Cycle Related Vertex Odd and Even Divisor Cordial Labeling for some Special Graphs

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***Abstract*: A vertex divisor cordial labeling of a graph G = (V,E) is a bijection f : V → {1,2,3,...2n-1} if odd and f : V → {1,2,3,...2n} if even such that if each edge uv is assigned the label 1 if f(u)/ f(v) or f(v)/ f(u) and the label 0 if f(u) does not divide f(v) then $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. A graph which admits vertex even divisor cordial labeling is called even divisor cordial graph.**

In this paper we proved that splitting of Friendship graph $S'(F_n)$, splitting of helm graph $S'(H_n)$,web graph Wb_n and umbrella graph U(m,n) ,(m=n+1) admits odd and even divisor cordial labeling. The gear graph G_n, switching of an apex vertex in $S(k_{1,n})$, the graph $P_2 + mK_1$, 1-weak shell graph C(n,n − 3), 2-weak shell graph C(n,n − 4) admits even divisor cordial labeling.

***Index Terms*: labeling, cordial labeling, divisor cordial labeling, vertex odd divisor cordial labeling, vertex even divisor cordial labeling**

1. INTRODUCTION

Graph theory has several interesting applications in system analysis, operations research and economics. The concept of labeling of graphs is an active research area and it has been widely studied by several researchers. In a wide area network (WAN), several systems are connected to the main server, the labeling technique plays a vital role to label the cables. The labeling of graphs have been applied in the fields such as circuit design, communication network, coding theory, and crystallography.

A graph labeling, is a process in which each vertex is assigned a value from the given set of numbers, the labeling of edges depends on the labels of its end vertices. Cordial labeling was introduced by Cahit1. A graph is called cordial if it is possible to label its vertices with 0's and 1's so that when the edges are labeled with the difference of the labels at their endpoints, the number of vertices (edges) labeled with ones and zeros differ at most by one.

2. BASIC DEFINITIONS

Definition 2.1 Let G = (V,E) be a graph. A mapping f : V → {0,1} is called the binary vertex labeling of G and f(v) is called the label of the vertex v ∈ V of G under f. The induced edge labeling f * : E → {0,1} is given by f *(e) = |f(u) − f(v)|, for all e = uv ∈ E. [3,11]

We denote $v_f(i)$ is the number of vertices of G having label i under f and $e_f(i)$ is the number of edges of G having label i under f.(where i=0,1)

Definition 2.2 Let G = (V,E) be a graph and f : V → {0,1} is called the binary vertex labeling of G. The map f is called a cordial labeling if $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits cordial labeling is called cordial graph.[11]

Definition 2.3 A divisor cordial labeling of a graph G = (V,E) is a bijection f : V → {1,2,3,...|v|} such that if each edge uv is assigned the label 1 if f(u)/ f(v) or f(v) /f(u) and the label 0 if f(u) does not divide f(v) then $|e_f(0) - e_f(1)| \leq 1$.[11]

Definition 2.4 A vertex odd divisor cordial labeling of a graph G =(V,E) is a bijection f : V → {1,2,3,...(2n − 1)} such that if each edge uv is assigned the label 1 if f(u)/ f(v) or f(v)/ f(u) and the label 0 if f(u) does not divide f(v) then $|e_f(0)-e_f(1)| \leq 1$.A graph which admits odd divisor cordial labeling is called vertex odd divisor cordial graph.[11]

Definition 2.5 A vertex even divisor cordial labeling of a graph G =(V,E) is a bijection f : V → {1,2,3,...2n} such that if each edge uv is assigned the label 1 if f(u)/ f(v) or f(v)/ f(u) and the label 0 if f(u) does not divide f(v) then $|e_f(0)-e_f(1)| \leq 1$.A graph which admits even divisor cordial labeling is called vertex even divisor cordial graph.

Definition 2.6 The splitting graph S'(G) of a graph G is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

Definition 2.7 Friendship graph is the graph obtained by taking m copies of the cycle graph C_3 with a vertex in common. It is also known as Dutch-Windmill graph

Definition 2.8 The Helm graph is the graph obtained from a wheel graph by adjoining a pendant edge at each node of the cycle.

