# SKEWNESS AND KURTOSIS ON VERTEX COLOURING OF CERTAIN GRAPHS

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***Abstract*: In this paper we found minimum vertex colouring sum based on a minimum proper colouring of a given graph G and we compute the statistical measures Mean, Variance, Median, Standard deviation, Skewness and Kurtosis.**

***Keywords*: Graph Colouring; colouring sum of graphs; colouring mean; colouring variance; colouring median; colouring standard deviation; colouring skewness; colouring kurtosis; $\chi$-chromatic.**

## INTRODUCTION

The vertex colouring or simply a colouring of a graph is an assignment of colours or labels to the vertices of a graph subject to certain conditions. In a proper colouring of a graph, its vertices are coloured in such a way that no two adjacent vertices in that graph have the same color.
We extend the concepts of mean, median, variance, standard deviation, skewness and kurtosis are important statistical measures, to the theory of graph colouring and determine the values of these parameters for a number of standard graphs.

## PRELIMINARIES:

- Let C = {$c_1$, $c_2$,...., $c_k$} be a particular type of proper k-colouring of a given graph $G$ and $\theta(c_i)$ denotes the number of times a particular color $c_i$ is assigned to the vertex of $G$. Then, the vertex colouring sum of a colouring C of a given graph $G$ denoted by $\omega_C(G)$ is defined to be [9,10,11,12,13,14,15,16,17],

$$\omega_C(G) = \sum_{i=1}^{k} i\theta(c_i)$$

- A graph in which one edge connecting every two consecutive vertices and there are no other edges, is called a path [4][6].
- A simple graph in which every vertex must be connected to all other vertices of the graph, is called as a complete graph [4][6].
- A simple graph with n vertices ($n \geq 3$) and n edges is called a cycle graph if the degree of each vertex in the graph is two and it is denoted by $C_n$ [4][6].
- A wheel graph is obtained from a cycle graph $C_n - 1$ by adding a new vertex adjacent to all the vertices of the cycle [4][6].

### Colouring Statistical Parameters of Graphs

We can identify the colouring of the vertices of a given graph $G$ with a random experiment. Let C = {$c_1$, $c_2$, $c_3$,..., $c_k$} be a proper k-colouring of $G$ and let X be the random variable (r.v) which denotes the number of vertices in $G$ having a particular color. Since the sum of all weights of colors of $G$ is the order of $G$, the real valued function f(i) is defined by, $f(i) = \left\{ \begin{array}{l} \dfrac{\theta(c_i)}{|V(G)|}; i = 1,2,....k \\ 0 \end{array} \right\}$ is the probability mass function (p.m.f) of the r.v X. If the context is clear, we can also say that f(i) is the p.m.f of the graph $G$ with respect to the given colouring C. Hence, analogous to the definitions of the mean, median, variance, standard deviation, skewness and kurtosis of random variables, those statistical parameters of a graph $G$, with respect to a general colouring of $G$ can be defined as follows.

**Definition .1.** Let C = {$c_1$, $c_2$, $c_3$,..., $c_k$} be a certain type of proper k-colouring of a given graph $G$ and $\theta(c_i)$ denotes the number of times a particular color $c_i$ is assigned to vertices of $G$. Then, the colouring mean of a colouring C of a given graph G, denoted by $\mu_C(G)$ is given by, $\mu_C(G) = \dfrac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}$

**Definition .2.** The colouring median of a colouring C of a given graph $G$, denoted by $M_C(G)$ and is defined to be

$$M_C(G) = \frac{\sum_{i=1}^{k} \theta(c_i)}{2}$$

**Definition .3.** The colouring variance of a colouring C of a given graph $G$, denoted by $\sigma^2{}_C(G)$ is given by,

$$\sigma^2{}_C(G) = \frac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} - \left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}\right]^2$$

**Definition .4.** The colouring standard deviation of a colouring C of a given graph $G$, denoted by by $\sigma_C(G)$ is given by,

$$\sigma_C(G) = \sqrt{\frac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} - \left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}\right]^2}$$

**Definition.5.** The colouring skewness of a colouring C of a given graph $G$, denoted by $\gamma_C(G)$ and is defined to be $\gamma_C(G)$ =

$$3\left(\frac{Mean - Median}{Standard\ \ Deviation}\right)$$

**Definition 6**. For a positive integer r, the $r^{th}$ moment of the colouring C is denoted by $\mu_{C^r}(G)$ is given by,

$$\mu_{C^r}(G) = \frac{\sum_{i=1}^{k} i^r\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}$$

We have various moments as follows:

**$1^{st}$ Moment:**

$$\mu_{C^1}(G) = \frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}$$

**$2^{nd}$ Moment:**

$$\mu_{C^2}(G) = \frac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} - \left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}\right]^2$$

**$3^{rd}$ Moment:**

$$\mu_{C^3}(G) = \frac{\sum_{i=1}^{k} i^3\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)} - 3\left[\frac{\sum_{i=1}^{k} i^2\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}\right]\left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}\right] + 2\left[\frac{\sum_{i=1}^{k} i^3\theta(c_i)}{\sum_{i=1}^{k} \theta(c_i)}\right]^3$$

