Find Labeling Numbers of All Powers of Paths Using an Interval Graph G

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***Abstract*: One of the principal topics in graph theory is labeling or coloring of graph. Graph labeling is motivated by the problems like, task assignment, traffic phasing, frequency assignment in radio communications, fleet maintenance. In this paper we have studied a generalization of optimal vertex coloring or labeling problem, namely L ($h_1, h_2, \ldots, h_m$)-labeling of graphs, which is also a generalization of L(h,k)labeling problem. Large amounts of work have been done for various classes of graphs. For various values of $h_1, h_2, \ldots, h_m$ different problems have been addressed by the researchers. Motivated by these, we investigated the L ($h_1, h_2, \ldots, h_m$)labeling of all powers of paths using an interval graph G.**

***Keywords*: Path, Power of Path, Interval Family, Interval Graph, Labeling Number**

Introduction: The field of Graph Theory plays vital role in various fields. One of the important areas in graph theory is Graph Labeling used in many applications like coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, data base management. Graph labeling is one of the fascinating areas of graph theory with wide ranging applications. Graph Labeling was first introduced in the 1960's. A graph labeling is an assignment of integers to the vertices or the edges, or both, subject to certain conditions. If the domain is the set of vertices we speak about the vertex labeling. If the domain is the set of edges, then the labeling is called the edge labeling. If the labels are assigned to the vertices and also to the edges of a graph, such a labeling is called total. An enormous body of literature has grown around graph labeling in the last four decades. Labeled graphs provide mathematical models for a broad range of applications. The qualitative labeling of a graph elements have been used in diverse fields such as conflict resolutions in social psychology, energy crises etc. Quantitative labeling of graph elements have been used in missile guidance codes, radar location codes, coding theory, x-ray crystallography, astronomy, circuit design, communication network. The frequency assignment problem is a problem where the task is to assign a frequency (non-negative integer) to a given group of televisions or radio transmitters so that interfering transmitters are assigned frequency with at least a minimum allowed separation. A frequency assignment problem is motivated from the distance labeling problem of graphs. It is to find a proper assignment of channels to transmitters in a wireless network. The level of interference between any two radio stations correlates with the geographic locations of the stations. Closer stations have a stronger interference and thus there must be a greater difference between their assigned channels.

The L (2, 1)-labeling of a graph G as a function f from the vertex set V to the set of non-negative integers such that $|f(x)-f(y)| \geq 2$ if d (x,y) = 1 and $|f(x)-f(y)| \geq 1$ if d (x,y) = 2, where d (x,y) represent the distance between the vertices x and y. The minimum span over all possible labeling functions of L (2, 1)-labeling is denoted by $\lambda_{2,1}(G)$ and is called L (2, 1) labeling number of G. Similarly, an L (3, 2, 1)-labeling of a graph G is a function f from its vertex set V to the set of non-negative integers such that $| f(x) - f(y) | \geq 3$ if d (x, y) = 1, $| f(x) - f(y) | \geq 2$ if d (x, y) = 2 and $| f(x) - f(y) | \geq 1$ if d (x , y) = 3. The L(3, 2, 1)-labeling number, $\lambda_{3,2,1}(G)$, of G is the smallest non-negative integer k such that G has a L (3, 2, 1)-labeling of span k. Also, an L (4, 3, 2, 1)-labeling of a graph G is a function f from its vertex set V to the set of non-negative integers such that $| f(x) - f(y) | \geq 4$ if d (x , y) = 1, $| f(x) - f(y) | \geq 3$ if d (x, y) = 2, $| f(x) - f(y) | \geq 2$ if d (x , y) = 3 and $| f(x) - f(y) | \geq 1$ if d (x , y) = 4. The L (4, 3, 2, 1)-labeling number, $\lambda_{4,3,2,1}(G)$, of G is the smallest non-negative integer k such that G has a L (4, 3, 2, 1)-labeling of span k. Similarly L (5, 4, 3, 2, 1) and L ($h_1, h_2, \ldots, h_m$) labeling.

Preliminaries: The power of graph G is donated as G^k and defined as 'u' and 'v' are two vertices in G^k if the distance $d(u,v) \leq k$ then 'u', 'v' are adjacent vertices in G^k. A graph G with 'n' vertices is said to be a path if v_i and v_{i+1} are adjacent for $1 \leq i \leq n-1$ denoted as P_n. The k^{th} power of path of order 'n' is denoted as P_n^k and defined as v_i is adjacent to $v_{i+1}, v_{i+2}, \ldots v_{i+k}$ for $1 \leq i \leq n-k$. The distance d(u, v) between two vertices u and v in G is the length of a shortest path joining them if any; otherwise d(u, v) = ∞. One of the principal topics in graph theory is labeling or coloring of graph. Graph labeling is motivated by the problems like, task assignment, traffic phasing, frequency assignment in radio communications, fleet maintenance. L($h_1, h_2, \ldots, h_m$)-labeling problem starts journey from vertex coloring problem. The problem is; discover a way to color the vertices of a graph such that no two adjoining vertices are colored using identical color; this is called a vertex coloring. In computer and mathematical representations, instead of colors non-negative integers are used. L($h_1, h_2, \ldots, h_m$)-labeling problem is other variation of usual vertex coloring problem where the adjoining vertices and the vertices at distance i, i = 1, 2, . . . , m are to be labeled under certain conditions. For the case of m = 2, h_1 = 1 and h_2 = 0, L($h_1, h_2, \ldots, h_m$)-labeling is L(1, 0) identical with vertex coloring problem. In that case for the vertices at distance i, $i = 1, 2, \ldots, m$ there is no restriction, so L(1, 0)-labeling problem is exactly same vertex coloring problem.

In graph theory one of the leading topics is graph coloring or graph labeling. In this paper, we have studied a generalization of optimal vertex coloring/labeling problem, namely $L(h_1, h_2, \ldots, h_m)$ labeling of graphs, which is also a generalization of L(h, k)-labeling problem. Large amounts of work have been done for various classes of graphs. For various values of $h_1, h_2, \ldots, h_m$ different problems have been addressed by the researchers. Motivated by these, we investigated the $L(h_1, h_2, \ldots, h_m)$-labeling problem on graphs for different values of $h_1, h_2, \ldots, h_m$. For different values of $h_1, h_2, \ldots, h_m$ many results related to $L(h_1, h_2, \ldots, h_m)$-labeling problem have been published for various graph classes. The computation of exact value of $\lambda_{h_1, h_2, \ldots, h_m}(G)$ for any graph G is very tenacious task. For this motive, researchers are trying to determine the upper bounds for various graph classes.

For any graph G and two non-negative integers h, k, an L (h, k)-labeling is a mapping f: V (G) → {0, 1, . . . ,}, so that $|f(u) - f(v)| \geq h$, if d(u, v) = 1, $|f(u) - f(v)| \geq k$ if d(u, v) = 2. $L(h_1, h_2, \ldots, h_m)$-labeling: For non-negative integers $h_1, h_2, \ldots, h_m$, an $L(h_1, h_2, \ldots, h_m)$-labeling of a graph G is defined as the mapping f: V(G)→{0, 1, . . . ,} such that $|f(u) - f(v)| \geq h_i$, if d(u, v) = i, $1 \leq i \leq m$, where d(u, v) is the distance between the vertices u and v. The $L(h_1, h_2, \ldots, h_m)$-labeling numbers of G, is denoted by $\lambda_{h_1, h_2, \ldots, h_m}(G)$ which is the difference between largest and least labels used in $L(h_1, h_2, \ldots, h_m)$-labeling.

Note: $h_1 = m, h_2 = m-1, h_3 = m-2, \ldots, h_{m-1} = 2, h_m = 1$
$h_1 > h_2 > h_3 > \ldots > h_{m-1} > h_m$

Main Theorems

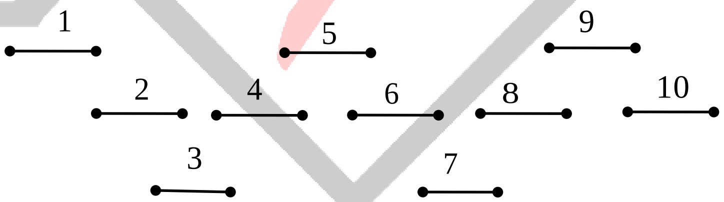
Theorem 1: if $I = \{i_1, i_2, \ldots, i_n\}$ be an interval family. We consider each interval intersects next interval only. Interval graph corresponding to an interval family 'I' is path with length n then the $L(h_1, h_2, h_3, \ldots, h_m)$ labeling number of G is

case:(1) if 'm' is even than $\lambda_{h1,h2,..hm} = \left(\frac{m}{2} + 1\right) m$,

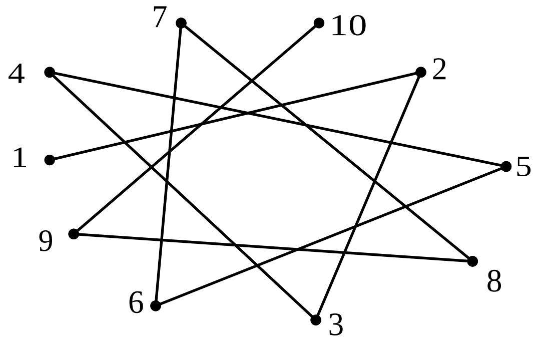
case:(2) if 'm' is odd than $\lambda_{h1,h2,\ldots,hm} = \left(\frac{m+1}{2}\right) m + \left(\frac{m-1}{2}\right)$ where 'm' is suffix of h.

