Rationalizing the Impact of Geometric Form towards Structural Efficiency

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Abstract: **Recent design trends indicate that the structural inputs are taken during the post-design stage (after finalizing the concept and geometric form by an architect) and this restricts the creativity of the structural engineer. Hence, there is a need to develop a process for designing the built mass/ geometry, which is not only governed by functional or aesthetic requirements, but also through strong principles derived from the physical forces and thus creating an efficient solution. This process, when exercised by a designer/ architect from the pre-design stage, forms a holistic design approach. To understand the impact of geometries on their structural behavior, the research analyses various geometric typologies, evolution, structural classifications affecting their stability. The research would investigate the effect of minor alterations to the geometry (such as curvature) and their effect on structural efficiency. It also discusses the technological advancements of the geometric form and the structural principles derived from various case studies. This paper showcases the application of engineering principles and material optimization, which results in a slender, lightweight and efficient structures.**

Keywords**: Shell Structures, Tensile structures, Long Span Curved Roofs, Barrel Roof, Folded Plate, Saddle Roof, Dome.**

1. INTRODUCTION

Since the evolution of primitive shelters, humans have built their spaces/shelters to suit their essential functional aspects with a primary motive to resist the forces from nature. All these primitive geometrical shapes were the crude solutions, as a response to adapt the natural environmental conditions. In the due course of time, as the needs of the users kept changing drastically, this resulted in a highly sophisticated form and architectural language. Which in turn was complemented by development in the techniques, technology, materials and innovations in designs. Due this sophistication in the technology and process for designing the built environment led to streamlining and dividing the roles among different professionals (such as Engineers, Architects, Property owners, Contractor, Construction Labours, etc) who address their particular scope of work and the coordination between these professionals is the key for a successful project. Two such distinct professionals include architects and structural designers. Often, structural inputs are taken as post-designing solutions, which would not integrate from the concept development and the pre-designing stage. When structural engineering is confined to just a mathematical approach, an architect's input towards the structural efficiency would be negligible. Hence, The architect who initiates the design process has to have a good knowledge of achieving structurally efficient forms. On the other hand, Engineers are not trained enough with the architectural/aesthetic part of a structure. Two essential members of construction talk a different language and there is a necessity to bridges the difference, which makes design a holistic process.

This gap created between the professionals provides a huge scope for the designer to address and create an efficient form, which is not just a solution to aesthetic and functional aspect but also, to explore a process in which form is governed by strong principles of its physical forces from the pre-design stage. Hence, the analysis and understanding of the contribution of geometric form towards structural efficiency become vital for every designer in this era to take a holistic design approach.

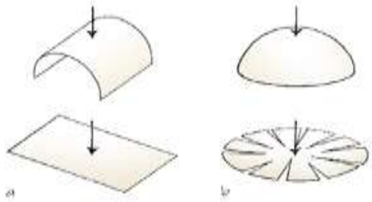
2. CONTRIBUTION OF GEOMETRY

The evolution of long-span structures began with a necessity to accommodate a large number of people during the gatherings and to accommodate this a roof structure was essential, which is only supported at its periphery giving a large clear space. Even till date, no roof structural system using natural materials or any artificial compressive material can be designed without curving them or using curvature, this was the reason why domes were used in past to span longer distances.

After the innovation and development of modern techniques in steel manufacturing methods to make them inexpensive and technological advancements in reinforced concrete made the design of flat roof possible and practical. They had an advantage as compared to domes or other curved structures as they do not waste excess space above the roof which is superflux and unnecessarily increases the load on mechanical heating and cooling system, also the erecting is much simpler as compared to the dome.

Developable and Non-Developable Surfaces:

An experiment performed by bending a straight sheet to take a form of a semi-cylinder and observing its displacement under its self-weight or gravity loads. We observe that the sheet would again develop to its original shape, i.e straight sheet. Whereas, if we consider a dome, it would not develop into a straight surface even if we try to push it in, until its surface would shear off by developing cracks or unless if we make some radial cuts. Here, the dome is non-developable surface and they are much more stable than the developable surface as they are difficult to flatten. Hence, they were also used in the past, as dome structure to span longer distances.



