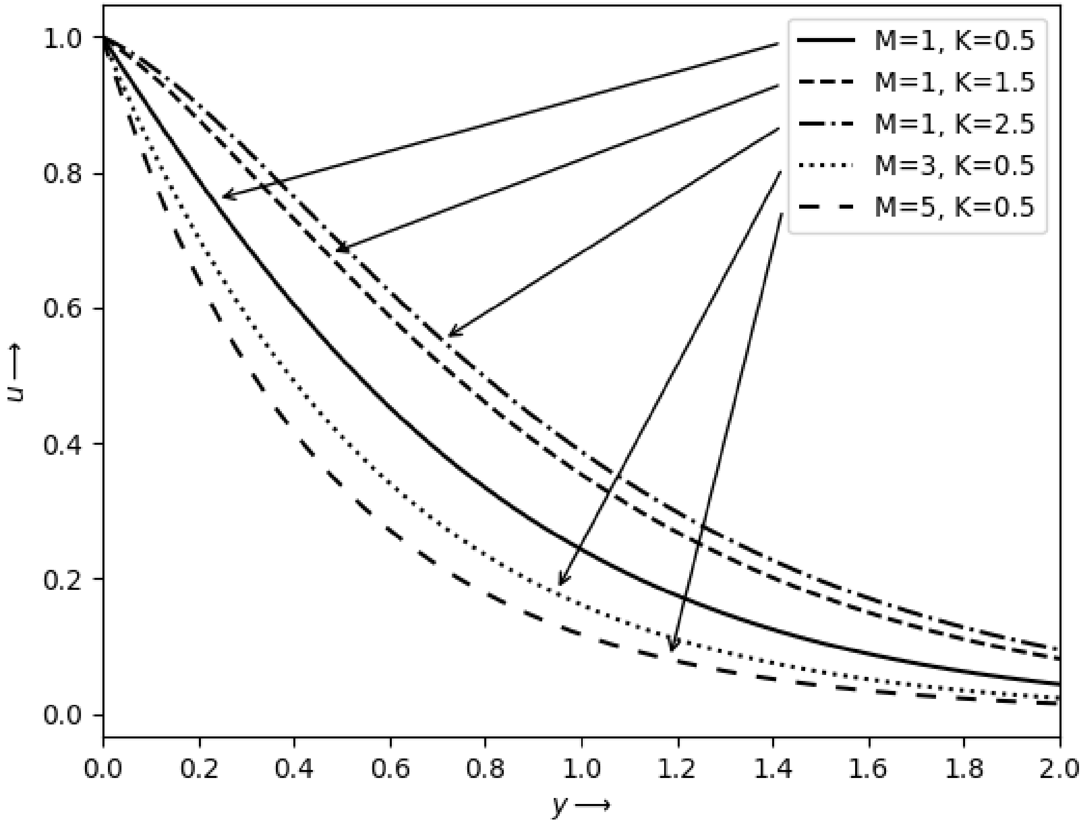

Fig-(1): Effect of M and K on velocity profile, when Gr = 2, Gm = 2, Sc = 2, Pr = 2, $\epsilon = 0.04$, a = 0.2 and b = 0.2.