Proof: Let $I = \{i_1, i_2, \ldots, i_n\}$ be an interval family. We consider each interval intersects next interval only. Interval graph corresponding to an interval family 'I' is path with length n. For non-negative integers $h_1, h_2, \ldots, h_m$, an $L(h_1, h_2, \ldots, h_m)$-labeling of a graph G is defined as the mapping f: V(G)→{0, 1, . . . ,} such that $|f(u) - f(v)| \geq h_i$, if d(u, v) = i, $1 \leq i \leq m$, where d(u, v) is the distance between the vertices u and v. The $L(h_1, h_2, \ldots, h_m)$-labeling numbers of G, is denoted by $\lambda_{h_1, h_2, \ldots, h_m}(G)$ which is the difference between largest and least labels used in $L(h_1, h_2, \ldots, h_m)$-labeling. We will prove the theorem by following experimental problems.

Experimental Problem



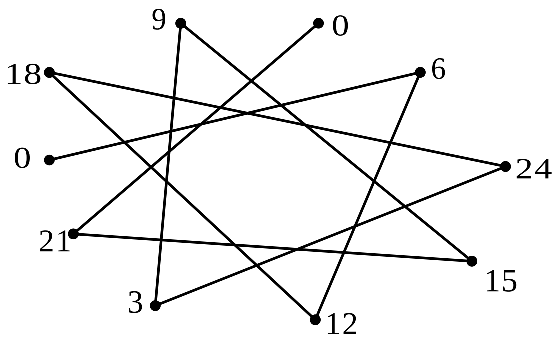
Interval family I



Interval graph G

Find the distance between u, v such that u, v are in V

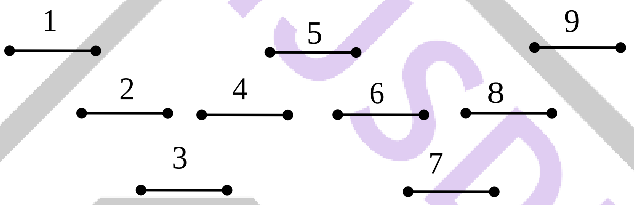
d(1,1)=0	d(2,1)= 1	d(3,1)=2
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=2	d(2,3)=1	d(3,3)=0
d(1,4)=3	d(2,4)=2	d(3,4)=1
d(1,5)=4	d(2,5)=3	d(3,5)=2
d(1,6)=5	d(2,6)=4	d(3,6)=3
d(1,7)=6	d(2,7)=5	d(3,7)=4
d(1,8)=7	d(2,8)=6	d(3,8)=5
d(1,9)=8	d(2,9)=7	d(3,9)=6
d(1,10)=9	d(2,10)=8	d(3,10)=7
d(4,1)=3	d(5,1)=4	d(6,1)=5
d(4,2)=2	d(5,2)=3	d(6,2)=4
d(4,3)=1	d(5,3)=2	d(6,3)=3
d(4,4)=0	d(5,4)=1	d(6,4)=2
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=2	d(5,6)=1	d(6,6)=0
d(4,7)=3	d(5,7)=2	d(6,7)=1
d(4,8)=4	d(5,8)=3	d(6,8)=2
d(4,9)=5	d(5,9)=4	d(6,9)=3
d(4,10)=6	d(5,10)=5	d(6,10)=4
d(7,1)=6	d(8,1)=7	d(9,1)=8
d(7,2)=5	d(8,2)=6	d(9,2)=7
d(7,3)=4	d(8,3)=5	d(9,3)=6
d(7,4)=3	d(8,4)=4	d(9,4)=5
d(7,5)=2	d(8,5)=3	d(9,5)=4
d(7,6)=1	d(8,6)=2	d(9,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=2	d(8,9)=1	d(9,9)=0
d(7,10)=3	d(8,10)=2	d(9,10)=1
d(10,1)=9		
d(10,2)=8		
d(10,3)=7		
d(10,4)=6		
d(10,5)=5		
d(10,6)=4		
d(10,7)=3		
d(10,8)=2		
d(10,9)=1		
d(10,10)=0		



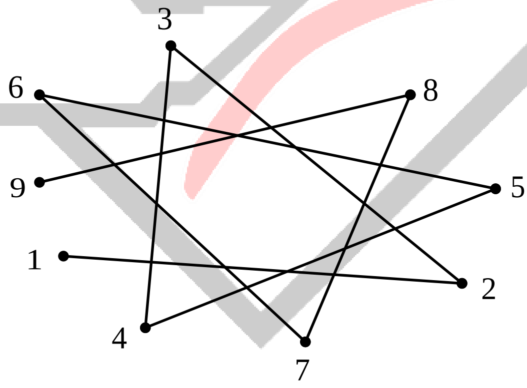
Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
Labels	0	6	12	18	24	3	9	15	21	0

Experimental Problem



Interval family I



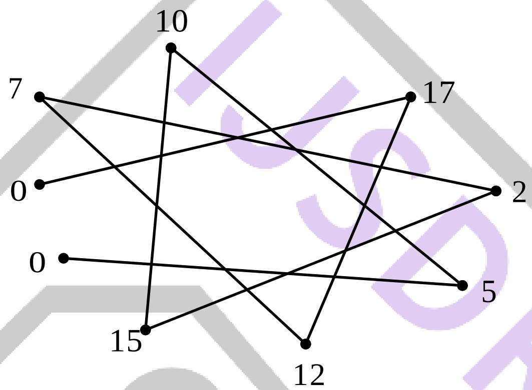
Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=2
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=2	d(2,3)=1	d(3,3)=0
d(1,4)=3	d(2,4)=2	d(3,4)=1
d(1,5)=4	d(2,5)=3	d(3,5)=2
d(1,6)=5	d(2,6)=4	d(3,6)=3
d(1,7)=6	d(2,7)=5	d(3,7)=4
d(1,8)=7	d(2,8)=6	d(3,8)=5
d(1,9)=8	d(2,9)=7	d(3,9)=6

d(4,1)=3	d(5,1)=4	d(6,1)=5
d(4,2)=2	d(5,2)=3	d(6,2)=4
d(4,3)=1	d(5,3)=2	d(6,3)=3
d(4,4)=0	d(5,4)=1	d(6,4)=2
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=2	d(5,6)=1	d(6,6)=0
d(4,7)=3	d(5,7)=2	d(6,7)=1
d(4,8)=4	d(5,8)=3	d(6,8)=2
d(4,9)=5	d(5,9)=4	d(6,9)=3

d(7,1)=6	d(8,1)=7	d(9,1)=8
d(7,2)=5	d(8,2)=6	d(9,2)=7
d(7,3)=4	d(8,3)=5	d(9,3)=6
d(7,4)=3	d(8,4)=4	d(9,4)=5
d(7,5)=2	d(8,5)=3	d(9,5)=4
d(7,6)=1	d(8,6)=2	d(9,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=2	d(8,9)=1	d(9,9)=0



Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Labels	**0**	**5**	**10**	**15**	**2**	**7**	**12**	**17**	**0**

Like this we did for different values of 'm' and tabled below L ($h_1, h_2, \ldots, h_m$) labeling number of G = P_n

'm' suffix of h	**'k' power of path**	**L($h_1, h_2, \ldots, h_m$) labeling number of** G = P_n
m=2	**k=1**	**4**
m=3	**k=1**	**7**
m=4	**k=1**	**12**
m=5	**k=1**	**17**
m=6	**k=1**	**24**
…	**…**	**…**
…	**…**	**…**
m is even	**k=1**	$\left(\frac{m}{2}+1\right)m$
m is odd	**k=1**	$\left(\frac{m+1}{2}\right)m+\left(\frac{m-1}{2}\right)$

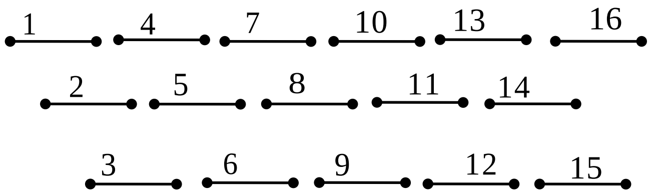
Theorem 2: if I = {$i_1, i_2, \ldots, i_n$} be an interval family. We consider each interval intersects next two consecutive intervals only. Interval graph corresponding to an interval family 'I' is square of path with length n then the L ($h_1, h_2, h_3, \ldots, h_m$) labeling number of G is

case:(1) if 'm' is even than $\lambda_{h1,h2,\ldots,hm} = (m+1)m$,

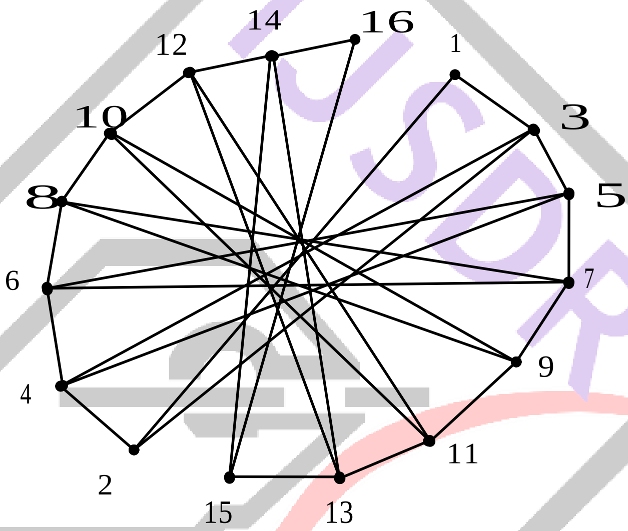
case:(2) if 'm' is odd than $\lambda_{h1,h2,\ldots,hm} = (m+1)m + \left(\frac{m-1}{2}\right)$ where 'm' is suffix of h.