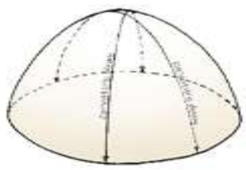
When a similar study is done to analyse a flat slab behaviour, consider a flat slab plate supported on 2 opposite sides, it bends down and creates an upward curvature. But, the same experiment performed when all the four sides are supported, this would result in a non-developable surface. External forces have to stretch the membrane surface, in addition, to bend or twist the slab membrane. This provides additional strength to the non-developable surface. If slabs need to stretch more than fifteen or twenty feet, stiffening them with ribs on their underside is economical, which can be organized in a variety of ways. Nervi used Ferro-cement, to pour in slabs stiffened by curved ribs, angled in the most rational directions to move the loads from the slab to the columns. In fact, these rounded ribs give the underside of the slabs great beauty.

3. STRENGTH THROUGH CURVATURE

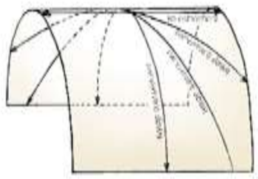
The rigidity of flat slabs, like that of beams, depends on its thickness and they become too elastic if they are too thin. Hence to counteract the issue occurred due to their slenderness and to achieve the stiffness and strength not just by increasing their thickness, as this would increase in the amount of material needed and increasing the self-weight. Instead, we can add an appropriate curvature to the slender members, which increases their strength. To visualise this, consider an experiment by holding the short side of a thin sheet of paper and we observe that the sheet drops down as sit's unable to bear even its own weight. However, if we hold it up with a slight curvature, the same sheet of paper is now stronger and capable of supporting its own weight and also can carry a small external weight of a pencil or pen too. This concept to attain strength by curvature can be applied to thin sheets of reinforced concrete and has been used successfully to create stadium roofs with a thickness of just a few inches which cantilever out thirty feet or even more. As we observe the form of such roofs is developable surface, such as cylinders, show sufficient strength due to their curvature (when properly supported) to allow their use as a structural element.

Types of Curvature:

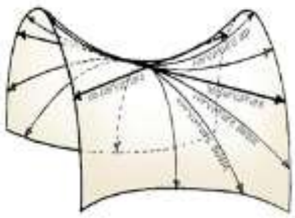
Karl F.Gauss (1777-1855) discovered that all the infinitely varying curved surfaces are found in nature or imagined belong only to three types of families, dome, cylinder or saddle. In all its radial directions, a domelike surface has curvatures downwards. Domes and hanging roofs are the first category of Gauss's classes, each with curvatures always in the same direction (either downwards or upwards). These are non-developable materials that have been used to cover large surfaces for centuries.



The second category for Gauss is the tubes (cylinder). We notice that all the curvature is in downward direction, except for one: cutting along the axis of the pipe has a straight line of curvature. The cylinder does not have a curvature in its axis direction.



The third of Gauss's categories is the saddle like surfaces technically called Hyperbolic paraboloid or Hypar Roofs. Saddle Roof is the most stable and structurally efficient geometric shape as they have two directions of curvature, one positive and another negative. A line where the curvature changes from positive to negative have a zero curvature. Saddle surfaces fall under the category of non-developable surfaces and have gained significance due the stiffness achieved. Architect Felix Candela designed a saddle surface, which is the thinnest concrete roof in the world of just one and a half an inch thick covering the Cosmic Rays Laboratory, Mexico.