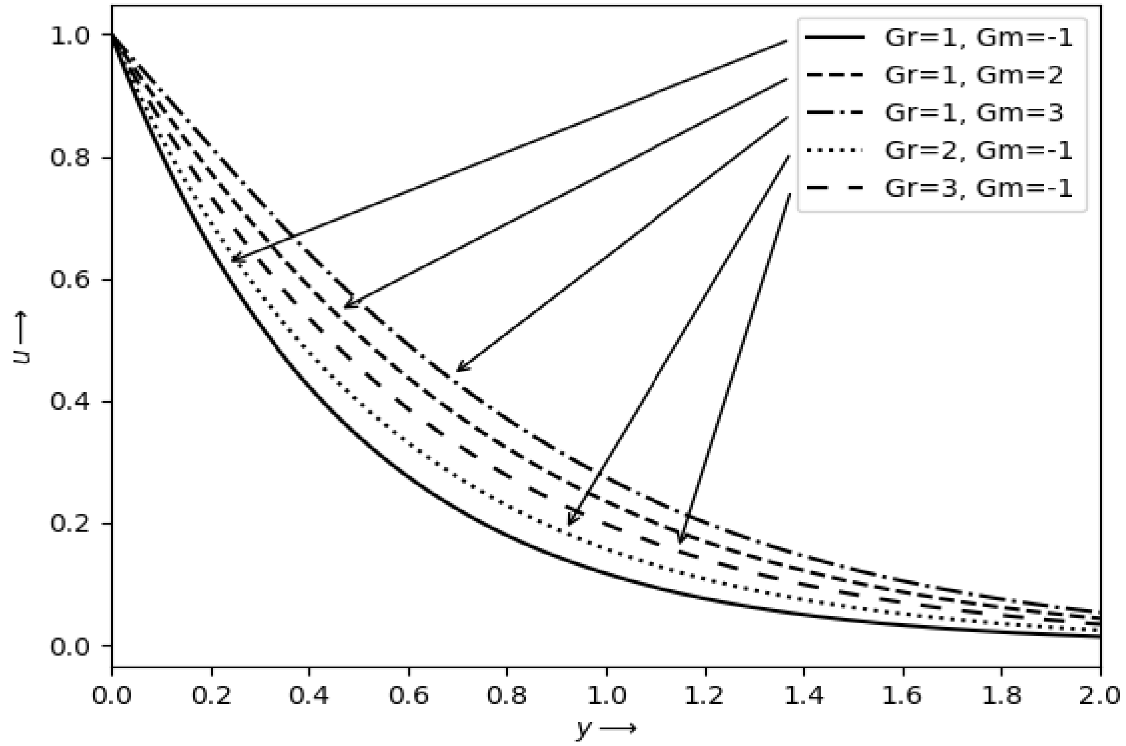

Fig-(2): Effect of Gr and Gm on velocity profile, when M = 2, K = 2, Sc = 2, Pr = 2, $\epsilon = 0.04$, a = 0.2 and b = 0.2.

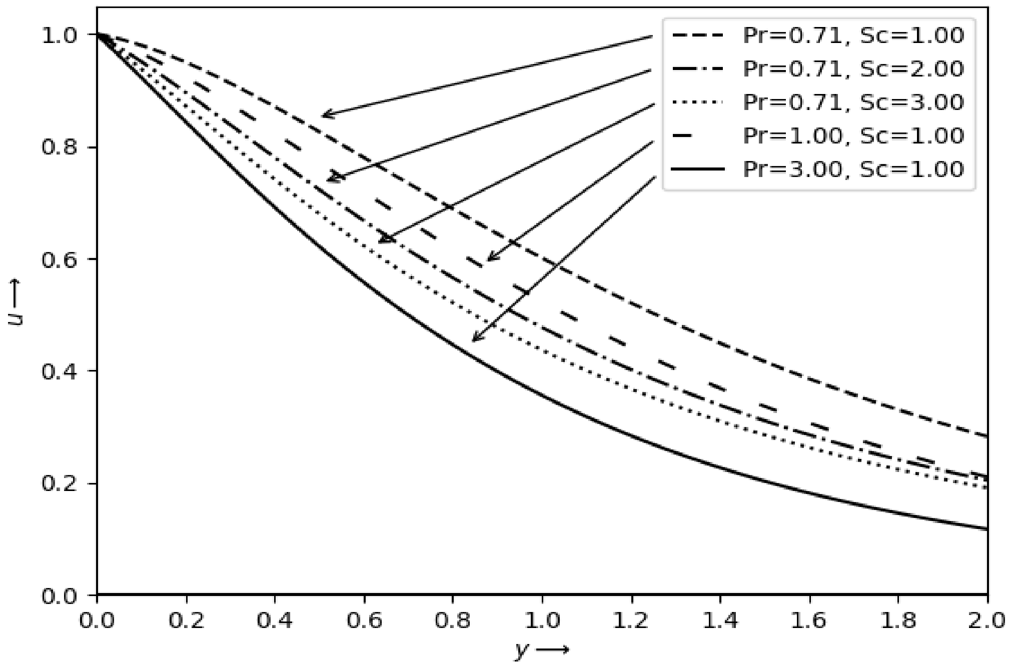

Fig-(3): Effect of Pr and Sc on velocity profile, when M = 2, K = 2, Gr = 2, Gm = 2, $\epsilon = 0.04$, a = 0.2 and b = 0.2.