Proof: Let I = {i_1, i_2, …., i_n} be an interval family. We consider each interval intersects next two consecutive intervals only. Interval graph corresponding to an interval family 'I' is square of path with length n. We will prove the theorem by following experimental problems.

Experimental Problem



Interval family I



Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3
d(1,9)=4	d(2,9)=4	d(3,9)=3
d(1,10)=5	d(2,10)=4	d(3,10)=4
d(1,11)=5	d(2,11)=5	d(3,11)=4
d(1,12)=6	d(2,12)=5	d(3,12)=5
d(1,13)=6	d(2,13)=6	d(3,13)=5
d(1,14)=7	d(2,14)=6	d(3,14)=6
d(1,15)=7	d(2,15)=7	d(3,15)=6
d(1,16)=8	d(2,16)=7	d(3,16)=7

d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=1	d(5,2)=2	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=4	d(5,11)=3	d(6,11)=3
d(4,12)=4	d(5,12)=4	d(6,12)=3
d(4,13)=5	d(5,13)=4	d(6,13)=4
d(4,14)=5	d(5,14)=5	d(6,14)=4
d(4,15)=6	d(5,15)=5	d(6,15)=5
d(4,16)=6	d(5,16)=6	d(6,16)=5

d(7,1)=3	d(8,1)=4	d(9,1)=4
d(7,2)=3	d(8,2)=3	d(9,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=3
d(7,4)=2	d(8,4)=2	d(9,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1
d(7,12)=3	d(8,12)=2	d(9,12)=2
d(7,13)=3	d(8,13)=3	d(9,13)=2
d(7,14)=4	d(8,14)=3	d(9,14)=3
d(7,15)=4	d(8,15)=4	d(9,15)=3
d(7,16)=5	d(8,16)=4	d(9,16)=4

d(10,1)=5	d(11,1)=5	d(12,1)=6
d(10,2)=4	d(11,2)=5	d(12,2)=5
d(10,3)=4	d(11,3)=4	d(12,3)=5
d(10,4)=3	d(11,4)=4	d(12,4)=4
d(10,5)=3	d(11,5)=3	d(12,5)=4
d(10,6)=2	d(11,6)=3	d(12,6)=3
d(10,7)=2	d(11,7)=2	d(12,7)=3
d(10,8)=1	d(11,8)=2	d(12,8)=2
d(10,9)=1	d(11,9)=1	d(12,9)=2
d(10,10)=0	d(11,10)=1	d(12,10)=1
d(10,11)=1	d(11,11)=0	d(12,11)=1
d(10,12)=1	d(11,12)=1	d(12,12)=0
d(10,13)=2	d(11,13)=1	d(12,13)=1
d(10,14)=2	d(11,14)=2	d(12,14)=1
d(10,15)=3	d(11,15)=2	d(12,15)=2
d(10,16)=3	d(11,16)=3	d(12,16)=2

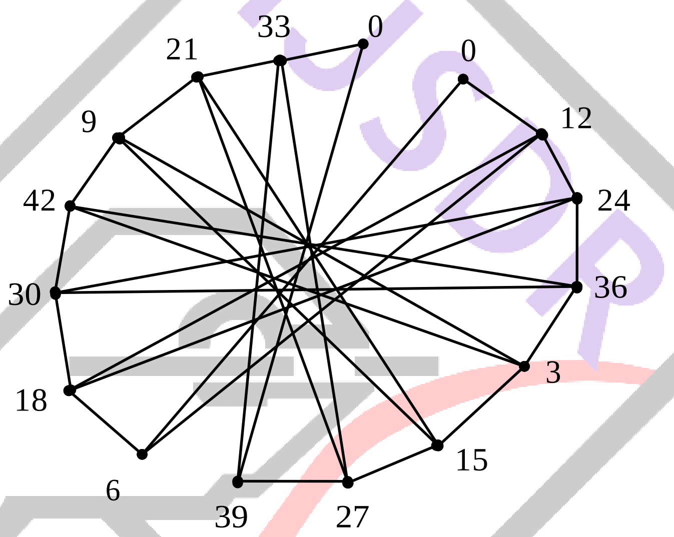
d(13,1)=6	d(14,1)=7	d(15,1)=7
d(13,2)=6	d(14,2)=6	d(15,2)=7
d(13,3)=5	d(14,3)=6	d(15,3)=6
d(13,4)=5	d(14,4)=5	d(15,4)=6
d(13,5)=4	d(14,5)=5	d(15,5)=5
d(13,6)=4	d(14,6)=4	d(15,6)=5
d(13,7)=3	d(14,7)=4	d(15,7)=4
d(13,8)=3	d(14,8)=3	d(15,8)=4
d(13,9)=2	d(14,9)=3	d(15,9)=3
d(13,10)=2	d(14,10)=2	d(15,10)=3
d(13,11)=1	d(14,11)=2	d(15,11)=2
d(13,12)=1	d(14,12)=1	d(15,12)=2

d(13,13)=0
d(13,14)=1
d(13,15)=1
d(13,16)=2

d(14,13)=1
d(14,14)=0
d(14,15)=1
d(14,16)=1

d(15,13)=1
d(15,14)=1
d(15,15)=0
d(15,16)=1

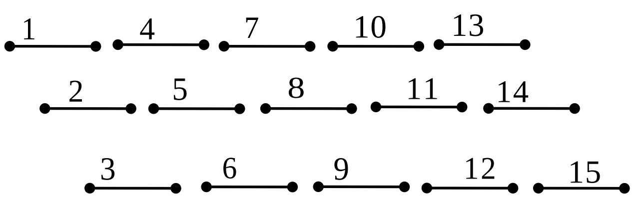
d(16,1)=8
d(16,2)=7
d(16,3)=7
d(16,4)=6
d(16,5)=6
d(16,6)=5
d(16,7)=5
d(16,8)=4
d(16,9)=4
d(16,10)=3
d(16,11)=3
d(16,12)=2
d(16,13)=2
d(16,14)=1
d(16,15)=1
d(16,16)=0



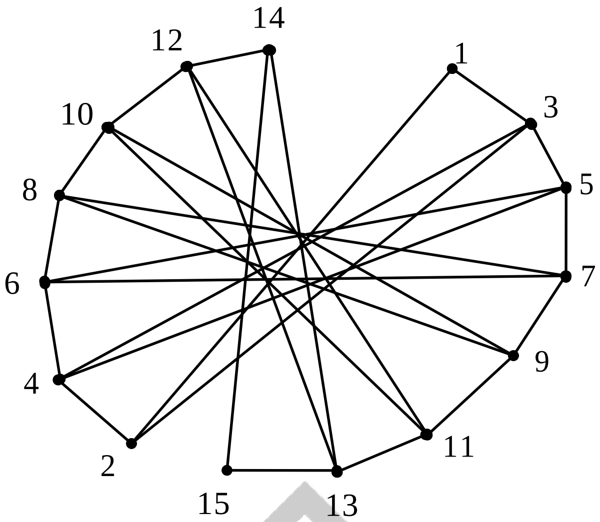
Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}
Labels	**0**	**6**	**12**	**18**	**24**	**30**	**36**	**42**	**3**	**9**	**15**	**21**	**27**	**33**	**39**	**0**

Experimental Problem



Interval family I



Interval graph G

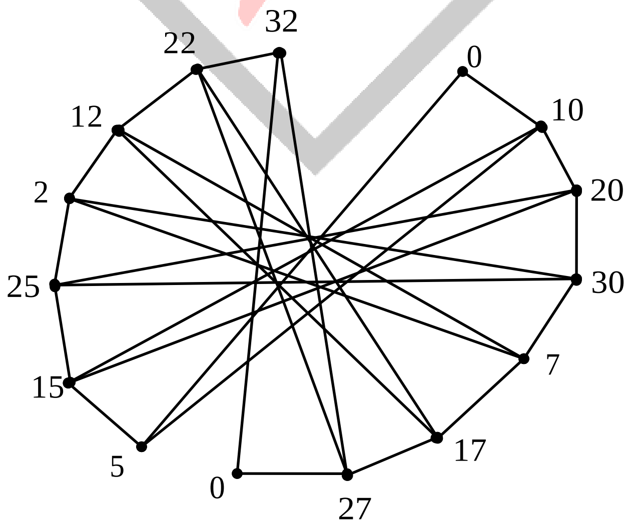
Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3
d(1,9)=4	d(2,9)=4	d(3,9)=3
d(1,10)=5	d(2,10)=4	d(3,10)=4
d(1,11)=5	d(2,11)=5	d(3,11)=4
d(1,12)=6	d(2,12)=5	d(3,12)=5
d(1,13)=6	d(2,13)=6	d(3,13)=5
d(1,14)=7	d(2,14)=6	d(3,14)=6
d(1,15)=7	d(2,15)=7	d(3,15)=6
d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=1	d(5,2)=2	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=4	d(5,11)=3	d(6,11)=3
d(4,12)=4	d(5,12)=4	d(6,12)=3
d(4,13)=5	d(5,13)=4	d(6,13)=4
d(4,14)=5	d(5,14)=5	d(6,14)=4
d(4,15)=6	d(5,15)=5	d(6,15)=5
d(7,1)=3	d(8,1)=4	d(9,1)=4
d(7,2)=3	d(8,2)=3	d(9,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=3
d(7,4)=2	d(8,4)=2	d(9,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1