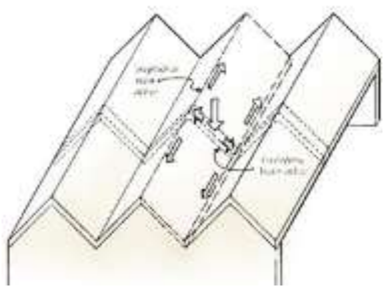


4. BARREL ROOFS

Even if some developable surfaces such as cylinders are less stiff as compared to non-developable surfaces such as domes saddle or domed surfaces, they are still used as roof surfaces. This is due to simplicity in erecting formwork and the same formwork can be moved to a different location and can be used multiple times due to less complexity of their curvature. Hence they are found mostly in industrial buildings. Location of supports influences the load carry mechanism of the barrel/vaulted roof. Consider, the roof supported along its longitudinal axis then the roof is equivalent to a series of arches placed beside each other. Hence, elements such as buttresses or tier-beams are used to absorb the developed outward thrust action of the arches. Considering the same vault supported on its curved or shorted face, they act as a beam and resembling the same neutral axis concept, where the concrete above neutral axis undergoes compression and below undergoes tension. Hence, the vaults must be supported on the longitudinal edges to take advantage of its geometric shape.

5. FOLDED PLATE ROOF

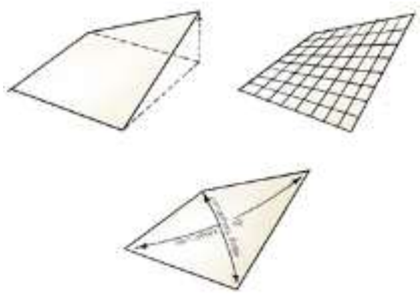
As we have discussed barrel vaults earlier, folded plates are similar geometric shape which is a series of zig-zag profiled flat plates along their shorter side. The cross-section has valley lines and ridge lines. They require a very simple, flat formwork or sometimes no formwork at all, as the flat slabs can be casted on the ground and then moved up by grouting and connecting the valley or ridges using reinforcement and mortar. Hence the monolithic nature can be achieved. The load transfer mechanism here follows "Twofold Path". Load act on the ridges, from which they get diverted to adjacent valley lines, from where the load is transferred to the supports along the longitudinal line acting as a beam. Here, the stiffness is attained through the fold pattern of the slab. The support should be placed at the ends and the plates along the supports have to be designed like a general slab.



6. SADDLE ROOF

An Anti-clastic shape is considered as a most stable geometric shape when compared to any other geometries. One of the best examples of anticlastic shape is the saddle roof. Saddle surfaces are supported along their longitudinal curved edge profile. To visualize the shape of the saddle surface, connect two rims of equal diameter placed apart with equal length strings vertically. Now, rotate the top rim fixing the bottom rim to the ground. We can visualise the shape attained by the strings, they are doubly curved and know as rotational hyperboloid, this shape generated has enormous strength and hence used as cooling towers.

The saddle shape can also be achieved through a rectangle, in which one corned is displaced to a distance vertically, whereas other vertices are placed on the ground level. Hence, this created a shape where two sides are horizontal and rest two sides have a curvature upward. We now connect the similar points along the surface in the opposite direction. One set of lines curve down and the other set curves up and technically called hyperbolic paraboloid or Hypar know in common. Hypar roof can be attained by simply tilting the saddle so that both the vertices rest on ground or column.



The structural strength of these roofs is attained by their curvature. Downward curvature along its section acts as an arch by absorbing the compressive stresses, hence to resist the thrust reaction exerted by downward curvature and their supports must be buttressed. Whereas the perpendicular curvature to the downward curvature undergoes tension or upward pull and hence must be reinforced to carry the tension (if built out of concrete).

With such a sophisticated and efficient load transfer mechanism of hypar roofs, the cross-section could be a thin profile with just a few inches thick to span long distances of 30-40 ft. The upward curvature pulls its perpendicular downwards curvature of the roof and thus preventing the buckling tendency of slender cross-sections of hypar roofs. Adding to this, under uniform loads such as dead load or snow load, they develop the same tension and compression throughout. With so many advantages present in it, the setback preventing it from vast usage is their cost for the formwork.