Definition 2.9 The Web graph Wb_n is a stacked prism graph $Y_{n+1,3}$ with the edges of the outer cycle removed.

Definition 2.10 An Umbrella graph U(m,n) is the graph obtained by joining a path P_n with the central vertex of a fan graph F_m. It is also known as Parapluie graph.[8,10]

Definition 2.11 The Gear graph G_n is obtained from the wheel by subdividing each of its rim edge.[11]

Definition 2.12 A Vertex Switching G_v of a graph is obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G. [11]

Definition 2.13 Suppose we remove k chords from a shell graph S_n , then the resulting graph is denoted by $C(n,n-2-k)$ and we call this graph as k –weak shell graph, where $1 \leq i < n$. If $\{v_1,v_2,...,v_n\}$ is vertex set of $C(n,n-2-k)$, then its edge set is $\{v_iv_{i+1};1 \leq i\ (n-1)\} \cup \{vnv1\} \cup \{v_iv_j;j = 3,4,...,n-1-k\}$. [11]

3. MAIN RESULTS

Theorem 3.1 The Splitting of Friendship graph $S'(F_n)$ is vertex odd divisor cordial.

Proof. Let G be the Splitting graph of friendship graph $S'(F_n)$.

Let $V(S'(F_n)) =\{v_1,v_{i+1},v'_i\ ;\ 1\leqslant i \leqslant n\}$, where v_1 is the common vertex of cycle nC_3 ,v_{i+1} are the outer vertices of the cycle, v'_i are the splitting vertices of F_n.

Then $|V(G)| = 4n+2$ and $|E(G)| = 9n$.

We define $f: V(G) \longrightarrow \{1,3,5,\ldots,8n+3\}$ as follows

$f(v_1) = 1$

$f(v_{i+1}) = 2i+1\ ,1\leqslant i\leqslant 2n$

$f(v'_i) = 2i+V_{2n+1}\ ,\ 1\leqslant i\leqslant 2n+1$

We observe that, from the above labeling pattern, we have $e_f(1) = e_f(0)$, when n is even and $e_f(1) = 1+e_f(0)$, when n is odd.

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex odd divisor cordial graph.

Example 3.2 Vertex odd divisor cordial labeling of splitting of friendship graph $S'(F_6)$ is shown in Fig 1.

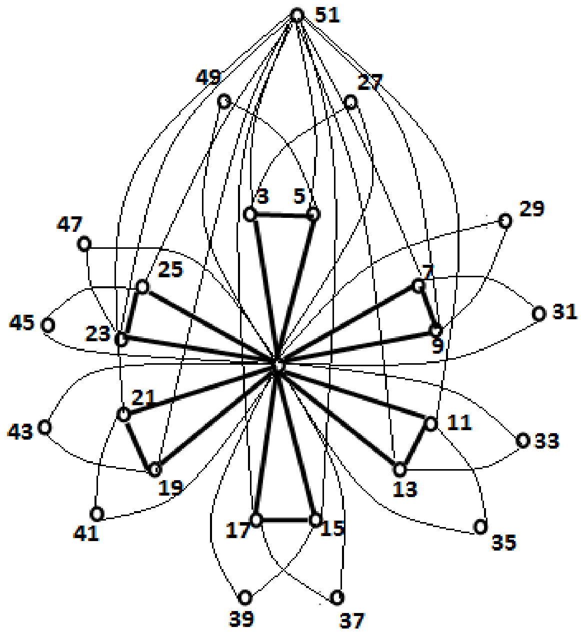

Figure :1 $S'(F_6)$

Theorem 3.3 The Splitting of Helm graph $S'(H_n)$ is vertex odd divisor cordial.

Proof. Let G be the Splitting graph of helm graph $S'(H_n)$.

