Interacting Dark matter and Holographic Dark Energy in Kantowski Sachs Universe with Constant Deceleration Parameter

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***Abstract*: In this paper, we deal with the Kantowski-Sachs universe filled with Interacting dark matter and Holographic dark energy. The solutions of the field equations are obtained by using constant deceleration parameter. The geometrical and physical aspect are studied in details.**

1] Introduction: -

The observations from distant type Ia supernova, the combination of the anisotropies of the Cosmic Microwave Background Radiation (CMBR) and the mass-energy density estimates from galaxy clusters, weak lensing and large-scale structure suggest that the universe is undergoing a phase of accelerated expansion [Perlmutter, S, *et. al*. 1998 ; Riess, A.G. *et. al*. 1998]. This expansion has been responsible exotic component, called dark energy, with negative pressure which can induce repulsive gravitational force causing the accelerated expansion. The wilkinson Microwave Anisotropy Probe (WMAP) shows that the Dark Energy (DE) has about 73% of the energy of our universe and Dark Matter (DM) occupies 23% also the baryon matter occupies only 4% of the total energy of the universe. Further investigate the property of Dark Energy, many researchers proposed the model such as quintessence with EoS $\omega > -1$ (Barreiro *et.al.* 2000), phantom with EoS $\omega < -1$(Caldwell 2003), tachyon (Bagla *et.al.* 2003; Sen 2002).
Another most talked about form of Dark Energy is the so-called Holographic dark energy (HDE) based on the holographic principle in quantum gravity theory. The holographic dark energy model was originally proposed by Nojiri and Odintsov (2007). Recently, the holographic dark energy model with Chaplygin gas [M.R. Setare, 2007] and with modified Chaplygin gas [B.C. Paul, 2007] have been investigated. In recent years, the holographic dark energy has been studied as a possible candidate for dark energy. It is motivated from the holographic principle which might lead to the quantum gravity to explain the events involving high energy scale. In the thermodynamics of black hole, there is a maximum entropy in a box of length L, commonly termed, the Bekenstein-Hawking entropy bound $S \square M_p^2 L^2$, which scales as the area of the box $A \square L^2$ [Bekenstein, 1973]. In consecutive researchers, Granda and Oliveros (2008) considered a holographic density of the form $\rho_\Lambda \approx \alpha H2 + \beta H^\cdot$ where H is the Hubble parameter and α, β are constants which must satisfy the restrictions imposed by the current observational data. This new model of dark energy represents the accelerated expansion of the universe and is consistent with the current observational data. Granda and Oliveros (2009) have also shown the correspondence between different scalar field dark energy models with this holographic dark energy model for a flat FRW universe. HDE model have been tested and constrained by various astronomical observations such as [Zhang and Wu, 2005; Enqvist, Hannestad and Sloth, 2005; Shen, Wang, Abdalla and Su, 2005; Chang, Wu and Zhang, 2006]. A special class are models in which holographic DE is allowed to interact with DM [Pavon and Zimdahl, 2005; Wang *et. al.* 2005; Carvalho and Saa, 2004; Perivolaropoulos, 2005; Gong, 2004; Wang *et. al.* 2006; Nojiri and Odintsov, 2006; Guberina, *et al.* 2005, 2006; Guo, *et. al.* 2005,2007; Hu and Ling, 2006; Li, *et. al.* 2006; Setare, 2006, 2007; Zimdahl and Pavon, 2007].
Sarkar (2014a, 2014b, 2014c) have studied non-interacting holographic dark energy with linearly varying deceleration parameter in Bianchi type-I & V and Kantowski-Sachs universe and interacting holographic dark energy in Bianchi type-II respectively and Adhav *et. al.* (2014) have been studied interacting dark matter and holographic dark energy in an anisotropic universe. In addition, the proposal of interacting dark energy is compatible with the current observations such as the SNIa and CMB data (Guo, Ohta and S. Tsujikawa, 2007).