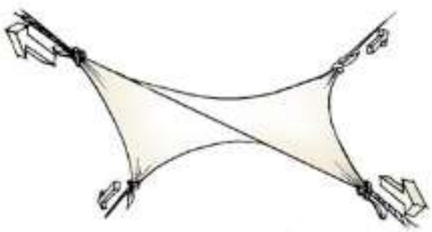
7. TENSILE STRUCTURE

Tents are one of the crude examples of pre-stressed form works and winds are the critical factors affecting the stability of the tents. If the tent is a simple unstiffen piece of cloth, a strong breeze could make the structure flap and tear away the structure. This issue could be solved by simply putting the structure in tension or by stretching it, which not only acquires strength but also can support various external loads without deformation. The same mechanism is used to catch people jumping from high windows or openings in time of fire escape and they just bounce over the trampoline instead of getting hurt.

Humans used these tents or tensile structures to harness to potential of light weight characteristic, but the material and fabric of the component in tension play a key role in such systems. The tauter the membrane, the stiffer the structure. Hence, Natural fibbers are generally not suited as they cannot take a large stretch and plastic fibres have evolved to address this purpose. They include the vinyl's that are inexpensive but is vulnerable upon exposure to ultraviolet rays of the sun. Alternatively, Glass fibres were introduced which consist of a woven network of woven glass threads in a thin layer of plastic. In recent times, the materials are coated with Teflon and if required by additional coats of reflective plastic. The shear-off strength for these materials is more than 800 pounds per inch and additionally, they have a self-cleaning capacity as dust particles repel from their surface material.

Secondly, the strength of the material is not sufficient for longer spans through a single bay. Hence, this thin membrane is segmented at intermediate positions by heavy boundary cables, which are supported on long column support. Frei Otto was a master in using these advance techniques and spanning these systems to longer spans in many of his projects, one of which is Munich Olympic Stadium.

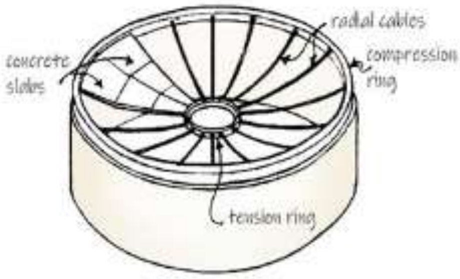
The Tensile roofs if stretched flat in the same plane has a high risk of deflection during the impact of any external loads such as snow, rain or wind pressure. Hence, a curvature (at least one downward curvature) in the roof profile is always preferred compared with a plain roof.



Saddle Roof is one of the best solutions for tensile structures. A simple saddle roof using a tensile membrane can be visualized by tying two opposite end at the top and the other two end pull-down and tied stiff. The surface complexity and strength achieved is due double curvature of which one is one positively curved and other negatively curved, providing maximum stretch through geometry.

Hanging Roofs:

The primary concept to the hanging roof is to suspend the inner rim through a series of cables which connects this inner rim to the outer rim. The outer rim is supported by the columns or a continuous shear wall. As the inner rim is stretched by the cables through which it hangs at a height less than the outer rim, the inner undergoes large tension due to the stretch created on it and hence must be made by a material which is capable of withstanding large tensile stresses, such as steel. Similarly, even the cables undergo large tensile stress due to the stretch created by hanging the inner rim and hence they must be made of a material able to withstand large wear and tear with high tensile strength, such as metal cables. Whereas outer rim undergoes a compression due to inward pull created by the cables, hence the outer rim must be made of material, which can undergo large compressive forces.



8. CASE STUDY

Various projects have been studied to understand the evolution of the structure with a change in the timeline and to determine criteria to design a particular roof structure. The projects selected for the research includes diversity, both in terms of typology and their span. Hence, this aims at giving a broader understanding.