**4th Moment:**

$$\mu_{C^4}(G)=\frac{\sum_{i=1}^{k} i^4\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}-4\left[\frac{\sum_{i=1}^{k} i^3\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]\left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]+6\left[\frac{\sum_{i=1}^{k} i^3\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]\left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]^2-3\left[\frac{\sum_{i=1}^{k} i\theta(c_i)}{\sum_{i=1}^{k}\theta(c_i)}\right]^4$$

**Definition .7.** The colouring kurtosis of a colouring C of a given graph $G$, denoted by $\beta_2(G)$ and defined by,

$$\beta_2(G)=\frac{\mu_{C^4}(G)}{(\mu_{C^2}(G))^2}$$

**χ - Chromatic Statistical Parameters of Graphs:**

Colouring mean, median, variance, standard deviation, skewness and kurtosis corresponding to a particular type of minimal proper colouring of the vertices of $G$ are defined as follows:

**Definition .8.** A colouring mean of a graph $G$, with respect to a proper vertex colouring C is said to be a χ - chromatic mean of $G$, if C is the minimum proper colouring of $G$ and the colouring sum $\omega_C(G)$ is also minimum. The χ -chromatic mean of a graph $G$ is denoted by $\mu_\chi(G)$.

**Definition .9.** A colouring median of a graph $G$, with respect to a proper vertex colouring C is said to be a χ - chromatic median of $G$. The χ -chromatic median of a graph $G$ is denoted by $M_\chi(G)$.

**Definition.10.** The χ chromatic variance of $G$, denoted by $\sigma_\chi^{\,2}(G)$, is a colouring variance of $G$ with respect to a minimal proper vertex colouring of $G$ which yields the minimum colouring sum.

**Definition .11.** The χ chromatic standard deviation of $G$, denoted by $\sigma_\chi(G)$, is a colouring standard deviation of $G$ with respect to a minimal proper vertex colouring of $G$ which yields the minimum colouring sum.

**Definition.12.** The χ chromatic skewness of $G$, denoted by $\gamma_\chi(G)$, is a colouring variance of $G$ with respect to a minimal proper vertex colouring of $G$ which yields the minimum colouring sum.

**Definition .13.** The χ chromatic kurtosis of $G$, denoted by $\beta_{2\chi}(G)$, is a colouring kurtosis of $G$ with respect to a minimal proper vertex colouring of $G$ which yields the minimum colouring sum. If $\beta_2 = 3$, then it is known as **MESOKURTIC** Curve, if $\beta_2 < 3$, then it is known as **PLATYKURTIC** Curve and if $\beta_2 > 3$, then it is known as **LEPTOKURTIC** Curve

Let us now determine the χ - chromatic mean, median, variance, standard deviation, skewness and kurtosis of certain standard graph classes. The following result discusses on Complete graph $K_n$.

**Proposition1.** The χ -Chromatic mean of a Complete graph $K_n$ is $\frac{n+1}{2}$, χ -chromatic variance is $\frac{n^2-1}{12}$, χ-chromatic median is $\frac{n}{2}$, χ- chromatic standard deviation, $\sqrt{\frac{n^2-1}{12}}$, χ- chromatic skewness $3\left(\sqrt{\frac{3}{n^2-1}}\right)$ and χ- chromatic kurtosis $\frac{9n^4-10n^2+7}{5n^4-10n^2-5}$.

**Proof.** Consider a Complete graph $K_n$ has different colours as they are adjacent to each other. (i.e) $\theta(c_i)=1$ for the color $c_i$; $1\le i\le n$

Therefore $\mu_\chi(K_n)=\frac{n+1}{2}$ $\sigma^2{}_\chi(K_n)=\frac{n^2-1}{12}$ $M_\chi(K_n)=\frac{n}{2}$, $\sigma_\chi(K_n)=\sqrt{\frac{n^2-1}{12}}$, $\gamma_\chi(K_n)=3\left(\sqrt{\frac{3}{n^2-1}}\right)$

We can obtain kurtosis by various moments as,

$$\mu'_1=\frac{n+1}{2},\ \mu'_2=\frac{(n+1)(2n+1)}{6}\ \mu'_3=\frac{n(n+1)^2}{4},\ \mu'_4=\frac{(n+1)(2n+1)(3n^2+3n+1)}{30}$$

$\mu_3 = 0 \quad \mu_4 = \dfrac{3n^4 - 10n^2 + 7}{240}$ And the $\chi$ - chromatic kurtosis of $\beta_{2\chi}(K_n) = \dfrac{9n^4 - 10n^2 + 7}{5n^4 - 10n^2 - 5}$

If $\beta_{2\chi}(K_n) < 3$, then it is known as PLATYKURTIC Curve.

**THEOREM 1:**

Any Proper colouring of a complete graph $K_n$ has the discrete uniform distribution on {1, 2, .., k} (D(U(k)) that is discrete uniform distribution.