Let $V(S'(H_n)) = \{v_1, v_i, v'_i \,;\, 1 \leqslant i \leqslant n\}$, where v_1 is the apex vertex ,v_i are the vertices of the helm H_n, v'_i are the splitting vertices of H_n.

Then $|V(G)| = 4n+2$ and $|E(G)| = 9n$.

We define $f: V(G) \longrightarrow \{1,3,5,\ldots,8n+3\}$ as follows

Label the vertex v_1 with label 1, $f(v_1) = 1$. Our aim is to generate $\left[\frac{9n}{2}\right]$ edges having label 1 and $\left[\frac{9n}{2}\right]$ edges having label 0. $f(v_1) =1$ generates n edges having label 1.Now it remains to generate $k=\left[\frac{9n}{2}\right]-n$ edges with label 1.For the vertices $v_2,v_3,\ldots,v_n$ and $v'_1,v'_2,\ldots,v'_n$ assign the vertex label till generating k edges with label 1.The remaining labels are assigned to the vertices in such a way that no two adjacent vertices are multiples of each other.

We observe that, from the above labeling pattern, we have $e_f(1) = \left[\frac{9n}{2}\right] = e_f(0)$, when n is even and $e_f(1) = 1+e_f(0)$, when n is odd.

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex odd divisor cordial graph.

Example 3.4 Vertex odd divisor cordial labeling of splitting of helm graph $S'(H_5)$ is shown in Fig 2.

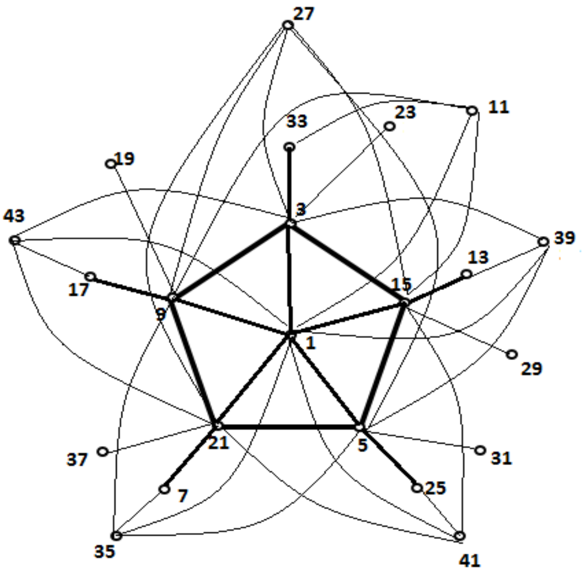

Figure :2 $S'(H_5)$

Theorem 3.5 Web graph Wb_n is vertex odd divisor cordial.

Proof. Let G be the web graph Wb_n. Let $V(Wb_n) = \{v_1, v_{i+1} ; 1 \leqslant i \leqslant n\}$, where v_1 is any one vertex having degree 4, v_{i+1} are the outer vertices of Wb_n.

Then $|V(G)| = 3n$ and $|E(G)| = 4n$.We define $f: V(G) \longrightarrow \{1,3,5,\dots,6n-1\}$ as follows

Label the vertex v_1 with label 1, $f(v_1) = 1$. Our aim is to generate 2n edges having label 1 and 2n edges having label 0. $f(v_1) = 1$ generates n edges having label 1.Now it remains to generate $k=2n-n=n$ edges with label 1.For the vertices $v_2, v_3, \dots, v_n$ assign the vertex label till generating k edges with label 1.The remaining labels are assigned to the vertices in such a way that no two adjacent vertices are multiples of each other.

We observe that, from the above labeling pattern, we have $e_f(1) = 2n = e_f(0)$, when n is even and $e_f(1) = 1 + e_f(0)$, when n is odd.

Therefore $|e_f(0) - e_f(1)| \leqslant 1$.

Hence G is a vertex odd divisor cordial graph.

Example 3.6 Vertex odd divisor cordial labeling of web graph Wb_{10} is shown in Fig 3.