Recently Shikha Srivastava *et. al.* (2019) studied New holographic dark energy in bianchi- III universe with k-essence; R. L. Naidu (2019) studied Bianchi type-II modified holographic Ricci dark energy cosmological model in the presence of massive scalar field; Hatkar *et. al.* (2020) studied Viscous holographic dark energy in Brans–Dicke theory of gravitation.

Motivated by the above discussion, in the present paper, we consider spatially homogeneous and anisotropic Kantowski-Sachs universe filled with interacting Dark matter and Holographic dark energy. The general solutions of the Einstein's field equations in Kantowski Sachs space-time have been obtained under the assumption of constant deceleration parameter. The physical and geometrical aspects of the models are also discussed.

2] Metric and Field Equation: -

The Kantowski-Sachs universe given by

$$ds^2 = dt^2 - a_1^2(t)dr^2 - a_2^2(t)(d\theta^2 + \sin^2\theta d\varphi^2) \quad , \quad (1.1)$$

where $a_1(t)$ and $a_2(t)$ are the cosmic scale factor.

The Einstein's field equation in natural limit ($8\pi G = 1$ *and* $c = 1$) is given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -({}^{DM}T_{ij} + {}^{DE}T_{ij}) \quad . \tag{2.2}$$

The energy momentum tensor for matter

$${}^{DM}T_{ij} = diag[\rho_{DM}, 0, 0, 0] \quad , \tag{2.3}$$

where ρ_{DM} is the energy density of dark matter (pressureless i.e. $\omega_{DM} = 0$).

Here we consider the energy momentum tensor for holographic dark energy in the form

$${}^{DE}T_{ij} = diag[\rho_{DE,}\ -P_{DEr},\ -P_{DE\theta},\ -P_{DE\varphi}] \quad , \tag{2.4}$$

where ρ_{DE} are the energy density and $-P_{DEr}, -P_{DE\theta}, -P_{DE\varphi}$ are pressure of holographic dark energy in the direction of $r, \theta, and\ \varphi$ respectively.

$${}^{DE}T_{ij} = diag[1, -\omega_r, -\omega_\theta, -\omega_\varphi]\rho_{DE} \quad ,$$

$${}^{DE}T_{ij} = diag[1, -\omega, -(\omega+\gamma), -(\omega+\gamma)]\rho_{DE} \quad , \tag{2.5}$$

where $\omega_r = \omega,\ \omega_\theta = (\omega+\gamma),\ \omega_\varphi = (\omega+\gamma)$ are the directional EoS parameters of holographic dark energy. The skewness parameter γ that is the deviation from ω. Also $\omega\, and\ \gamma$ are not necessarily constant and might be function of time.

In comoving coordinate systems, the Einstein's field equations (2.2) for the metric (2.1) with the help of Eq. (2.3) can be written as

$$2\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 + \frac{1}{a^2{}_2} = \rho_{DM} + \rho_{DE} \quad , \tag{2.6}$$

$$2\frac{\ddot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 + \frac{1}{a_2^2} = -\omega\rho_{DE} \quad , \tag{2.7}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = -(\omega+\gamma)\rho_{DE} \quad , \tag{2.8}$$

where an overhead dot (·) represents derivative with respect to time *t*.
The Directional Hubble Parameter in the direction of $r\ ,\theta\ \ and\ \ \varphi$ respectively and Average Hubble parameter is defined as

$$H_r = \frac{\dot{a}_1}{a_1}\ ,\ H_\theta = \frac{\dot{a}_2}{a_2}\ \ and\ \ H_\varphi = \frac{\dot{a}_2}{a_2} \quad , \tag{2.9}$$

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_r + 2H_\theta) \quad . \tag{2.10}$$

The Volume V and Average Scale factor a is defined as

$$V = a^3 = a_1 a_2^{\ 2}\ , \qquad a = \left(a_1 a_2^{\ 2}\right)^{1/3} \quad . \tag{2.11}$$

