Hypothesis and Applications of Numerical Analysis

[1]U.SUJATHA, [2]G.VEERESH, [3]Y.APARNA

[1]ASSISTANT PROFESSOR, [2]ASSISTANT PROFESSOR, [3]ASSISTANT PROFESSOR
[1]HUMANITIES AND SCIENCE,
[1]Dr.K.V.SUBBA REDDY COLLEGE OF ENGINEERING FOR WOMEN
(Approved by AICTE, Affiliated to JNTUA)
KURNOOL, INDIA

***Abstract*: Hypothesis and Applications of Numerical Analysis is an independent Second Edition, giving an early because of the fundamental themes in numerical examination. The book underlines both the hypotheses which demonstrate the fundamental thorough arithmetic and the calculations which characterize exactly how to program the numerical strategies. Both hypothetical and viable models are incorporated.**

***Index Terms*: Numerical Algorithms, Polynomials, Ordinary Differential equations, Partial Differential equations, Computer programs.**

I. INTRODUCTION

What is Numerical Analysis? Numerical Algorithms. Appropriately Posed and Well-Conditioned Problems. Essential Analysis: Functions. Points of confinement and Derivatives. Arrangements and Series. Incorporation. Logarithmic and Exponential Functions. Taylor's Polynomial and Series: Function Approximation. Taylor's Theorem. Assembly of Taylor Series. Taylor Series in Two Variables. Power Series. The Interpolating Polyomial: Linear Interpolation. Polynomial Interpolation. Exactness of Interpolation. The Neville–Aitken Algorithm. Backwards Interpolation. Separated Differences. Similarly Spaced Points. Subsidiaries and Differences. Impact of Rounding Error. Decision of Interpolation Points. Instances of Bernstein and Runge. "Best" Approximation: Norms of Functions. Best Approximations. Least Squares Approximations. Symmetrical Functions. Symmetrical Polynomials. Minimax Approximation. Chebyshev Series. Streamlining of Power Series. The Remez Algorithms. Further Results on Minimax Approximation. Splines and Other Approximations: Introduction. B-Splines. Similarly Spaced Knots. Hermite Interpolation. Pade and Rational Approximation. Numerical Integration and Differentiation: Numerical Integration. Romberg Integration. Gaussian Integration. Inconclusive Integrals. Inappropriate Integrals. Numerous Integrals. Numerical Differentiation. Impact of Errors. Arrangement of Algebraic Equations of One Variable: Introduction. The Bisection Method. Addition Methods. One-Point Iterative Methods. Quicker Convergence. Higher Order Processes. The Contraction Mapping Theorem. Direct Equations: Introduction. Lattices. Straight Equations. Rotating. Examination of Elimination Method. Grid Factorization. Minimal Elimination Methods. Symmetric Matrices. Tridiagonal Matrices. Adjusting Errors in Solving Linear Equations. Grid Norms and Applications: Determinants, Eigenvalues, and Eigenvectors. Vector Norms. Lattice Norms. Molding. Iterative Correction from Residual Vectors. Iterative Methods. Network Eigenvalues and Eigenvectors: Relations between Matrix Norms and Eigenvalues; Gerschgorin Theorems. Basic and Inverse Iterative Method. Sturm Sequence Method. The QR Algorithm. Decrease to Tridiagonal Form: Householder's Method. Frameworks ofNon-Linear Equations: Contraction Mapping Theorem. Newton's Method. Conventional Differential Equations: Introduction. Distinction Equations and Inequalities. One-Step Methods. Truncation Errors of One-Step Methods. Combination of One-Step Methods. Impact of Rounding Errors on One-Step Methods. Strategies Based on Numerical Integration; Explicit Methods. Techniques Based on Numerical Integration; Implicit Methods. Repeating with the Corrector. Milne's Method of Estimating Truncation Errors. Numerical Stability. Frameworks and Higher Order Equations. Examination of Step-by-Step Methods. Limit Value and Other Methods for Ordinary Differential Equations: Shooting Method for Boundary Value Problems. Limit Value Problem. Extrapolation to the Limit. Conceded Correction. Chebyshev Series Method. Informative supplements. Answers for Selected Problems. References. Subject Index. Area of mathematics and computer science that creates, analyzes, and implements algorithms for obtaining numerical solutions to problems involving continuous variables. Such problems arise throughout the natural sciences, social sciences, engineering, medicine, and business. Since the mid-20th century, the growth in power and availability of digital computers has led to an increasing use of realistic mathematical models in science and engineering, and numerical analysis of increasing sophistication is needed to solve these more detailed models of the world. The formal academic area of numerical analysis ranges from quite theoretical mathematical studies to computer science issues. With the increasing availability of computers, the new discipline of scientific computing, or computational science, emerged during the 1980s and 1990s. The discipline combines numerical analysis, symbolic mathematical computations, computer graphics, and other areas of computer science to make it easier to set up, solve, and interpret complicated mathematical models of the real world.
Numerical analysis continues this long tradition: rather than exact symbolic answers, which can only be applied to real-world measurements by translation into digits, it gives approximate solutions within specified error bounds.
Common Perspectives in Numerical Analysis