d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1
d(7,12)=3	d(8,12)=2	d(9,12)=2
d(7,13)=3	d(8,13)=3	d(9,13)=2
d(7,14)=4	d(8,14)=3	d(9,14)=3
d(7,15)=4	d(8,15)=4	d(9,15)=3

d(10,1)=5	d(11,1)=5	d(12,1)=6
d(10,2)=4	d(11,2)=5	d(12,2)=5
d(10,3)=4	d(11,3)=4	d(12,3)=5
d(10,4)=3	d(11,4)=4	d(12,4)=4
d(10,5)=3	d(11,5)=3	d(12,5)=4
d(10,6)=2	d(11,6)=3	d(12,6)=3
d(10,7)=2	d(11,7)=2	d(12,7)=3
d(10,8)=1	d(11,8)=2	d(12,8)=2
d(10,9)=1	d(11,9)=1	d(12,9)=2
d(10,10)=0	d(11,10)=1	d(12,10)=1
d(10,11)=1	d(11,11)=0	d(12,11)=1
d(10,12)=1	d(11,12)=1	d(12,12)=0
d(10,13)=2	d(11,13)=1	d(12,13)=1
d(10,14)=2	d(11,14)=2	d(12,14)=1
d(10,15)=3	d(11,15)=2	d(12,15)=2

d(13,1)=6	d(14,1)=7	d(15,1)=7
d(13,2)=6	d(14,2)=6	d(15,2)=7
d(13,3)=5	d(14,3)=6	d(15,3)=6
d(13,4)=5	d(14,4)=5	d(15,4)=6
d(13,5)=4	d(14,5)=5	d(15,5)=5
d(13,6)=4	d(14,6)=4	d(15,6)=5
d(13,7)=3	d(14,7)=4	d(15,7)=4
d(13,8)=3	d(14,8)=3	d(15,8)=4
d(13,9)=2	d(14,9)=3	d(15,9)=3
d(13,10)=2	d(14,10)=2	d(15,10)=3
d(13,11)=1	d(14,11)=2	d(15,11)=2
d(13,12)=1	d(14,12)=1	d(15,12)=2
d(13,13)=0	d(14,13)=1	d(15,13)=1
d(13,14)=1	d(14,14)=0	d(15,14)=1
d(13,15)=1	d(14,15)=1	d(15,15)=0



Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
Labels	0	5	10	15	20	25	30	2	7	12	17	22	27	32	0

Like this we did for different values of 'm' and tabled below L ($h_1, h_2, \dots, h_m$) labeling number of G=P_n^2

'm' suffix of h	'k' power of path	L($h_1, h_2, \dots, h_m$) labeling number of G=P_n^2
m=2	k=2	6
m=3	k=2	13
m=4	k=2	20
m=5	k=2	32
m=6	k=2	42
..	..	..
..	..	..
m is even	k=2	(m+1)m
m is odd	K=2	(m+1)m+$\left(\frac{m-1}{2}\right)$

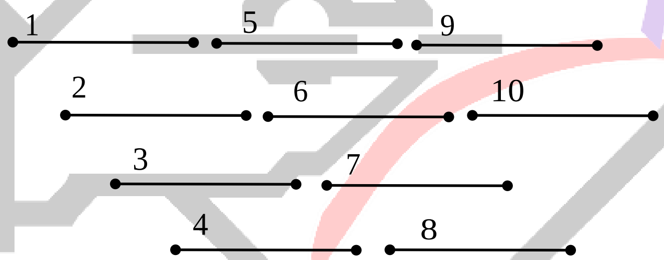
Theorem 3: if I = {$i_1, i_2, \dots, i_n$} be an interval family. We consider each interval intersects next three consecutive intervals only. Interval graph corresponding to an interval family 'I' is power three of path with length n then the L ($h_1, h_2, h_3, \dots, h_m$) labeling number of G is **case:(1**) if 'm' is even than $\lambda_{h1, h2, \dots, hm}$ (P_n^3) = $\left(3\frac{m}{2}+1\right)m$,

case:(2) if 'm' is odd than $\lambda_{h1, h2, \dots, hm}$ (P_n^3) = $3\left(\frac{m+1}{2}\right)$ m + $\left(\frac{m-1}{2}\right)$ where 'm' is suffix of h.

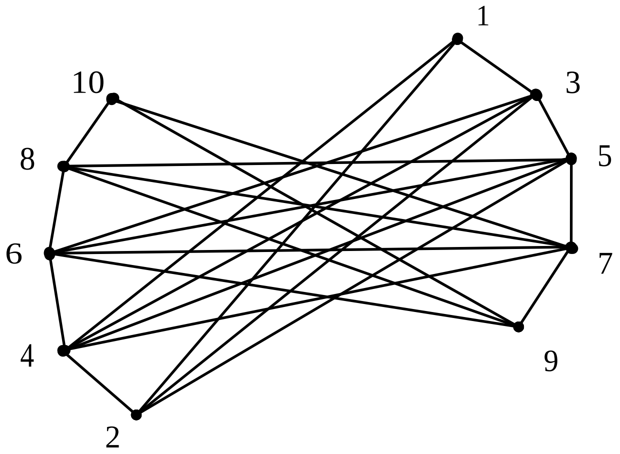
Proof: Let I = {$i_1, i_2, \dots, i_n$} be an interval family. We consider each interval intersects next three consecutive intervals only. Interval graph corresponding to an interval family 'I' is power three of path with length n. For nonnegative integers $h_1, h_2, \dots, h_m$, an L($h_1, h_2, \dots, h_m$) labeling of a graph G is defined as the mapping f: V(G)→{0, 1, . . .,} such that $|f(u)-f(v)| \ge h_i$, if d(u, v) = i, $1 \le i \le m$, where d(u, v) is the distance between the vertices u and v. The L ($h_1, h_2, \dots, h_m$)-labeling numbers of G, is denoted by $\lambda_{h_1,h_2,\dots,h_m}$ (G) which is the difference between largest and least labels used in L($h_1, h_2, \dots, h_m$)-labeling. We can prove this theorem by following experimental problems.

Experimental Problem

Find the L(2, 1) labeling of G = P_n^3



Interval family I



Interval graph G

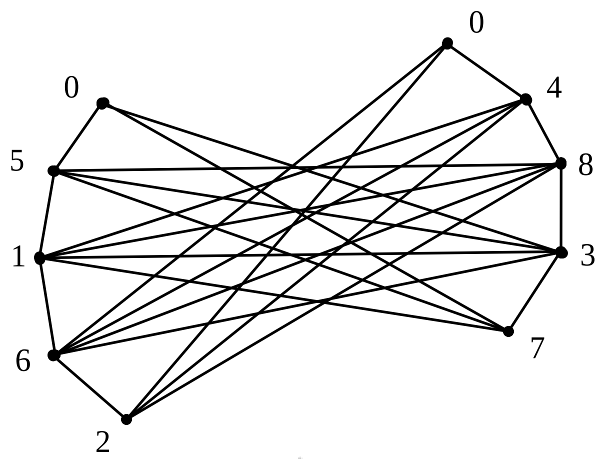
Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=1	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=1	d(3,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=1
d(1,7)=2	d(2,7)=2	d(3,7)=2
d(1,8)=3	d(2,8)=2	d(3,8)=2
d(1,9)=3	d(2,9)=3	d(3,9)=2
d(1,10)=3	d(2,10)=3	d(3,10)=3

d(4,1)=1	d(5,1)=2	d(6,1)=2
d(4,2)=1	d(5,2)=1	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=1
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=1	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=1	d(6,8)=1
d(4,9)=2	d(5,9)=2	d(6,9)=1
d(4,10)=2	d(5,10)=2	d(6,10)=2

d(7,1)=2	d(8,1)=3	d(9,1)=3
d(7,2)=2	d(8,2)=2	d(9,2)=3
d(7,3)=2	d(8,3)=2	d(9,3)=2
d(7,4)=1	d(8,4)=2	d(9,4)=2
d(7,5)=1	d(8,5)=1	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=1
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=1	d(8,10)=1	d(9,10)=1

d(10,1)=3
d(10,2)=3
d(10,3)=3
d(10,4)=2
d(10,5)=2
d(10,6)=2
d(10,7)=1
d(10,8)=1
d(10,9)=1
d(10,10)=0