Project	Agnelli Exhibition Hall B	Algeciras Market Hall	Bacardi Rum Factory
Location	Turin, Italy	Algeciras, Spain	Cuautitlan, Mexico
Engineer/Designer	Pier Luigi Nervi	Eduardo Torroja	Luis Torrres Landa / Felix Candela
Year of Completion	1948	1934	1960
Type of structure	Barrel vault	Thin shell dome	Thin shell hypar
Material	Ferro-cement	Pre-stressed concrete	Reinforced concrete
Dimensions	Main hall is 328 ft by 262 ft and 60 ft tall	The diameter is 47.6 m and is 90 mm thick	Six vaults of a square plan of 30 m side and 12 m in height

Project	Berlin Hauptbahnhof	Stock Exchange Hall	BP Station
Location	Berlin, Germany	Mexico City, Mexico	Deitingen, Switzerland
Engineer/Designer	Ürgen Hillmenr, Meinhard Von Gerkan, Marg & Partners, Hamburgo Gerkan	Felix Candela, Enrique de La Mora Lopez, and Fernando Carmona	Heinz Isler
Year of Completion	2006	1955	1968
Type of structure	Grids with cable network	Thin shell hypar	Thin shell dome
Material	Glass and steel roof; Reinforced concrete tower	Reinforced concrete	Reinforced concrete
Dimensions	Length is 182.7m and 46m height. Each span is 8.7m wide.	Length is 25.5 m and the width is 14.1 m	Roof is 31.6m long and 26m wide. The slab thickness is about 10-12 cm thick.

Project	Dutch Maritime Museum	Heimberg Tennis Centre	Flaminio Stadium
Location	Amsterdam, Holland	Heimberg, Switzerland	Rome, Italy
Engineer/Designer	Laurent Ney & Dok Architecten	Heinz Isler	Pier Luigi Nervi
Year of Completion	2011	1979	1960
Type of structure	Dome	Thin shell dome	Pleated beam cantilever
Material	Glass and steel	Reinforced concrete	Ferro cement
Dimensions	A square plan of side 34m, with a maximum height of 5m.	Length of the roof is 48 m, the width 18.6 m, height is 9.9m	Length is 85.8 m and width is 153.3 m; roof beam is 1.42 m wide, 27.7 m long

Project	Naturtheater Grotzingen	Norfolk Scope Arena	Our Lady of the Miraculous Medal Church
Location	Grotzingen, Switzerland	Norfolk, VA	Narvarte, Mexico City, Mexico
Engineer/Designer	Heinz Isler	Pier Luigi Nervi	Felix Candela
Year of Completion	1977	1971	1955
Type of structure	Thin shell dome	Shell	Hyperbolic Paraboloid
Material	Reinforced concrete	Reinforced concrete	Concrete
Dimensions	5 point supported shell with a thickness of 9-12cm, Over a maximum length of 28m and width of 42m.	The diameter of the roof is 99m and a height of 35.5m. The thickness of the slab varies from 14.5cm to 19cm	Dimensions of the roof are 31m in length, 53m in width and 20 m high.

Project	Restaurant Los Manantiales in Xochimilco	Rio's Warehouse	San Antonio de las Huertas
Location	Xochimilco, Mexico City, Mexico	Mexico City, Mexico	Mexico City, Mexico
Engineer/Designer	Felix Candela	Felix Candela	Felix Candela
Year of Completion	1958	1954	1956
Type of structure	Hyperbolic Paraboloid (Hypar)	Umbrella shell	Thin shell hypar
Material	Concrete	Concrete	Reinforce concrete
Dimensions	The diameter of the roof is 139 ft and Groins spanning at 106 ft.	Solid umbrellas measuring 10m X 15m; thickness of 4cm.	Roof consist of three squares of 1mX10m

Project	Zarzuela Hippodrome	School roof next to "Sagrada Família''	Subway Station Candelaria
Location	Madrid, Spain	Barcelona, Spain	Mexico City, Mexico
Engineer/Designer	Eduardo Torroja	Antoni	Felix Candela
Year of Completion	1935	1943	1967
Type of structure	Thin shell horizontal cantilever	Tile vaulted school roof in a conoid structure.	Folded hypar
Material	Reinforced concrete	Layered bricks	Reinforce concrete
Dimensions	The slab of 6 cm thickness which is supported by ribs of 20X 10cm. The slab is cantilevered by 12.75m and supports are placed 5.35m apart.	The roof consists of 3 layers of bricks and the Catalan vault is 24m long and 12m wide, 8cm thick.	The roof spans 66m in length and 29m in width with each 22 umbrella members of dimension 13X6m each.