The deceleration parameter $q(t)$ is defined by

$$q = -\frac{a\,\ddot{a}}{\dot{a}^2}\ . \tag{2.12}$$

The mean anisotropy parameter of expansion (Δ) is defined by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 . \tag{2.13}$$

Using equation (2.9)-(2.11), the Anisotropy parameter can be written as

$$\Delta = \frac{2}{9H^2}(H_\theta - H_r)^2 \quad . \tag{2.14}$$

Subtracting equation (2.7) from (2.8), we get

$$\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1} = H_\theta - H_r = \frac{\lambda}{V} + \frac{1}{V}\int\left(\gamma\rho_{DE} - \frac{1}{a_2^2}\right)Vdt \quad , \tag{2.15}$$

where λ is the constant of integration and term γ arise due to the possible intrinsic anisotropy of the fluid.

Using (2.15) in (2.14), we obtain anisotropy parameter as

$$\Delta = \frac{2}{9}\frac{1}{H^2}\left[\lambda + \int\left(\gamma\rho_{DE} - \frac{1}{a_2^2}\right)Vdt\right]^2 V^{-2} \quad . \tag{2.16}$$

The integral term in (3.8) vanishes

$$\gamma = \frac{1}{\rho_{DE}a_2^2} \quad . \tag{2.17}$$

Using (3.9) in (3.8) we obtain anisotropy parameter as

$$\Delta = \frac{2}{9}\frac{\lambda^2}{H^2}V^{-2} \quad . \tag{2.18}$$

The Einstein's field equation (2.6)-(2.8) with the use of (2.17) arises as

$$2\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 = \rho_{DM} + (1-\gamma)\rho_{DE} \quad , \tag{2.19}$$

$$2\frac{\ddot{a}_2}{a_2}+\left(\frac{\dot{a}_2}{a_2}\right)^2=-(\omega+\gamma)\rho_{DE} \quad , \tag{2.20}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}=-(\omega+\gamma)\rho_{DE} \quad . \tag{2.21}$$

Using equation (2.19)-(2.21) and (2.11) the scale factors $a_1(t)$ *and* $a_2(t)$ can be written explicitly as

$$a_1(t)=D_2^2V^{1/3}\exp\left(2D_1\int V^{-1}dt\right) \quad , \tag{2.22}$$

$$a_2(t)=D_2^{-1}V^{1/3}\exp\left(-D_1\int V^{-1}dt\right) \quad , \tag{2.23}$$

where D_1 *and* D_2 are constant.

The holographic dark energy density is given by

$$\rho_{DE}=3\left(\alpha_1 H^2+\beta_1\dot{H}\right), \tag{2.24}$$

i.e. $\rho_{DE}=3\left(\alpha_1 H^2+\beta_1\dot{H}\right)$ with $M_p^{-2}=8\pi G=1$ (Granda and Oliveros 2008).

For the universe, where dark energy and dark matter are interacting to each other the total energy density $(\rho=\rho_{DM}+\rho_{DE})$ satisfies the equation of continuity as

$$\dot{\rho}_{DM}+\dot{\rho}_{DE}+3H(\rho_{DM}+\rho_{DE}+p_{DE})=0. \tag{2.25}$$

Assuming that the dark matter component is interacting with the dark energy component through an interaction term *Q*, the continuity equation of matter and dark energy can be obtained as

$$\dot{\rho}_{DM}+\left(\frac{\dot{V}}{V}\right)\rho_{DM}=Q\ , \tag{2.26}$$

$$\dot{\rho}_{DE}+\left(\frac{\dot{V}}{V}\right)(1+\omega)\rho_{DE}=-Q\ , \tag{2.27}$$