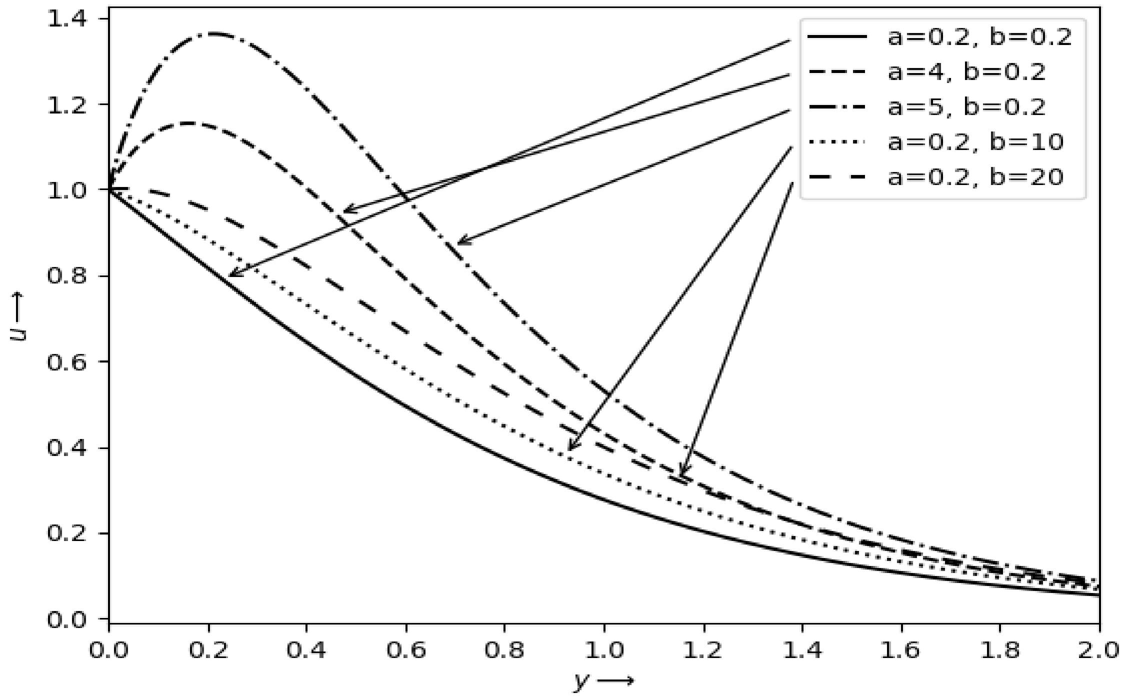

Fig-(4): Effect of a and b on velocity profile, when M = 2, K = 2, Gr = 2, Gm = 2, $\epsilon = 0.04$, Sc = 2 and Pr = 2.

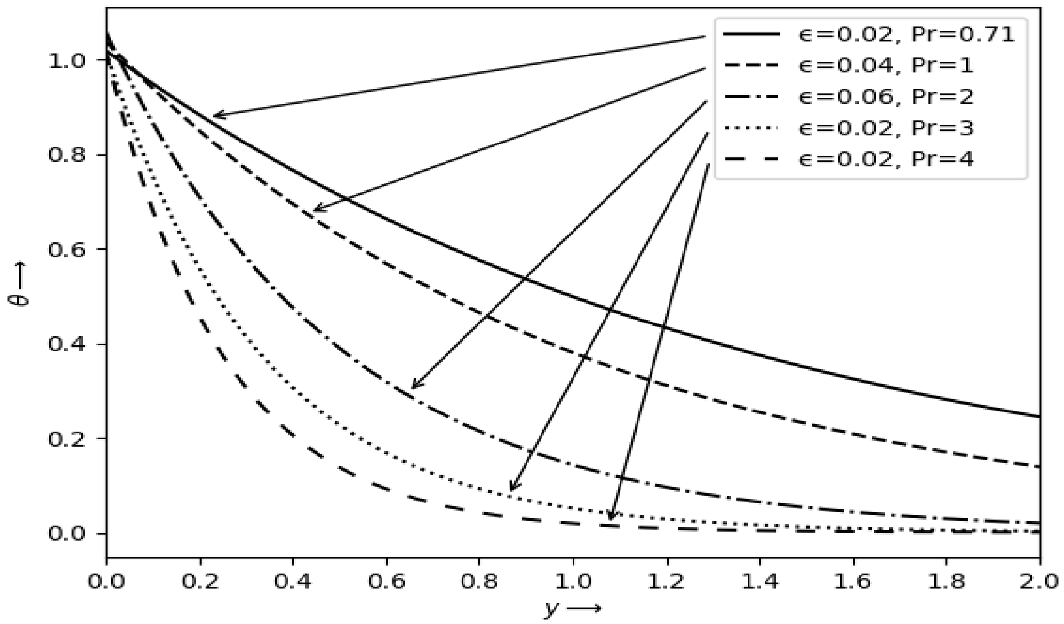

Fig-(5): Effect of ϵ and Pr on Temperature profile, when b = 0.2 and in the absence of other parameters.