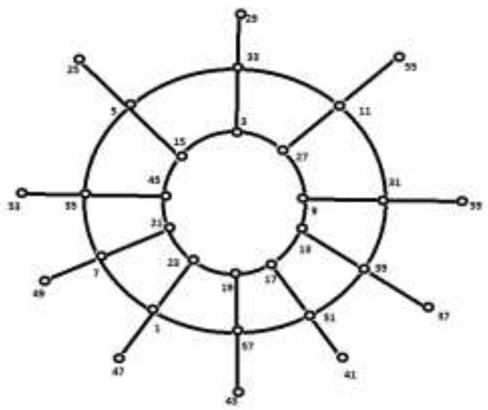

Figure :3 Wb_{10}

Theorem 3.7 Umbrella graph U(m,n) (when m=n+1) is a vertex odd divisor cordial.

Proof. Let G be the umbrella graph U(m,n). Let V(U(m,n)) ={v_1, v_{i+1}, v'_i ; $1 \leqslant i \leqslant n$ } , where v_1 is the central vertex of fan F_m ,v_{i+1} are the vertices of F_m , v'_i are the vertices of P_n.

Then |V(G)| = 2n+1 and |E(G)| = 3n.

We define f: V(G) ⟶ {1,3,5,…,4n+1} as follows

Case 1: n < 6

$f(v_1) = 1$

$f(v_{i+1}) = 2i+1$,$1 \leqslant i \leqslant n+1$

$f(v'_i) = 2i+V_{n+2}$, $1 \leqslant i \leqslant n-1$

Case 2: n ⩾ 6

$f(v_1) = 1$

Label the vertices of V_{i+1} such that there are equal number of edges having label 0 and label 1.

$f(v'_i) = 2i+V_{n+2}$, $1 \leqslant i \leqslant n-1$

We observe that, from the above labeling pattern, we have $e_f(1) = e_f(0)$, when n is even and $e_f(1) = 1+e_f(0)$, when n is odd.

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex odd divisor cordial graph.

Example 3.8 Vertex odd divisor cordial labeling of umbrella graph U(6,5) and U(8,7) is shown in Fig 4(a) and 4(b).

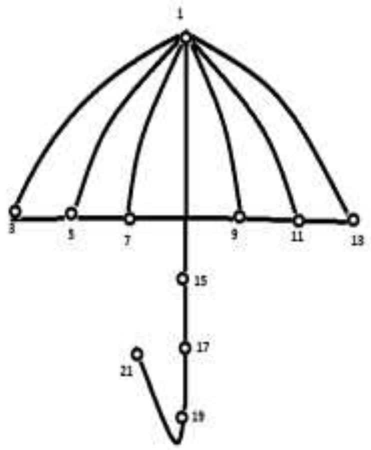

Figure :4(a) U(6,5)

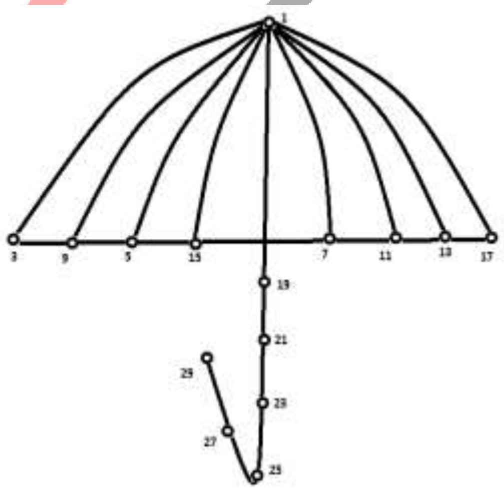

Figure :4(b) U(8,7)

Theorem 3.9 The Splitting of Friendship graph $S'(F_n)$ is vertex even divisor cordial.

Proof. Let G be the Splitting graph of friendship graph $S'(F_n)$. Let $V(S'(F_n))$ ={v_1, v_{i+1}, v'_i ; $1 \leqslant i \leqslant n$ } , where v_1 is the common vertex of cycle nC_3 ,v_{i+1} are the outer vertices of the cycle, v'_i are the splitting vertices of F_n.