Numerical analysis is concerned with all aspects of the numerical solution of a problem, from the theoretical development and understanding of numerical methods to their practical implementation as reliable and efficient computer programs. Most numerical analysts specialize in small subfields, but they share some common concerns, perspectives, and mathematical methods of analysis. These include the following:

1. When presented with a problem that cannot be solved directly, they try to replace it with a "nearby problem" that can be solved more easily. Examples are the use of interpolation in developing numerical integration methods and root-finding methods.
2. There is widespread use of the language and results of linear algebra, real analysis, and functional analysis (with its simplifying notation of norms, vector spaces, and operators).
3. There is a fundamental concern with error, its size, and its analytic form. When approximating a problem, it is prudent to understand the nature of the error in the computed solution. Moreover, understanding the form of the error allows creation of extrapolation processes to improve the convergence behaviour of the numerical method.
4. Numerical analysts are concerned with stability, a concept referring to the sensitivity of the solution of a problem to small changes in the data or the parameters of the problem. Consider the following example.
5. The polynomial $p(x)=(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)$, or expanded, $p(x) = x^7 - 28x^6 + 322x^5 - 1{,}960x^4 - 6{,}769x^3 - 13{,}132x^2 + 13{,}068x - 5{,}040$has roots that are very sensitive to small changes in the coefficients. If the coefficient of x^6 is changed to -28.002, then the original roots 5 and 6 are perturbed to the complex numbers 5.459 0.540i—a very significant change in values. Such a polynomial $p(x)$ is called unstable or ill-conditioned with respect to the root-finding problem. Numerical methods for solving problems should be no more sensitive to changes in the data than the original problem to be solved. Moreover, the formulation of the original problem should be stable or well-conditioned.
6. Numerical analysts are very interested in the effects of using finite precision computer arithmetic. This is especially important in numerical linear algebra, as large problems contain many rounding errors.
7. Numerical analysts are generally interested in measuring the efficiency (or "cost") of an algorithm. For example, the use of Gaussian elimination to solve a linear system $Ax = b$ containing n equations will require approximately ${}^{2}n^{3}/_{3}$ arithmetic operations. Numerical analysts would want to know how this method compares with other methods for solving the problem.

Modern Applications And Computer Software

Numerical analysis and mathematical modeling are essential in many areas of modern life. Sophisticated numerical analysis software is commonly embedded in popular software packages (e.g., spreadsheet programs) and allows fairly detailed models to be evaluated, even when the user is unaware of the underlying mathematics. Attaining this level of user transparency requires reliable, efficient, and accurate numerical analysis software, and it requires problem-solving environments (PSE) in which it is relatively easy to model a given situation. PSEs are usually based on excellent theoretical mathematical models, made available to the user through a convenient graphical user interface.

Applications:

Computer-aided engineering (CAE), in industry, the integration of design and manufacturing into a system under the direct control of digital computers. CAE combines the use of computers in industrial-design work, computer-aided design (CAD), with their use in manufacturing operations, computer-aided manufacturing (CAM). This integrated process is commonly called CAD/CAM. CAD systems generally consist of a computer with one or more terminals featuring video monitors and interactive graphics-input devices; they can be used to design such things as machine parts, patterns for clothing, or integrated circuits. CAM Systems involve the use of numerically controlled machine tools and high-performance, programmable industrial robots. In a CAE system, drawings developed and revised during the design process are converted directly into instructions for the production machines that will manufacture the desired object. CAE systems reduce the time needed to develop new products and increase productivity by optimizing production flow and scheduling and by providing greater flexibility in altering machine operations.

Another important application is atmospheric modeling. In addition to improving weather forecasts, such models are crucial for understanding the possible effects of human activities on the Earth's climate. In order to create a useful model, many variables must be introduced. Fundamental among these are the velocity $V(x, y, z, t)$, pressure $P(x, y, z, t)$, and temperature $T(x, y, z, t)$, all given at position (x, y, z) and time t. In addition, various chemicals exist in the atmosphere, including ozone, certain chemical pollutants, carbon dioxide, and other gases and particulates, and their interactions have to be considered. The underlying equations for studying $V(x, y, z, t)$, $P(x, y, z, t)$, and $T(x, y, z, t)$ are partial differential equations; and the interactions of the various chemicals are described using some quite difficult ordinary differential equations. Many types of numerical analysis procedures are used in atmospheric modeling, including computational fluid mechanics and the numerical solution of differential equations. Researchers strive to include ever finer detail in atmospheric models, primarily by incorporating data over smaller and smaller local regions in the atmosphere and implementing their models on highly parallel supercomputers.