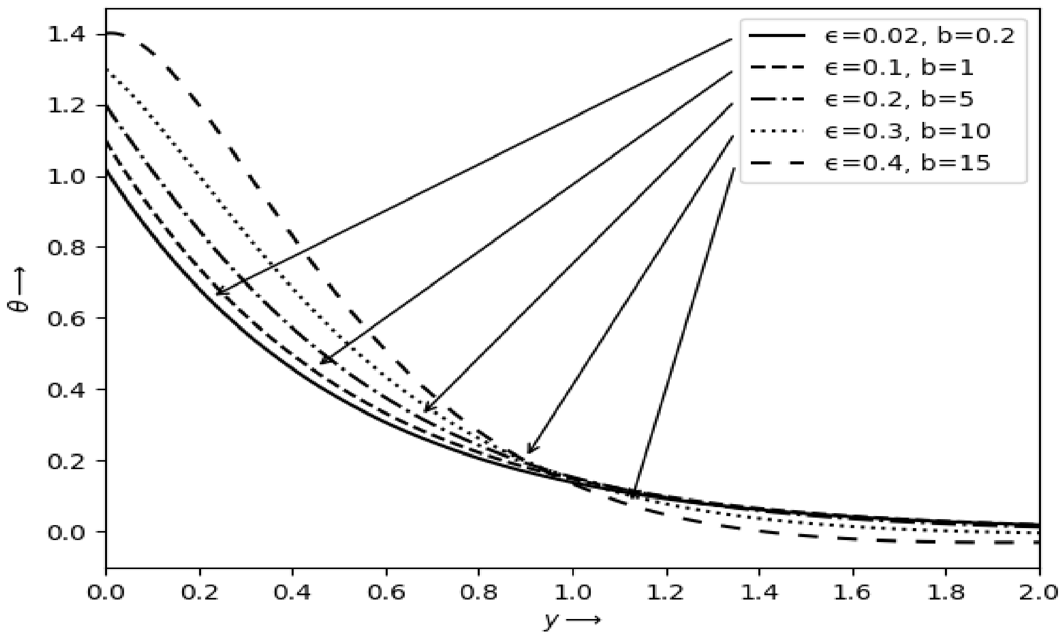

Fig-(6): Effect of ϵ and b on Temperature profile, when Pr = 2 and in the absence of other parameters.

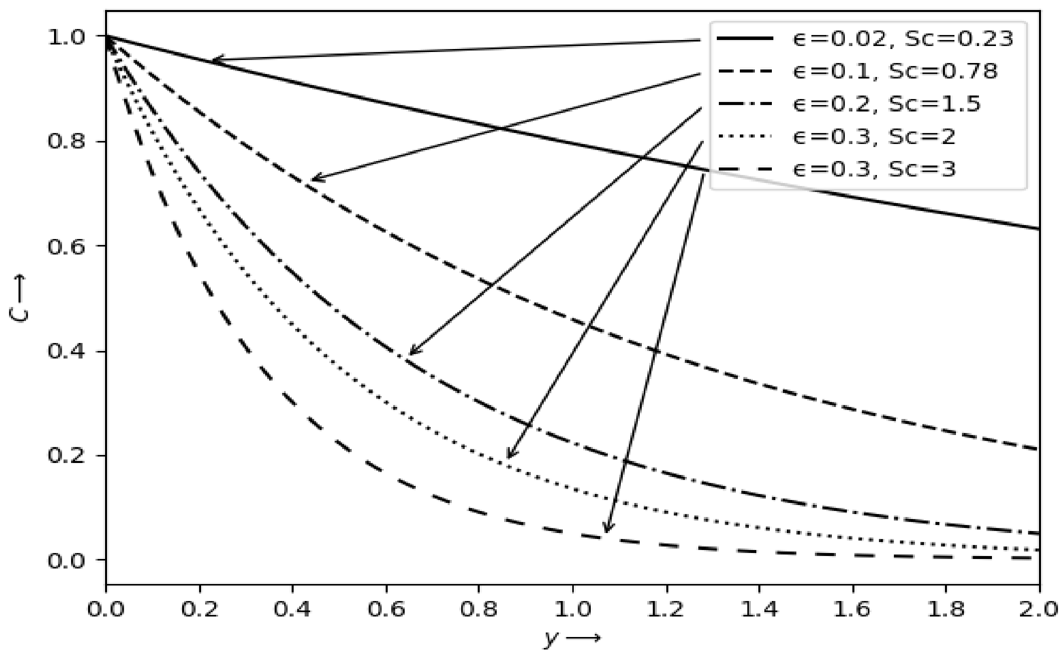

Fig-(7): Effect of ϵ and Sc on Mass concentration profile in the absence of other parameters.

1.5 Conclusion:

In this investigation, "Effect of variable viscosity and thermal conductivity of free convective fluid along a vertical plate through porous medium", the following points are set out:

1. Lorentz force exerted due to transverse magnetic field tends to resist the fluid motion.
2. Also by increasing the permeability parameter, drag force decreases which leads to velocity increase.
3. Convective amount due to cooling/heating at the plate decreases the concentration, which enhances the magnitude of velocity.
4. Increase of thermal parameter and variable viscosity parameter increases the velocity, but plays opposite role in skin friction.
5. Thermal conductivity parameter leads to rise of temperature initially and afterwards, temperature decreases.

6. Increase in thermal conductivity decreases the Nusselt number.
7. Mass concentration and temperature decreases by the increase of ϵ, Schmidt number (Sc) and Prandtl number (Pr), respectively.

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