Then $|V(G)| = 4n+2$ and $|E(G)| = 9n$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,8n+4\}$ as follows

$f(v_1) = 2$

$f(v_{i+1}) = 2i+2$,$1 \leqslant i \leqslant 2n$

$f(v'_i) = 2i+V_{2n+1}$, $1 \leqslant i \leqslant 2n+1$

We observe that, from the above labeling pattern, we have $e_f(1) = e_f(0)$, when n is even and $e_f(1) = 1+e_f(0)$, when n is odd.

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.10 Vertex even divisor cordial labeling of splitting of friendship graph $S'(F_4)$ is shown in Fig 5.

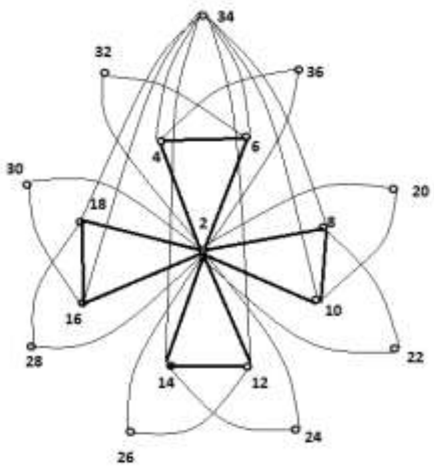

Figure :5 $S'(F_4)$

Theorem 3.11 Splitting of Helm graph $S'(H_n)$ is vertex even divisor cordial.

Proof. Let G be the Splitting graph of helm graph $S'(H_n)$. Let $V(S'(H_n)) = \{v_1, v_i, v'_i \; ; 1 \leqslant i \leqslant n\}$, where v_1 is the apex vertex ,v_i are the vertices of the helm H_n, v'_i are the splitting vertices of H_n.

Then $|V(G)| = 4n+2$ and $|E(G)| = 9n$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,8n+4\}$ as follows

Label the vertex v_1 with label 2, $f(v_1) = 2$. Our aim is to generate $\left[\frac{9n}{2}\right]$ edges having label 1 and $\left[\frac{9n}{2}\right]$ edges having label 0. $f(v) = 2$ generates n edges having label 1.Now it remains to generate $k = \left[\frac{9n}{2}\right] - n$ edges with label 1.For the vertices $v_2, v_3, \ldots, v_n$ and $v'_1, v'_2, \ldots, v'_n$ assign the vertex label till generating k edges with label 1.The remaining labels are assigned to the vertices in such a way that no two adjacent vertices are multiples of each other.

We observe that, from the above labeling pattern, we have $e_f(1) = \left[\frac{9n}{2}\right] = e_f(0)$, when n is even and $e_f(1) = 1+e_f(0)$, when n is odd.

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.12 Vertex even divisor cordial labeling of splitting of helm graph $S'(H_5)$ is shown in Fig 6.

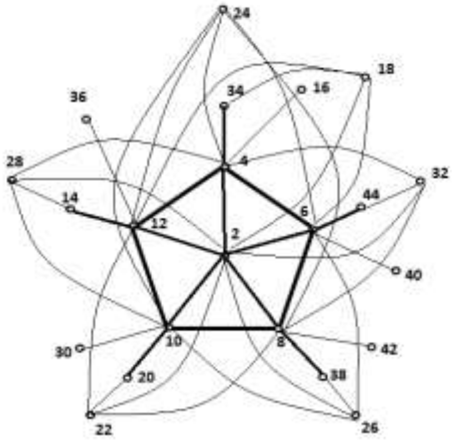

Figure :6 $S'(H_5)$

Theorem 3.13 Web graph Wb_n is vertex even divisor cordial.

Let G be the web graph Wb_n. Let $V(Wb_n) = \{v_1, v_{i+1} \; ; 1 \leqslant i \leqslant n\}$, where v_1 is any one vertex having degree 4, v_{i+1} are the outer vertices of Wb_n.