where $\omega=\frac{p_{DE}}{\rho_{DE}}$ is the equation of state parameter for holographic dark energy and $Q>0$ measures the strength of the interaction.
A vanishing *Q* implies that dark matter and dark energy are separately conserved. In view of continuity equations, the interaction between dark energy and dark matter must be a function of the energy density multiplied by a quantity with units of inverse of time, which can be chosen as the Hubble parameter *H*. There is freedom to choose the form of the energy density, which can be any combination of dark energy and dark matter. Thus, the interaction between dark energy and dark matter could be expressed phenomenologically in the forms as [Guo *et. al.* 2007a, 2007b; Amendola *et. al.* 2007]

$$Q=3b^2H\rho_{DM}=b^2\frac{\dot{V}}{V}\rho_{DM} \quad , \tag{2.28}$$

where b^2 is coupling constant.
Cai and Wang (2005) have taken same relation for interacting dark matter and phantom dark energy in order to avoid the coincidence problem.

Using Eqs. (2.26) and (2.28), we get the energy density of dark matter as

$$\rho_{DM}=\rho_0V^{\left(b^2-1\right)} \quad , \tag{2.29}$$

where $\rho_0 > 0$ is real constant of integration.

Using Eqs. (2.28) and (2.29), we get the interacting term Q as

$$Q = 3\rho_0 b^2 H V^{(b^2-1)} . \qquad (2.30)$$

3] Cosmological Solutions for Constant Deceleration Parameter:-

In order to obtain the solutions of the Eqs. (2.19) -(2.21), we assume the special law of variation for the Hubble parameter which yields the constant value of deceleration parameter (Berman, 1983). According to this law the variation of the mean Hubble parameter is given by

$$H = k\, a^{-n} \quad , \qquad (3.1)$$

where $k > 0$ and $n \geq 0$.

Here we obtain a cosmological models (i) Model for $n = 0$.

Model for $n = 0$ [Exponential Volumetric Expansion Model]:

For $n = 0$, Eq. (3.1) gives the volume scale factor as

$$V = c_1 e^{3kt} \quad , \qquad (3.2)$$

where $c_1 > 0$ is a constant of integration.

Using Eq. (3.2) in Eqs. (2.22)-(2.23), we obtain the exact values of scale factors as

$$a_1(t) = D_2^2 c_1^{1/3} \exp\left(kt - \frac{2D_1}{3c_1 k} e^{-3kt} \right) , \qquad (3.3)$$

$$a_2(t) = D_2^{-1} c_1^{1/3} \exp\left(kt + \frac{D_1}{3c_1 k} e^{-3kt} \right) . \qquad (3.4)$$

Using Eq. (3.2) in Eqs. (2.29) and (2.30), we get

$$\rho_{DM} = \rho_0 (c_1)^{b^2-1}\, e^{3k(b^2-1)t} , \qquad (3.5)$$

$$Q = 3kb^2 \rho_0 (c_1)^{b^2-1}\, e^{3k(b^2-1)t} . \qquad (3.6)$$

Using Eqs. (3.3)-(3.4) and (3.5) in Eq. (2.19), we obtain the energy density of holographic dark energy as

$$\rho_{DE} = 4k^2 - \frac{3D_1^2}{c_1^2} e^{-6kt} + \frac{D_2^2}{\left[\exp\left(kt + \frac{D_1}{3c_1 k} e^{-3kt} \right)\right]^2} - \rho_0 (c_1)^{b^2-1}\, e^{3k(b^2-1)t} . \qquad (3.7)$$

Using Eqs. (3.3)-(3.4) in Eq. (2.20), we obtain the pressure of holographic dark energy as

$$P_{DE} = -3k^2 + \frac{2D_1 k}{c_1} e^{-3kt} - \frac{D_1^2}{c_1^2} e^{-6kt} . \qquad (3.8)$$