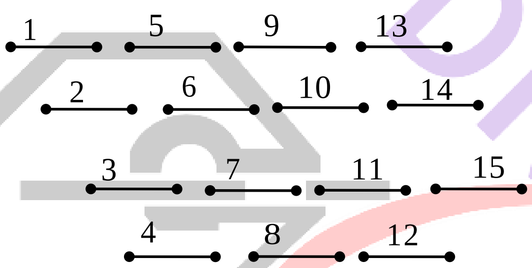


Labeled graph G

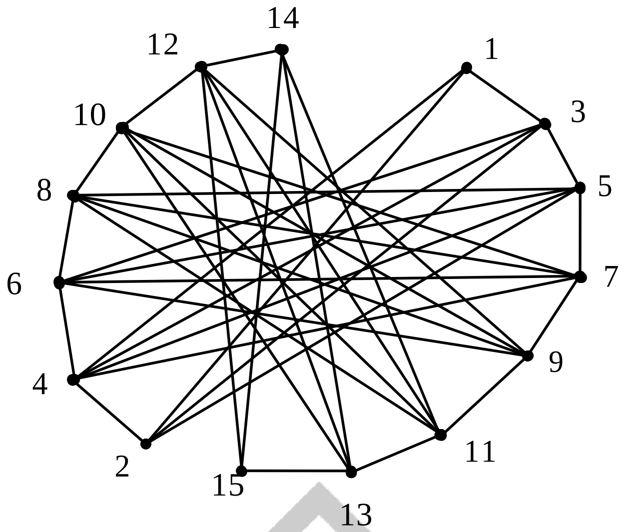
Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
Labels	0	2	4	6	8	1	3	5	7	0

Experimental Problem

Find the L(3, 2, 1) labeling of G = P_n^3



Interval family I



Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=1	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=1	d(3,5)=1
d(1,6)=2	d(2,6)=2	d(3,6)=1
d(1,7)=2	d(2,7)=2	d(3,7)=2
d(1,8)=3	d(2,8)=2	d(3,8)=2
d(1,9)=3	d(2,9)=3	d(3,9)=2
d(1,10)=3	d(2,10)=3	d(3,10)=3
d(1,11)=4	d(2,11)=3	d(3,11)=3
d(1,12)=4	d(2,12)=4	d(3,12)=3
d(1,13)=4	d(2,13)=4	d(3,13)=4
d(1,14)=5	d(2,14)=4	d(3,14)=4
d(1,15)=5	d(2,15)=5	d(3,15)=4

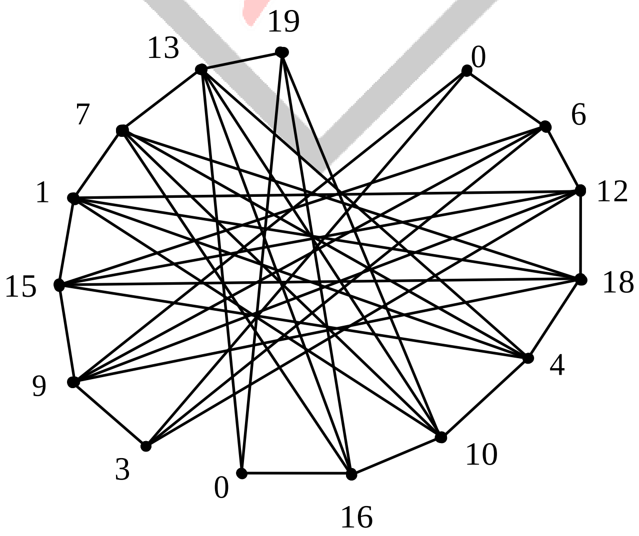
d(4,1)=1	d(5,1)=2	d(6,1)=2
d(4,2)=1	d(5,2)=1	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=1
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=1	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=1	d(6,8)=1
d(4,9)=2	d(5,9)=2	d(6,9)=1
d(4,10)=2	d(5,10)=2	d(6,10)=2
d(4,11)=3	d(5,11)=2	d(6,11)=2
d(4,12)=3	d(5,12)=3	d(6,12)=2
d(4,13)=3	d(5,13)=3	d(6,13)=3
d(4,14)=4	d(5,14)=3	d(6,14)=3
d(4,15)=4	d(5,15)=4	d(6,15)=3

d(7,1)=2	d(8,1)=3	d(9,1)=3
d(7,2)=2	d(8,2)=2	d(9,2)=3
d(7,3)=2	d(8,3)=2	d(9,3)=2
d(7,4)=1	d(8,4)=2	d(9,4)=2
d(7,5)=1	d(8,5)=1	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=1
d(7,7)=0	d(8,7)=1	d(9,7)=1

d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=1	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=1
d(7,12)=2	d(8,12)=2	d(9,12)=1
d(7,13)=2	d(8,13)=2	d(9,13)=2
d(7,14)=3	d(8,14)=2	d(9,14)=2
d(7,15)=3	d(8,15)=3	d(9,15)=2

d(10,1)=3	d(11,1)=4	d(12,1)=4
d(10,2)=3	d(11,2)=3	d(12,2)=4
d(10,3)=3	d(11,3)=3	d(12,3)=3
d(10,4)=2	d(11,4)=3	d(12,4)=3
d(10,5)=2	d(11,5)=2	d(12,5)=3
d(10,6)=2	d(11,6)=2	d(12,6)=2
d(10,7)=1	d(11,7)=2	d(12,7)=2
d(10,8)=1	d(11,8)=1	d(12,8)=2
d(10,9)=1	d(11,9)=1	d(12,9)=1
d(10,10)=0	d(11,10)=1	d(12,10)=1
d(10,11)=1	d(11,11)=0	d(12,11)=1
d(10,12)=1	d(11,12)=1	d(12,12)=0
d(10,13)=1	d(11,13)=1	d(12,13)=1
d(10,14)=2	d(11,14)=1	d(12,14)=1
d(10,15)=2	d(11,15)=2	d(12,15)=1

d(13,1)=4	d(14,1)=5	d(15,1)=5
d(13,2)=4	d(14,2)=4	d(15,2)=5
d(13,3)=4	d(14,3)=4	d(15,3)=4
d(13,4)=3	d(14,4)=4	d(15,4)=4
d(13,5)=3	d(14,5)=3	d(15,5)=4
d(13,6)=3	d(14,6)=3	d(15,6)=3
d(13,7)=2	d(14,7)=3	d(15,7)=3
d(13,8)=2	d(14,8)=2	d(15,8)=3
d(13,9)=2	d(14,9)=2	d(15,9)=2
d(13,10)=1	d(14,10)=2	d(15,10)=2
d(13,11)=1	d(14,11)=1	d(15,11)=2
d(13,12)=1	d(14,12)=1	d(15,12)=1
d(13,13)=0	d(14,13)=1	d(15,13)=1
d(13,14)=1	d(14,14)=0	d(15,14)=1
d(13,15)=1	d(14,15)=1	d(15,15)=0



Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
Labels	0	3	6	9	12	15	18	1	4	7	10	13	16	19	0

Like this we did for different values of 'm' and tabled below L ($h_1, h_2, \ldots, h_m$) labeling number of G=P_n^3

'm' suffix of h	'k' power of path	L($h_1, h_2, \ldots, h_m$) labeling number of G=P_n^3
m=2	k=3	8
m=3	k=3	19
m=4	k=3	28
m=5	k=3	47
m=6	k=3	60
..	..	..
..	..	..
m is even	k=3	$\left(3\frac{m}{2}+1\right)m$
m is odd	k=3	$3\left(\frac{m+1}{2}\right)m+\left(\frac{m-1}{2}\right)$

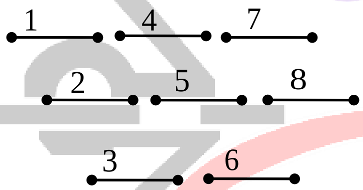
Theorem 4: if I = {$i_1, i_2, \ldots, i_n$} be an interval family. We consider each interval intersects next 'k' consecutive intervals only. Interval graph corresponding to an interval family 'I' is power 'k' of path with length n then the L (2, 1) labeling number of G is (k + 1)2 where 'k' is power of path.

Proof: Let I = {$i_1, i_2, \ldots, i_n$} be an interval family. We consider each interval intersects next k consecutive intervals only. Interval graph corresponding to an interval family 'I' is power k of path with length n.

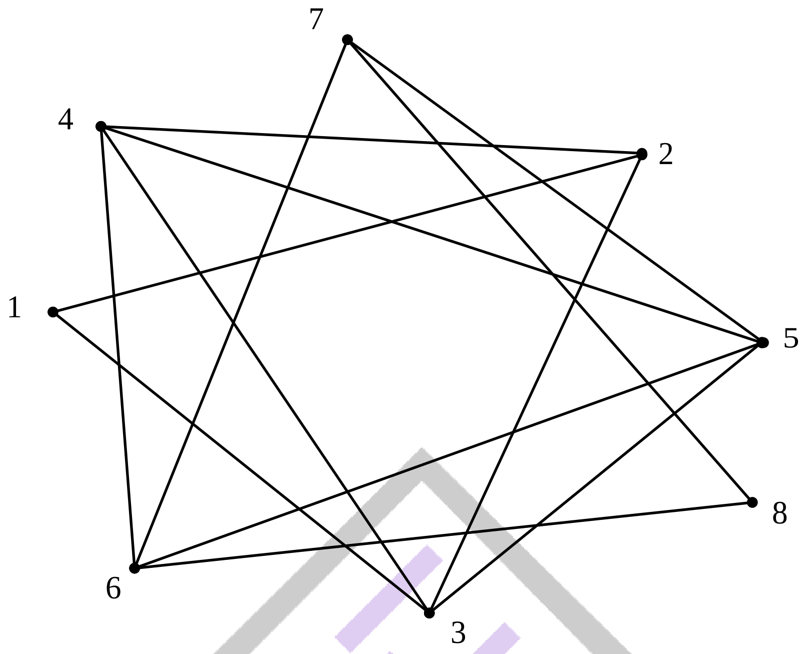
For any graph G the L (2, 1)-labeling is a mapping f: V (G) → {0, 1, . . . ,}, so that |f (u) − f (v)| ≥ 2 if d(u, v) = 1, |f (u) − f (v)| ≥ 1 if d(u, v) = 2. We will prove the theorem by following experimental problem.