Then $|V(G)| = 3n$ and $|E(G)| = 4n$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,6n\}$ as follows

Label the vertex v_1 with label 2, $f(v_1) = 2$. Our aim is to generate 2n edges having label 1 and 2n edges having label 0. $f(v_1) = 2$ generates n edges having label 1.Now it remains to generate k=2n–n= n edges with label 1.For the vertices $v_2, v_3, \ldots, v_n$ assign the vertex label till generating k edges with label 1.The remaining labels are assigned to the vertices in such a way that no two adjacent vertices are multiples of each other.

We observe that, from the above labeling pattern, we have $e_f(1) = 2n = e_f(0)$, when n is even and $e_f(1) = 1 + e_f(0)$, when n is odd.

Therefore $|e_f(0) - e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.14 Vertex even divisor cordial labeling of web graph Wb_6 is shown in Fig 7.

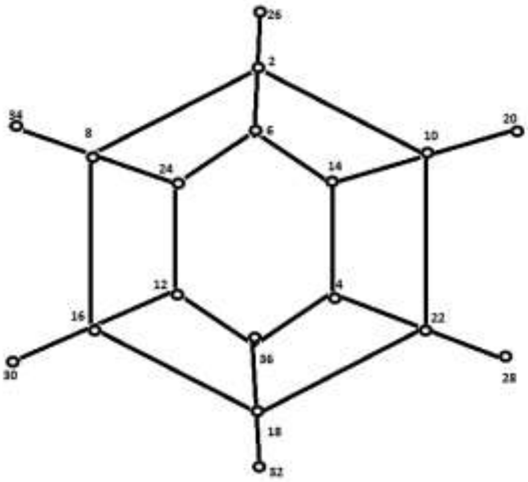

Figure :7 Wb_6

Theorem 3.15 Umbrella graph U(m,n) (when m=n+1) is vertex even divisor cordial

Proof. Let G be the umbrella graph U(m,n). Let $V(U(m,n)) = \{v_1, v_{i+1}, v'_i \,;\, 1 \leqslant i \leqslant n\}$, where v_1 is the central vertex of fan F_m ,v_{i+1} are the vertices of F_m , v'_i are the vertices of P_n.

Then $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,4n+2\}$ as follows

Case 1: $n < 6$

$f(v_1) = 2$

$f(v_{i+1}) = 2i+2$,$1 \leqslant i \leqslant n+1$

$f(v'_i) = 2i+V_{n+2}$, $1 \leqslant i \leqslant n-1$

Case 2: $n \geqslant 6$

$f(v_1) = 2$

Label the vertices of V_{i+1} such that there are equal number of edges having label 0 and label 1.

$f(v'_i) = 2i+V_{n+2}$, $1 \leqslant i \leqslant n-1$

We observe that, from the above labeling pattern, we have $e_f(1) = e_f(0)$, when n is even and

$e_f(1) = 1+e_f(0)$, when n is odd. Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.16 Vertex even divisor cordial labeling of umbrella graph U(6,5) and U(9,8) is shown in Fig 8(a) and 8(b).

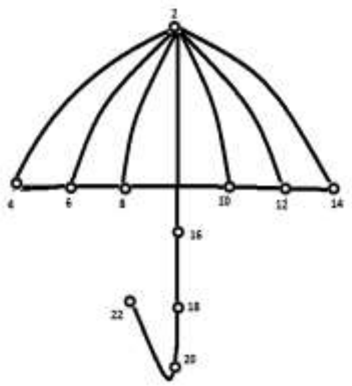

Figure : 8(a) U(6,5)

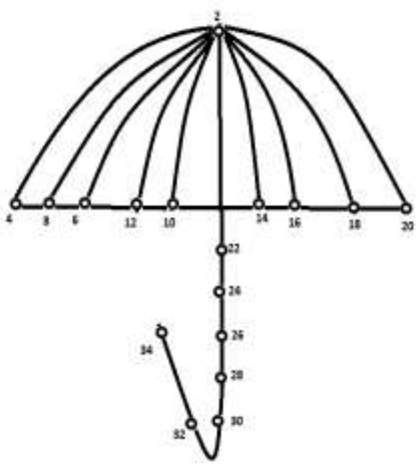

Figure : 8(a) U(9,8)

Theorem 3.17 Gear graph G_n is vertex even divisor cordial.