The EoS parameter of holographic dark energy is given by

$$\omega = \frac{-3k^2 + \frac{2D_1 k}{c_1} e^{-3kt} - \frac{D_1^2}{c_1^2} e^{-6kt}}{4k^2 - \frac{3D_1^2}{c_1^2} e^{-6kt} + \frac{D_2^2}{\left[\exp\left(kt + \frac{D_1}{3c_1 k} e^{-3kt} \right)\right]^2} - \rho_0 (c_1)^{b^2-1}\, e^{3k(b^2-1)t}} . \qquad (3.9)$$

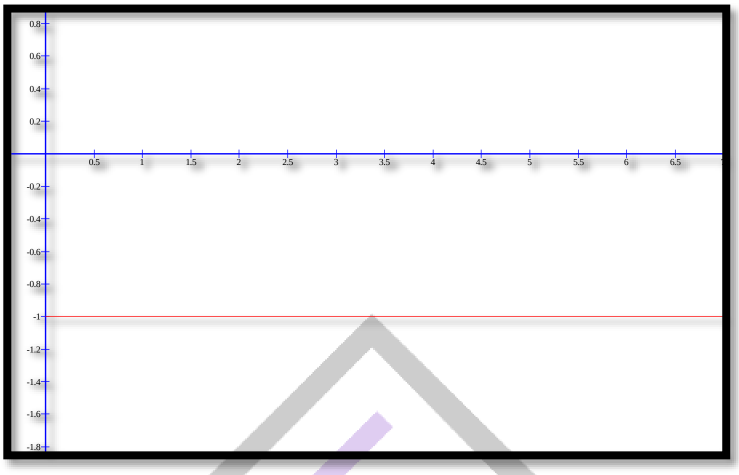
The mean Hubble parameter H, deceleration parameter q and mean anisotropy parameter of expansion Δ are given by respectively

$$H = k , \qquad (3.10)$$

$$q = -1 , \qquad (3.11)$$

$$\Delta = \frac{2\lambda^2}{9c_1^2k^2}e^{-6kt} \quad . \qquad (3.12)$$

4] Discussion:-



The Deceleration Parameter q (Fig1)

Fig (1), the sign of q imdicate that, where model accelerte or decelerate. The deceleration parameter for model is $q = -1$, as model inflates.

Anisotropy parameter of expansion Δ (Fig 2)

In Fig. 2 we plot anisotropy parameter of expansion (Δ) against cosmic time *t*. It is observed that in model anisotropy decreases as time increases then it becomes zero after some time and remains zero after some finite time. Hence, the model reaches to isotropy after some finite time which matches with the recent observation as the universe is isotropic at large scale.

EoS parameter ω (Fig 3)

In a Fig 3, (w_Λ) starts from phantom region ($w_\Lambda < -1$) and after some finite time ($w_\Lambda = -1$) after some finite t i.e. the model approaches to ΛCDM model after some finite t.

The investigators in cosmology, the SNe Ia data (Riess *et. al.* 2004, 2007; Astier *et. al.* 2006), the SDSS data (Eisenstein *et. al.* 2005), the three-year WMAP data (Spergel *et. al.* 2003) and Bamba *et. al.* (2012) all indicate that the ΛCDM model or the model that reduces to ΛCDM are a standard model in cosmology, is an excellent model to describe the cosmological evolution.

6] Conclusion: -

In this paper we have studied homogeneous and anisotropic Kantowski-Sachs universe filled with interacting DM and HDE. Our result shows that universe was anisotropic in the early stage and at the late time universe become isotropic.

I] It is observed that the deceleration parameter is an accelerating function as $q = -1$. Therefore, the model describes accelerate expansion of the universe.

II] The anisotropy parameter for large cosmic time t, $\Delta \to 0$. Therefore, for large cosmic time the anisotropy of universe damps out and approaches to an isotropic universe.

III] The value of EoS parameter ω_{DE} lies between $(-1 > \omega_{DE} < -1)$ starts with quintessence and for large cosmic time EoS parameter enters in the phantom region. Which matches the observation with [Wang *et. al.* 2005, 2006].

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