Modern businesses rely on optimization methods to decide how to allocate resources most efficiently. For example, optimization methods are used for inventory control, scheduling, determining the best location for manufacturing and storage facilities, and investment strategies.

Theory Of Numerical Analysis

The following is a rough categorization of the mathematical theory underlying numerical analysis, keeping in mind that there is often a great deal of overlap between the listed areas.

Numerical linear and nonlinear algebra

Many problems in applied mathematics involve solving systems of linear equations, with the linear system occurring naturally in some cases and as a part of the solution process in other cases. Linear systems are usually written using matrix-vector notation, $Ax = b$ with A the matrix of coefficients for the system, x the column vector of the unknown variables $x_1,\ldots, xn$, and b a

given column vector. Solving linear systems with up to 1,000 variables is now considered relatively straightforward in most cases. For small to moderately sized linear systems (say, $n \leq 1{,}000$), the favoured numerical method is Gaussian elimination and its variants; this is simply a precisely stated algorithmic variant of the method of elimination of variables that is introduced in elementary algebra. For larger linear systems, there is a variety of approaches depending on the structure of the coefficient matrix A. Direct methods lead to a theoretically exact solution x in a finite number of steps, with Gaussian elimination the best-known example. In practice, there are errors in the computed value of x due to rounding errors in the computation, arising from the finite length of numbers in standard computer arithmetic. Iterative methods are approximate methods that create a sequence of approximating solutions of increasing accuracy.
Nonlinear problems are often treated numerically by reducing them to a sequence of linear problems. As a simple but important example, consider the problem of solving a nonlinear equation $f(x) = 0$. Approximate the graph of $y = f(x)$ by the tangent line at a point $x^{(0)}$ near the desired root (use of parentheses is a common notational convention to distinguish successive iterations from exponentiation), and use the root of the tangent line to approximate the root of the original nonlinear function $f(x)$. This leads to Newton's iterative method for finding successively better approximations to the desired root: $x^{(k+1)} = x^{(k)} - f(x^{(k)})/f'(x^{(k)})$, $k = 0, 1, 2, \ldots$,

where $f'(x)$ indicates the first derivative of the original function.

This generalizes to handling systems of nonlinear equations. Let $f(x) = 0$ denote a system of n nonlinear equations in n unknowns $x = (x_1, \ldots, xn)$. Newton's method for solving this system is given by

$$x^{(k+1)} = x^{(k)} + \delta^{(k)}$$

$$f'(x^{(k)})\delta^{(k)} = -f(x^{(k)}), \; k = 0, 1, 2, \ldots$$

In this, $f'(x)$ is a generalization of the derivative known as the Jacobian matrix of $f(x)$, and the second equation is a linear system of order n. There are numerous other approaches to solving nonlinear systems, most based on using some type of approximation involving linear functions.

An important related class of problems occurs under the heading of optimization. Given a real-valued function $f(x)$ with x a vector of unknowns, a value of x that minimizes $f(x)$ is sought. In some cases x is allowed to vary freely, and in other cases there are constraints on x. Such problems occur frequently in business applications.
Approximation Theorey:
This category includes the approximation of functions with simpler or more tractable functions and methods based on using such approximations. When evaluating a function $f(x)$ with x a real or complex number, it must be kept in mind that a computer or calculator can only do a finite number of operations. Moreover, these operations are the basic arithmetic operations of addition, subtraction, multiplication, and division, together with comparison operations such as determining whether $x > y$ is true or false. With the four basic arithmetic operations, it is possible to evaluate polynomials

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + anxn$$

as well as rational functions (polynomials divided by polynomials). By including the comparison operations, it is possible to evaluate different polynomials or rational functions on different sets of real numbers x. The evaluation of all other functions—e.g., $f(x)$ = Square root of$\sqrt{x}$ or $2x$—must be reduced to the evaluation of a polynomial or rational function that approximates the given function with sufficient accuracy. All function evaluations on calculators and computers are accomplished in this manner.
One common method of approximation is known as interpolation. Consider a set of points (xi,yi) where $i = 0, 1, \ldots, n$, and then find a polynomial that satisfies $p(xi) = yi$ for all $i = 0, 1, \ldots, n$. The polynomial $p(x)$ is said to interpolate the given data points. Interpolation can be performed with functions other than polynomials (although these are most common), with important cases being rational functions, trigonometric polynomials, and spline functions (made by connecting several polynomial functions at their endpoints—they are commonly used in statistics and computer graphics).
Interpolation has a number of applications. If a function $f(x)$ is known only at a discrete set of data points $x_0, \ldots, xn$, with $yi = f(xi)$, then interpolation can be used to extend the definition to nearby points x. If n is at all large, spline functions are generally preferable to simple polynomials. Most numerical methods for the approximation of integrals and derivatives of a given function $f(x)$ are based on interpolation. For example, begin by constructing an interpolating function $p(x)$, often a polynomial, that approximates $f(x)$, and then integrate or differentiate $p(x)$ to approximate the corresponding integral or derivative of $f(x)$.