Proof. Let G be the gear graph G_n. Let v be the apex vertex of G and let $v_1,v_2,v_3,\ldots,v_{2n}$ be the rim vertices of G.

Then $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,4n+2\}$ as follows

Label the vertex v_1 with label 2, $f(v_1) = 2$. Our aim is to generate $\left[\frac{3n}{2}\right]$ edges having label 1 and $\left[\frac{3n}{2}\right]$ edges having label 0. $f(v) = 2$ generates n edges having label 1.Now it remains to generate $k=\left[\frac{3n}{2}\right]-n$ edges with label 1.For the vertices $v_1,v_2,v_3,\ldots,v_n$ assign the vertex label till generating k edges with label 1.The remaining labels are assigned to the vertices in such a way that no two adjacent vertices are multiples of each other. We observe that, from the above labeling pattern, we have $e_f(1) = \left[\frac{3n}{2}\right] = e_f(0)$, when n is even and $e_f(1) = 1+e_f(0)$, when n is odd.

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.18 Vertex even divisor cordial labeling of gear graph G_8 is shown in Fig 9.

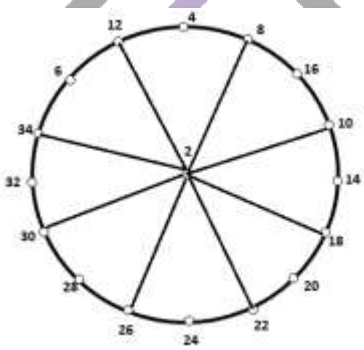

Figure :9 G_8

Theorem 3.19 The Switching of an apex vertex in $S(k_{1,n})$ is vertex even divisor cordial.

Proof. Let G be the Switching of an apex vertex in $S(k_{1,n})$. Let $V(S(k_{1,n})) = \{v,v_i,v'_i \,;\, 1 \leqslant i \leqslant n\}$, where v is the apex vertex ,v_i are the pendant vertices of $S(k_{1,n})$, v'_i are the vertices of degree 2.

Then $|V(G)| = 2n+1$ and $|E(G)| = 2n$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,4n+2\}$ as follows

$f(v) = 2$

$f(v_i) = 4i$,$1 \leqslant i \leqslant n$

$f(v'_i) = 4i+2$, $1 \leqslant i \leqslant n$

We observe that, from the above labeling pattern, we have $e_f(1) = n = e_f(0)$.Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.20 Vertex even divisor cordial of switching of an apex vertex in $S(k_{1,7})$ is shown in Fig 10.

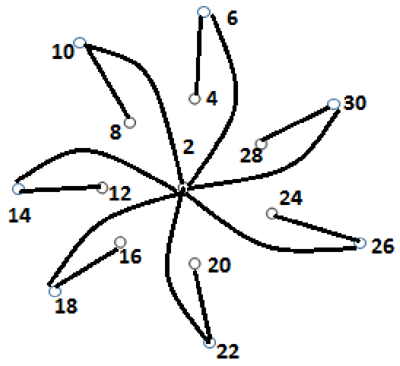

Figure:10 Switching of an apex vertex in $S(k_{1,7})$

Theorem 3.21 The graph P_2+mK_1 is vertex even divisor cordial.

Proof. Let G be the graph P_2+mK_1. Let u and v be vertices of P_2 and let $v_1,v_2,v_3,\ldots,v_m$ be vertices of mK_1 respectively

Then $|V(G)| = m+2$ and $|E(G)| = 2m+1$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,2m+4\}$ as follows

$f(u) = 2$

$f(v_i) = 2(i+1)$,$1 \leqslant i \leqslant m$

$f(v) = v_m+2$

We observe that, from the above labeling pattern, we have $e_f(0) = n$ and $e_f(1) = n+1$

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.22 Vertex even divisor cordial of the graph P_2+7K_1 is shown in Fig 11.