Experimental Problem

Find the L(2, 1) labeling of G = P_n^2



Interval family I



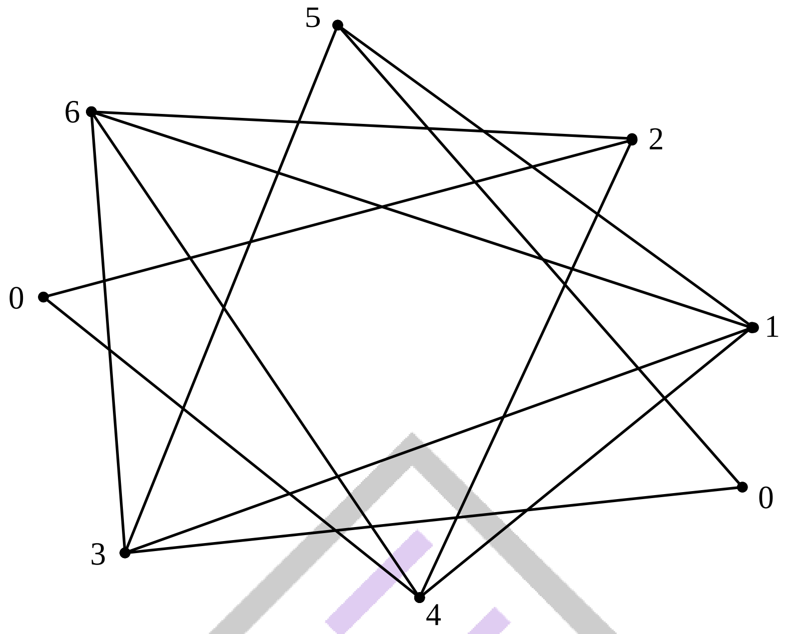
Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3

d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=1	d(5,2)=2	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1

d(7,1)=3	d(8,1)=4
d(7,2)=3	d(8,2)=3
d(7,3)=2	d(8,3)=3
d(7,4)=2	d(8,4)=2
d(7,5)=1	d(8,5)=2
d(7,6)=1	d(8,6)=1
d(7,7)=0	d(8,7)=1
d(7,8)=1	d(8,8)=0



Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
Labels	**0**	**2**	**4**	**6**	**1**	**3**	**5**	**0**

Like this we did for different powers of paths ‘k’ and tabled below L (2, 1) labeling number of G=P_n^k

‘m’ suffix of h	**‘k’ power of path**	**L(2, 1) labeling number of** G=P_n^k
m=2	**k=1**	**4**
m=2	**k=2**	**6**
m=2	**k=3**	**8**
m=2	**k=4**	**10**
m=2	**k=5**	**12**
m=2	**k=6**	**14**
…	**…**	**…**
…	**…**	**…**
m=2	**k is even k=2s**	**(2s+1)2**
m=2	**k is odd k=2s+1**	**(2s+2)2**
m=2	**k is power of path**	**(k+1)2**

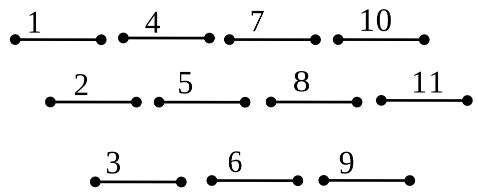
Theorem 5: If I = {i_1, i_2, …., i_n} be an interval family. We consider each interval intersects next ‘k’ consecutive intervals only. Interval graph corresponding to an interval family ‘I’ is power ‘k’ of path with length n then the L (3, 2, 1) labeling number of G is 6k+1 where ‘k’ is power of path.

Proof: Let I = {i_1, i_2, …., i_n} be an interval family. We consider each interval intersects next k consecutive intervals only. Interval graph corresponding to an interval family ‘I’ is power k of path with length n.

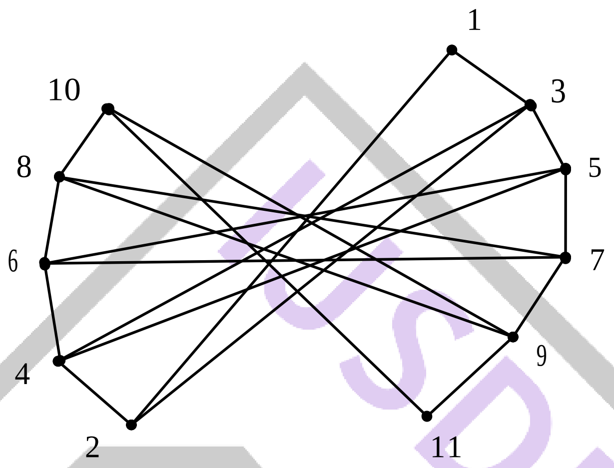
For any graph G the L (3, 2, 1)-labeling is a mapping f: V (G) → {0, 1, . . . ,}, so that |f (u) − f (v)| ≥ 3 if d(u, v) = 1, |f (u) − f (v)| ≥ 2 if d(u, v) = 2, |f (u) − f (v)| ≥ 1 if d(u, v) = 3. We will prove the theorem by following experimental problem

Experimental Problem

Find the L(3, 2, 1) labeling of G = P_n^2



Interval family I



Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3
d(1,9)=4	d(2,9)=4	d(3,9)=3
d(1,10)=5	d(2,10)=4	d(3,10)=4
d(1,11)=5	d(2,11)=5	d(3,11)=4

d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=1	d(5,2)=2	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=4	d(5,11)=3	d(6,11)=3

d(7,1)=3	d(8,1)=4	d(9,1)=4
d(7,2)=3	d(8,2)=3	d(9,2)=4

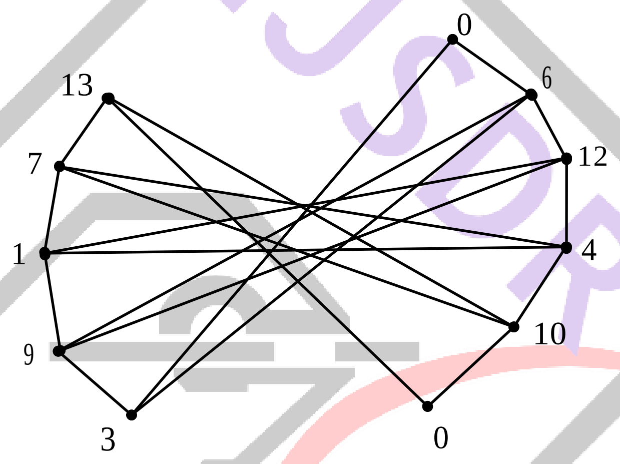
d(7,3)=2
d(7,4)=2
d(7,5)=1
d(7,6)=1
d(7,7)=0
d(7,8)=1
d(7,9)=1
d(7,10)=2
d(7,11)=2

d(8,3)=3
d(8,4)=2
d(8,5)=2
d(8,6)=1
d(8,7)=1
d(8,8)=0
d(8,9)=1
d(8,10)=1
d(8,11)=2

d(9,3)=3
d(9,4)=3
d(9,5)=2
d(9,6)=2
d(9,7)=1
d(9,8)=1
d(9,9)=0
d(9,10)=1
d(9,11)=1

d(10,1)=5
d(10,2)=4
d(10,3)=4
d(10,4)=3
d(10,5)=3
d(10,6)=2
d(10,7)=2
d(10,8)=1
d(10,9)=1
d(10,10)=0
d(10,11)=1

d(11,1)=5
d(11,2)=5
d(11,3)=4
d(11,4)=4
d(11,5)=3
d(11,6)=3
d(11,7)=2
d(11,8)=2
d(11,9)=1
d(11,10)=1
d(11,11)=0



Labeled graph G

Vertices	**v1**	**v2**	**v3**	**v4**	**v5**	**v6**	**v7**	**v8**	**v9**	**v10**	**v11**
Labels	**0**	**3**	**6**	**9**	**12**	**1**	**4**	**7**	**10**	**13**	**0**

Like this we did for different powers of paths 'k' and tabled below L (3, 2, 1) labeling number of G=P_n^k

'm' suffix of h	**'k' power of path**	**L(3, 2, 1) labeling number of** G=P_n^k
m=3	**k=1**	**7**
m=3	**k=2**	**13**
m=3	**k=3**	**19**
m=3	**k=4**	**25**
m=3	**k=5**	**31**
m=3	**k=6**	**37**
…	**…**	**…**
…	**…**	**…**
m=3	**k is even k=2s**	**12s**
m=3	**k is odd k=2s+1**	**12s+6**
m=3	**k is power of path**	**6k+1**

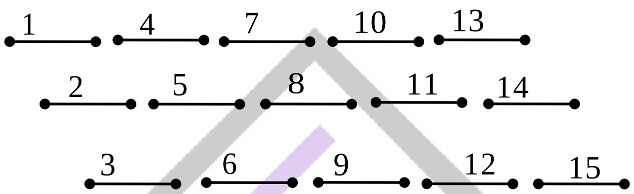
Theorem 6: if I = {i_1, i_2, …., i_n} be an interval family. We consider each interval intersects next 'k' consecutive intervals only. Interval graph corresponding to an interval family 'I' is power 'k' of path with length n then the L (h_1, h_2, h_3, …… , h_m) labeling number of G is **case:(1)** if 'm' is even than $\lambda_{h1,\, h2,\ldots,\, hm}\,(P_n^k) = \left(k\frac{m}{2}+1\right)m$,

case:(2) if 'm' is odd than $\lambda_{h1,\, h2,\ldots,\, hm}\,(P_n^k) = k\left(\frac{m+1}{2}\right)m + \left(\frac{m-1}{2}\right)$ where 'm' is suffix of h and 'k' is power of path.