Solving differential and integral equations
Most mathematical models used in the natural sciences and engineering are based on ordinary differential equations, partial differential equations, and integral equations. Numerical methods for solving these equations are primarily of two types. The first type approximates the unknown function in the equation by a simpler function, often a polynomial or piecewise polynomial (spline) function, chosen to closely follow the original equation. The finite element method discussed above is the best known approach of this type. The second type of numerical method approximates the equation of interest, usually by approximating the derivatives or integrals in the equation. The approximating equation has a solution at a discrete set of points, and this solution approximates that of the original equation. Such numerical procedures are often called finite difference methods. Most initial value problems for ordinary differential equations and partial differential equations are solved in this way. Numerical methods for solving differential and integral equations often involve both approximation theory and the solution of quite large linear and nonlinear systems of equations.

What is the use of Numerical analysis:
Numerical analysis, area of mathematics and computer science that creates, analyzes, and implements algorithms for obtaining numerical solutions to problems involving continuous variables. Such problems arise throughout the natural sciences, social sciences, engineering, medicine, and business.

Importance of Numerical methods
In order to develop efficient means of calculating a numerical solution, it is important to understand the characteristics of the computer being used. For example, the structure of the computer memory is often very important in devising efficient algorithms for large linear algebra problems. In numerical analysis, a numerical method is a mathematical tool designed to solve numerical problems. The implementation of a numerical method with an appropriate convergence check in a programming language is called a numerical algorithm.

Applications of Numerical Analysis:
Numerical methods are algorithms used for computing numeric data. They are used to provide 'approximate' results for the problems being dealt with and their necessity is felt when it becomes impossible or extremely difficult to solve a given problem analytically.

Numerical analysis is needed to solve engineering problems that lead to equations that cannot be solved analytically with simple formulas. Examples are solutions of large systems of algebraic equations, evaluation of integrals, and solution of differential equations.

Conclusion:
Numerical Analysis and Applications exists for the discussion and dissemination of algorithms and computational methods in mathematics, mathematical physics, and other applied fields. The emphasis should be on mathematical models and new computational methods, or the application of existing methods in a novel way. Whereas some papers are relevant to particular problems of elasticity theory, hydrodynamics, fluid dynamics, and geophysics, others form the basis for further developments in the area of study.

References:
K. Atkinson: An Introduction to Numerical Analysis, Wiley, (2nd ed.), 1989.
P.G. Ciarlet and J. L. Lions (eds), Handbook of Numerical Analysis, North Holland, 1990.
E. W. Cheney and D. R. Kincaid: Numerical Mathematics and Computing, Brooks Cole, 6 edition, 2007.
S. Conte and C. de Boor: Elementary Numerical Analysis, McGraw-Hill, 1980.
G. Dahlquist and A. Bjorck: Numerical Methods, Prentice Hall, 1974.
P. Deuflhard and A. Hohmann, Numerical Analysis in Modern Scientific Computing, 2nd ed., Springer, 2003.
W. Gautschi: Numerical Analysis: an introduction, Birkhauser, 1997.
M. T. Heath, Scientific Computing: An Introductory Survey, 2nd ed., McGraw-Hill, 2002.
E. Isaacson and H. Keller: Analysis of Numerical Methods, Wiley, 1966 (or Dover 1994).
D. Kahaner, C. Moler, and S. Nash: Numerical Methods and Software, Prentice-Hall, 1989.
D. Kincaid and W. Cheney: Numerical Analysis: Mathematics of Scientific Computing, American Mathematical Society, 3rd edition, 2001.
A. Quarteroni, R. Sacco, and F. Saleri: Numerical Mathematics, 2nd Edition, Springer, 2004.
A. Ralston and P. Rabinowitz: A First Course in Numerical Analysis, McGraw-Hill, 1978.
L. Shampine, Allen, and Pruess: Fundamentals of Numerical Computing,
G. W. Stewart: Afternotes on Numerical Analysis, SIAM, 1996.
G. W. Stewart: Afternotes Goes to Graduate School: Lectures on Advanced Numerical Analysis, SIAM, 1998
J. Stoer and R. Bulirsch: Introduction to Numerical Analysis, Springer, 3rd edition, 2002.
E. Suli and D. Mayers: An Introduction to Numerical Analysis, Cambridge, 2003.