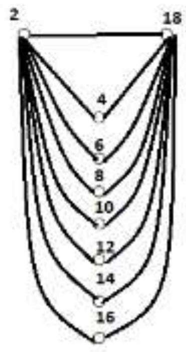

Figure :11 P_2+7K_1

Theorem 3.23 1-Weak shell graph C(n,n-3) is vertex even divisor cordial.

Proof. Let G be the graph C(n,n-3).

Then $|V(G)| = n$ and $|E(G)| = 2n-4$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,2n\}$ as follows

$f(v_1) = 2$

$f(v_{i+1}) = 2(i+1)$,$1 \leqslant i \leqslant n-1$

We observe that, from the above labeling pattern, we have $e_f(0) = n = e_f(1)$

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.24 Vertex even divisor cordial of 1-weak shell graph C(10,7) is shown in Fig 12.

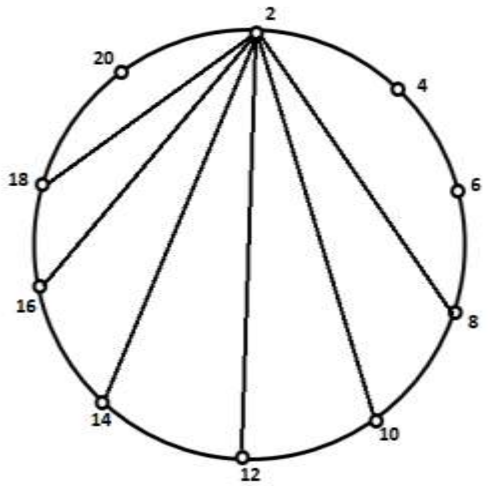

Figure :12 1-Weak shell graph

Theorem 3.25 2-Weak shell graph C(n,n-4) is vertex even divisor cordial

Proof Let G be the graph C(n,n-4).

Then $|V(G)| = n$ and $|E(G)| = 2n-5$.

We define $f: V(G) \longrightarrow \{2,4,6,\ldots,2n\}$ as follows

$f(v_1) = 2$

$f(v_{i+1}) = 2(i+1)$,$1 \leqslant i \leqslant n-1$

We observe that, from the above labeling pattern, we have $e_f(0) = n-2$ and $e_f(1) = n-3$

Therefore $|e_f(0)-e_f(1)| \leqslant 1$.

Hence G is a vertex even divisor cordial graph.

Example 3.26 Vertex even divisor cordial of 2-weak shell graph C(10,6) is shown in Fig 13.

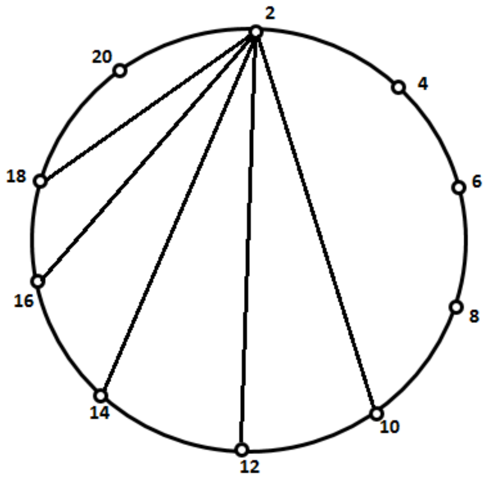

Figure :13 2-weak shell graph C(10,6).

4. CONCLUSION

In this paper, we have investigated certain cycle-related graphs satisfying odd and even divisor cordial labeling. Every graph do not admit vertex odd and even divisor cordial labeling and not all odd cordial graphs are even cordial.

Thus it is interesting to investigate graphs which satisfies the condition for odd and even divisor cordial labeling and also for both. Further the result can be extended to other graphs and graph families.

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