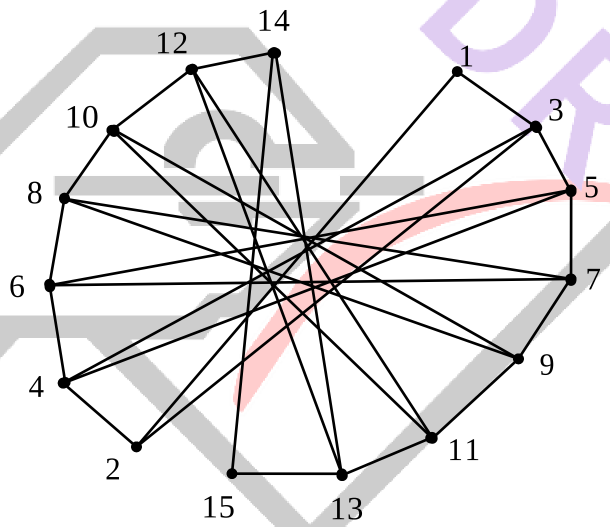
Proof: Let I = {i_1, i_2, …., i_n} be an interval family. We consider each interval intersects next k consecutive intervals only. Interval graph corresponding to an interval family 'I' is power k of path with length n.

For non-negative integers h_1, h_2, . . . , h_m , an L(h_1, h_2, . . . , h_m)-labeling of a graph G is defined as the mapping f: V(G)→{0, 1, . . . ,} such that $|f(u) - f(v)| \geq h_i$, if d(u, v) = i, $1 \leq i \leq m$, where d(u, v) is the distance between the vertices u and v. The L (h_1, h_2, . . . , h_m)-labeling numbers of G, is denoted by $\lambda_{h_1,h_2,\ldots,h_m}$ (G) which is the difference between largest and least labels used in L(h_1, h_2, . . . , h_m)-labeling. We will prove the theorem by following experimental problems.

Experimental Problem
Find the L(5, 4, 3, 2, 1) labeling of G = P_{15}^2



Interval family I



Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3
d(1,9)=4	d(2,9)=4	d(3,9)=3
d(1,10)=5	d(2,10)=4	d(3,10)=4
d(1,11)=5	d(2,11)=5	d(3,11)=4
d(1,12)=6	d(2,12)=5	d(3,12)=5

d(1,13)=6	d(2,13)=6	d(3,13)=5
d(1,14)=7	d(2,14)=6	d(3,14)=6
d(1,15)=7	d(2,15)=7	d(3,15)=6

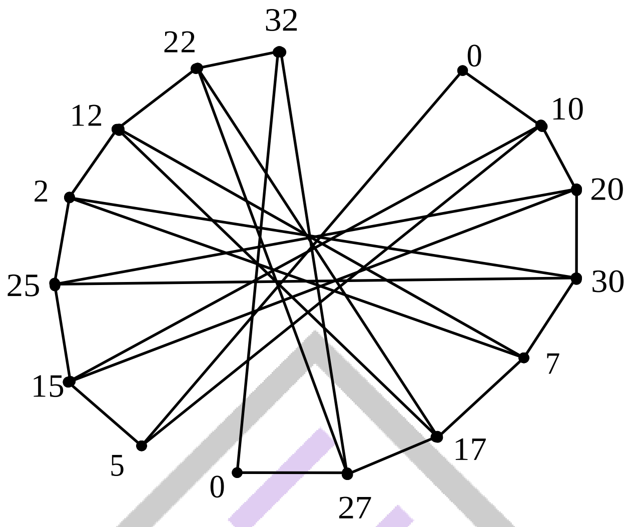
d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=1	d(5,2)=2	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=4	d(5,11)=3	d(6,11)=3
d(4,12)=4	d(5,12)=4	d(6,12)=3
d(4,13)=5	d(5,13)=4	d(6,13)=4
d(4,14)=5	d(5,14)=5	d(6,14)=4
d(4,15)=6	d(5,15)=5	d(6,15)=5

d(7,1)=3	d(8,1)=4	d(9,1)=4
d(7,2)=3	d(8,2)=3	d(9,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=3
d(7,4)=2	d(8,4)=2	d(9,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1
d(7,12)=3	d(8,12)=2	d(9,12)=2
d(7,13)=3	d(8,13)=3	d(9,13)=2
d(7,14)=4	d(8,14)=3	d(9,14)=3
d(7,15)=4	d(8,15)=4	d(9,15)=3

d(10,1)=5	d(11,1)=5	d(12,1)=6
d(10,2)=4	d(11,2)=5	d(12,2)=5
d(10,3)=4	d(11,3)=4	d(12,3)=5
d(10,4)=3	d(11,4)=4	d(12,4)=4
d(10,5)=3	d(11,5)=3	d(12,5)=4
d(10,6)=2	d(11,6)=3	d(12,6)=3
d(10,7)=2	d(11,7)=2	d(12,7)=3
d(10,8)=1	d(11,8)=2	d(12,8)=2
d(10,9)=1	d(11,9)=1	d(12,9)=2
d(10,10)=0	d(11,10)=1	d(12,10)=1
d(10,11)=1	d(11,11)=0	d(12,11)=1
d(10,12)=1	d(11,12)=1	d(12,12)=0
d(10,13)=2	d(11,13)=1	d(12,13)=1
d(10,14)=2	d(11,14)=2	d(12,14)=1
d(10,15)=3	d(11,15)=2	d(12,15)=2

d(13,1)=6	d(14,1)=7	d(15,1)=7
d(13,2)=6	d(14,2)=6	d(15,2)=7
d(13,3)=5	d(14,3)=6	d(15,3)=6
d(13,4)=5	d(14,4)=5	d(15,4)=6
d(13,5)=4	d(14,5)=5	d(15,5)=5
d(13,6)=4	d(14,6)=4	d(15,6)=5
d(13,7)=3	d(14,7)=4	d(15,7)=4
d(13,8)=3	d(14,8)=3	d(15,8)=4
d(13,9)=2	d(14,9)=3	d(15,9)=3
d(13,10)=2	d(14,10)=2	d(15,10)=3
d(13,11)=1	d(14,11)=2	d(15,11)=2

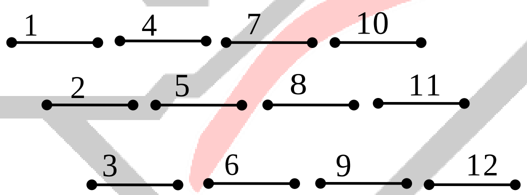
d(13,12)=1 d(14,12)=1 d(15,12)=2
d(13,13)=0 d(14,13)=1 d(15,13)=1
d(13,14)=1 d(14,14)=0 d(15,14)=1
d(13,15)=1 d(14,15)=1 d(15,15)=0



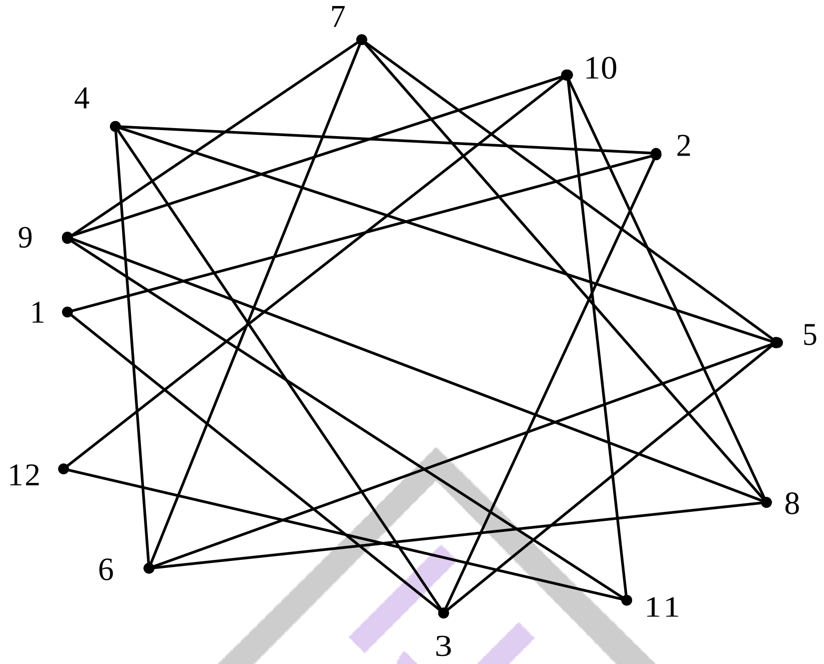
Labeled graph G

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
Labels	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**2**	**7**	**12**	**17**	**22**	**27**	**32**	**0**