Project	The Kresge Auditorium	The Little Sports Palace	The Wyss Garden Centre
Location	MIT, Cambridge, MA	Rome, Italy	Solothurn, Switzerland
Engineer/Designer	Ammann & Whitney	Pier Luigi Nervi	Heinz Isler
Year of Completion	1955	1957	1961
Type of structure	Dome	Ribbed dome	Thin shell Dome
Material	Reinforced concrete	Reinforced concrete	Reinforce concrete
Dimensions	The roof spans about 34.50m and a height of 15.24m.	The roof is of 60m in diameter and a height of 21m.	The roof spans 28m in length and 25m in width. The slab is 8cm in thick.

Project	Fronton Recoletos	Gatti Wool Factory	HighLife Textile Factory
Location	Madrid, Spain	Turin, Italy	Mexico city, Mexico
Engineer/Designer	Secundino Zuazo, Eduardo Torroja	Pier Luigi Nervi along with Aldo Arcangeli, Carlo Cestelli Guidi	Felix Candela

Year of Completion	1935	1951	1955
Type of structure	Barrel shell	Ribbed slabs	Perforated umbrella shell
Material	Reinforced concrete	Reinforced concrete	concrete
Dimensions	The diameter of 6.40 and 12.20 m; length of 55 by width 32.50 m	Roof slabs of dimensions 5m by 5m each.	Umbrella's length of 12 m, the width of 12 m, and a thickness of 4 cm

Project	Chapel Lomas De Cuernavaca	Turin Exhibition Hall	Cosmic Rays Laboratory
Location	Curenavaca, Mexico	Turin, Italy	Mexico City, Mexico
Engineer/Designer	Felix Candela; collaborated with Guillermo Rosell and Manuel Larrosa	Pier Luigi Nervi	Felix Candela
Year of Completion	1959	1954	1955
Type of structure	Grids with cable network	Vault	Hyperbolic paraboloid
Material	Rei::nforced concrete	Ferro cement	Reinforced concrete
Dimensions	A single equilateral hypar of 30m span and 24m height.	The shell is 3cm thick with a diameter of 40m.	shell is 12 m long, 10.75 m wide; thickness is 1.5-2 cm

Project	St. Louis Airport Terminal	The Sicli Building	The Food Court: Infosys Campus
Location	St. Louis, Missouri	Geneva, Switzerland	Hyderabad, India
Engineer/Designer	Minoru Yamasaki	Heinz Isler	Sundaram Architects
Year of Completion	1956	1970	2003
Type of structure	Thin shell	Thin shell dome .	Dome
Material	Reinforced concrete	Thin shell dome	Reinforce concrete
Dimensions	The shell is 3.5 inches thick and spans 12m.	Roof is of 33m x 54 m, which has 7 supports.	Roof is of 40m in diameter.

Project	Hyperbolic Paraboloid Shell	Kingdome	Municipal Stadium
Location	Denver, Colorado	Seattle, Washington	Florence, Italy
Engineer/Designer	Anton Tedesko	NBBJ (Seattle)	Pier Luigi Nervi
Year of Completion	1935	1976	1932
Type of structure	Hypar	Dome	Horizontal cantilever
Material	Reinforced concrete	Concrete and structural steel	Reinforced concrete
Dimensions	The roof consists of four hyperbolic paraboloid surfaces and they supported through steel hinges on buttresses in the four corners of a rectangle 112 x 132 ft the 3 in shell rises to a height of 28 ft.	One of the thinnest shell roof in the world, which spans over 200m and rising 76m high.	78 m by 204.6 m playing field; consists of 15 cantilevers, each 14.6 m long; floor slab thickness of 25 cm