Experimental Problem
Find the L(4, 3, 2, 1) labeling of G = P_{12}^{2}



Interval family I



Interval graph G

Find the distance between u, v such that u, v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=2	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=3	d(3,8)=3
d(1,9)=4	d(2,9)=4	d(3,9)=3
d(1,10)=5	d(2,10)=4	d(3,10)=4
d(1,11)=5	d(2,11)=5	d(3,11)=4
d(1,12)=6	d(2,12)=5	d(3,12)=5

d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=1	d(5,2)=2	d(6,2)=2
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=4	d(5,11)=3	d(6,11)=3
d(4,12)=4	d(5,12)=4	d(6,12)=3

d(7,1)=3	d(8,1)=4	d(9,1)=4
d(7,2)=3	d(8,2)=3	d(9,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=3
d(7,4)=2	d(8,4)=2	d(9,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1

d(7,9)=1
d(7,10)=2
d(7,11)=2
d(7,12)=3

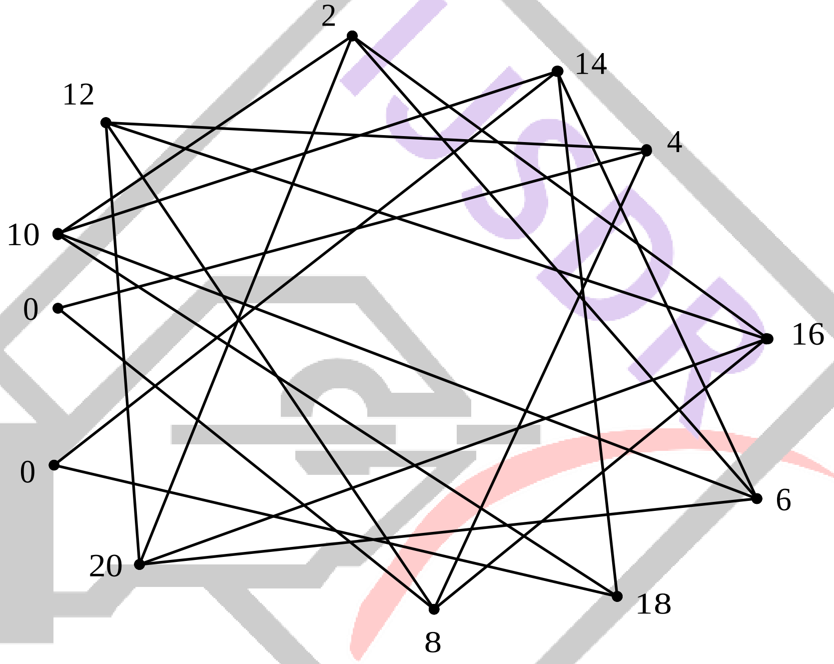
d(8,9)=1
d(8,10)=1
d(8,11)=2
d(8,12)=2

d(9,9)=0
d(9,10)=1
d(9,11)=1
d(9,12)=2

d(10,1)=5
d(10,2)=4
d(10,3)=4
d(10,4)=3
d(10,5)=3
d(10,6)=2
d(10,7)=2
d(10,8)=1
d(10,9)=1
d(10,10)=0
d(10,11)=1
d(10,12)=1

d(11,1)=5
d(11,2)=5
d(11,3)=4
d(11,4)=4
d(11,5)=3
d(11,6)=3
d(11,7)=2
d(11,8)=2
d(11,9)=1
d(11,10)=1
d(11,11)=0
d(11,12)=1

d(12,1)=6
d(12,2)=5
d(12,3)=5
d(12,4)=4
d(12,5)=4
d(12,6)=3
d(12,7)=3
d(12,8)=2
d(12,9)=2
d(12,10)=1
d(12,11)=1
d(12,12)=0



Labeled graph G

Vertices	**v_1**	**v_2**	**v_3**	**v_4**	**v_5**	**v_6**	**v_7**	**v_8**	**v_9**	**v_{10}**	**v_{11}**	**v_{12}**
Labels	**0**	**4**	**8**	**12**	**16**	**20**	**2**	**6**	**10**	**14**	**18**	**0**

'm' suffix of h	**'k' power of path**	**$L(h_1, h_2, \ldots, h_m)$ labeling number of G=P_n^k**
m=2	**k=2**	**6**
m=3	**k=2**	**13**
m=4	**k=2**	**20**
m=5	**k=2**	**32**
m=6	**k=2**	**42**
..	**..**	**..**
..	**..**	**..**
m is even	**k=2**	**(m+1)m**
m is odd	**K=2**	**$(m+1)m+\left(\frac{m-1}{2}\right)$**

‘m’ suffix of h	‘k’ power of path	L($h_1, h_2, . . . , h_m$) labeling number of G=P_n^k
m=2	k=3	8
m=3	k=3	19
m=4	k=3	28
m=5	k=3	47
m=6	k=3	60
..	..	..
..	..	..
m is even	k=3	$\left(3\frac{m}{2}+1\right)m$
m is odd	k=3	$3\left(\frac{m+1}{2}\right)m+\left(\frac{m-1}{2}\right)$

‘m’ suffix of h	‘k’ power of path	L($h_1, h_2, . . . , h_m$) labeling number of G=P_n^k
m=2	k=4	10
m=3	k=4	25
m=4	k=4	36
m=5	k=4	62
m=6	k=4	78
..	..	..
..	..	..
m is even	k=4	$(2m+1)m$
m is odd	k=4	$2(m+1)m+\left(\frac{m-1}{2}\right)$

‘m’ suffix of h	‘k’ power of path	L($h_1, h_2, . . . , h_m$) labeling number of G=P_n^k
m=2	k=5	12
m=3	k=5	31
m=4	k=5	44
m=5	k=5	77
m=6	k=5	96
..	..	..
..	..	..
m is even	k=5	$\left(5\frac{m}{2}+1\right)m$
m is odd	k=5	$5\left(\frac{m+1}{2}\right)m+\left(\frac{m-1}{2}\right)$

‘m’ suffix of h	‘k’ power of path	L($h_1, h_2, . . . , h_m$) labeling number of G=P_n^k
m=2	k=6	14
m=3	k=6	37
m=4	k=6	52
m=5	k=6	92
m=6	k=6	114
..	..	..
..	..	..
m is even	k=6	$(3m+1)m$
m is odd	k=6	$3(m+1)m+\left(\frac{m-1}{2}\right)$

'm' suffix of h	'k' power of path	$L(h_1, h_2, \ldots, h_m)$ labeling number of $G=P_n^k$
m is even	k is even k=2s	$(sm+1)m$
m is even	k is odd k=2s+1	$((2s+1)\frac{m}{2}+1)m$
m is odd	k is even k=2s	$s(m+1)m+\left(\frac{m-1}{2}\right)$
m is odd	k is odd k=2s+1	$(2s+1)\left(\frac{m+1}{2}\right) m+\left(\frac{m-1}{2}\right)$

In general $L(h_1, h_2, \ldots, h_m)$ labeling number of $G=P_n^k$ for any power of path 'k' and suffix of h 'm'

$$\textbf{m is even } \lambda_{h_1,h_2,\ldots,h_m}(P_n^k) = \left(k\frac{m}{2}+1\right)m$$

$$\textbf{m is odd } \lambda_{h_1,h_2,\ldots,h_m}(P_n^k) = \left(k\left(\frac{m+1}{2}\right)m+\left(\frac{m-1}{2}\right)\right)$$

Conclusion: In this chapter we find the $L(h_1, h_2, h_3, \ldots, h_m)$ labeling number of paths using an interval graph G and formulated as **case (1)** if 'm' is even than $\lambda_{h1,h2,\ldots,hm}(P_n) = \left(\frac{m}{2}+1\right)m$, **case(2)** if 'm' is odd than $\lambda_{h1,h2,\ldots,hm}(P_n) = \left(\frac{m+1}{2}\right)m + \left(\frac{m-1}{2}\right)$. $L(h_1, h_2, h_3, \ldots, h_m)$ labeling number of squares of paths and formulated as **case(1**) if 'm' is even than $\lambda_{h1,h2,\ldots,hm}(P_n^2) = (m+1)m$, **case(2)** if 'm' is odd than $\lambda_{h1,h2,\ldots,hm}(P_n^2) = (m+1)m + \left(\frac{m-1}{2}\right)$. $L(h_1, h_2, h_3, \ldots, h_m)$ labeling number of power three of paths and formulated as **case(1**) if 'm' is even than $\lambda_{h1,h2,\ldots,hm}(P_n^3) = \left(3\frac{m}{2}+1\right)m$, **case(2)** if 'm' is odd than $\lambda_{h1,h2,\ldots,hm}(P_n^3) = 3\left(\frac{m+1}{2}\right)m + \left(\frac{m-1}{2}\right)$. Finally $L(h_1, h_2, h_3, \ldots, h_m)$ labeling number of all powers of paths **case1**) if 'm' is even than $\lambda_{h1,h2,\ldots,hm}(P_n^k) = \left(k\frac{m}{2}+1\right)m$, **case(2)** if 'm' is odd than $\lambda_{h1,h2,\ldots,hm}(P_n^k) = k\left(\frac{m+1}{2}\right)m + \left(\frac{m-1}{2}\right)$ Where 'm' is suffix of h and 'k' is power of path.

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