9. ANALYSIS OF RESULTS

From the case studies, we can thus deduce certain principles for some of the most commonly used typologies such as:

Barrel Vault

- Long - barrel shells are structurally effective when the chord width is small compared to the length of the span. Cantilevers are achieved easily; thus the visible edge can be as thin as the basic shell.
- Chord widths vary to about 60 feet and shell thickness normally is about 3 inches.
- Height is determined by the span length. Areas from 40 to 160 feet may be spanned.
- Impressive interior effects, including many interesting lighting systems, can be achieved with exposed barrel shells.

Folded Plate

- The three basic types of folded plate shells are: V- shape, Z-shape, and modified W- shape.
- The usual depth to span ratio varies from 1:10 to 1:15.
- The thickness of shells is 3 to 6 inches. Spans range from 40 to 150 feet.
- The slope of a fold is from 25 to 45 degrees maximum unless there are special provisions
- Ridges and valleys can be sharp or blunted.
- A flat deck enclosing the channels formed by the folds may be used to integrate mechanical and electrical systems.

Domes

- A tension ring is usually required at the circumference of the shell.
- Thickness ranges from 3 to 4-1/2 inches.
- Spans range from 50 to 200 feet or more.
- Domes may be supported uniformly or may touch the earth at as few as three points.
- They may be pierced as desired for natural light, or appropriate light fixtures may be used.

Hyperbolic Paraboloid Umbrellas

- The low stresses in a hyperbolic paraboloid shell require only a minimum thickness of concrete and this thickness depends upon the concrete cover required for the reinforcement which varies from 3 to 3-1/4 inches
- Edge and ridge beams are required.
- The ceiling of each umbrella becomes a clean, unobstructed curved surface that is both attractive and strong.
- They are ideal for warehouses since they minimize exterior wall height.
- Multiple re-uses of forms make them practical.

Hyperbolic Paraboloid Saddle Roofs

- Generally are used for one-story buildings where large spans are required for both length and width.
- H/P saddle-shaped shell is a three - dimensional slab in which strength and rigidity are accomplished by curving it in space, not by increasing the thickness of the slab.
- Shell thickness is from 2 3/4 to 4 inches.
- Spans range from 50 to 160 feet.
- Shell projection is from 50 to 220 feet.
- A single-saddle roof requires only two abutments for support.

10. CONCLUSION

The research primarily focuses on the significance of the geometry and its contribution to structural strength. These innovations through shells and efficient structural systems help in designing light and efficient structures with the least material usage (as there is a significant reduction in the cross-section). For any project, the primary motive for any designer would be to reduce the cost and making yet economical and yet to retain its aesthetic and functional statements. Hence to span longer distances or larges spaces, conventional structural systems will turn out to be expensive and unpleasing aesthetically due to their huge structural members. To address these attempts were made to resist these loads more effectively and to derive an efficient geometric form in comparison to the conventional techniques. This led to the experimentation and innovation of structural efficient geometries like a shell or tensile structures. The efficiency of long-span structures depends on various factors like the economy of construction, formwork, utility, location and innovative design.

Apart from the look and feel of the slender nature, this reduction in material use age also contributes to efficient use of resources making the new door open for sustainability. The structural innovations should not be just restricted to the domain of engineers, as it would cause to lose a valuable opportunity for architects to innovate with the structural form. The design process and brainstorming to generate an efficient structure has to begin from the pre-design development stage, where architects work on the conceptual design forms. As we observed that simple changes in the geometry such as through curvature or folds, can cause a large difference in the stiffness/strength can take much greater loads. Hence, the input of geometric contribution towards structural efficiency is essential and must be considered as a conceptual idea, rather than post-design structural interventions.

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