**Proof:** Let X be the r.v representing the number of colors in a proper k-colouring of a complete graph $K_n$ . For any proper k-colouring C of the complete graph $K_n$, $\theta(c_i) = 1$ and $k = n$. Hence, the corresponding $p.m.f$ is $f(i) = \begin{cases} \frac{1}{n}; n = 1,2,....n \\ 0 \ ; elsewhere \end{cases}$ which is that of the discrete uniform distribution on {1, 2… k}. Hence, X follows DU (k)

**Proposition 2:** The $\chi$ – chromatic mean of a path $P_n$ is

$$\mu_\chi(P_n) = \begin{cases} \frac{3}{2}; if\ n\ is\ even \\ \frac{3n-1}{2n}; if\ n\ is\ odd \end{cases}$$

the $\chi$ – chromatic variance of a path $P_n$ is

$$\sigma^2{}_\chi(P_n) = \begin{cases} \frac{1}{4}; if\ n\ \ is\ even \\ \frac{n^2-1}{4n^2}; if\ n\ is\ odd \end{cases}$$

the $\chi$ – chromatic median of a path $P_n$ is $\frac{n}{2}$,

the $\chi$ – chromatic standard deviation of path $P_n$ is

$$\sigma_\chi(P_n) = \begin{cases} \frac{1}{2}; if\ n\ is\ even \\ \sqrt{\frac{n^2-1}{4n^2}}; if\ n\ is\ odd \end{cases}$$

the $\chi$ –chromatic skewness of $P_n$ is

$$\gamma_\chi(P_n) = \begin{cases} 3(3-n); if\ n\ is\ even \\ 3(\frac{-n^2+3n+1}{\sqrt{n^2+-1}}); if\ n\ is\ odd \end{cases}$$

the $\chi$ – chromatic kurtosis of a path $P_n$ is,

$$\beta_{2\chi}(P_n) = \begin{cases} 1; if\ n\ is\ even \\ \frac{n^4+2n^2-3}{n^4\_2n^2+1}; if\ n\ is\ odd \end{cases}$$

**Proof:**. Consider a path $P_n$ on n vertices. Then, we have the following cases.

(1) If n is even and is 2-colourable then $P_n$ has exactly $\frac{n}{2}$ vertices having colour $c_1$ and $c_2$ each. Then,

the $\chi$ -chromatic mean of $P_n$ is $\mu_\chi(P_n)=\frac{3}{2}$, the $\chi$ chromatic variance of $P_n$ is $\sigma^2{}_\chi(P_n)=\frac{1}{4}$, the $\chi$ chromatic median of $P_n$ is $\frac{n}{2}$ ,the $\chi$ -chromatic standard deviation of $P_n$ is $\sigma_\chi(P_n)=\frac{1}{2}$, the $\chi$ chromatic skewness of $P_n$ is $\gamma_\chi(P_n)=3(3-n)$ we can obtain kurtosis by various moments $\mu'_1=\frac{3}{2},\mu'_2=\frac{5}{2},\mu'_3=\frac{9}{2},\mu'_4=\frac{17}{2},\mu_3=0,\mu_4=\frac{1}{16}$, And the $\chi$ - chromatic kurtosis of $\beta_{2\chi}(P_n)=1$ If $\beta_{2\chi}(P_n)<3$ then it is known as PLATYKURTIC Curve.

(2) If n is odd, Then, the p:m:f of the corresponding r:v:X is given by , $f(i)=\begin{cases}\frac{n+1}{2n};i=1\\ \frac{n-1}{2n};i=2\\ 0;otherwise\end{cases}$ the $\chi$ - chromatic mean of $P_n$ is $\mu_\chi(P_n)=\frac{3n-1}{2n}$, the $\chi$ -chromatic variance of $P_n$ is $\sigma^2{}_\chi(P_n)=\frac{n^2-1}{4n^2}$, the $\chi$ -chromatic median of $P_n$ is $\frac{n}{2}$, the $\chi$ -chromatic standard deviation of $P_n$ is $\sigma_\chi(P_n)=\sqrt{\frac{n^2-1}{4n^2}}$, the $\chi$ - chromatic skewness of $P_n$ is $\gamma_\chi(P_n)=3(\frac{-n^2+3n-1}{\sqrt{n^2-1}})$, we can obtain kurtosis by various moments,

$\mu'_1=\frac{3n-1}{2n},\mu'_2=\frac{5n-3}{2n},\mu'_3=\frac{9n-7}{2n},\mu'_4=\frac{17n-15}{2n},\ \mu_3=\frac{n^2-1}{4n^3},\mu_4=\frac{n^4+2n^2-3}{16n^4}$, And the $\chi$ - chromatic kurtosis of $\beta_{2\chi}(P_n)=\frac{n^4+2n^2-3}{n^4-2n^2+1}$ If $\beta_{2\chi}(P_n)<3$, then it is known as PLATYKURTIC Curve.

**Proposition 3:** The $\chi$ – chromatic mean of a cycle $C_n$ is

$$\mu_\chi(C_n)=\begin{cases}\frac{3}{2};if\ n\ is\ even\\ \frac{3n+3}{2n};if\ n\ is\ odd\end{cases}$$

And the $\chi$ – chromatic variance of a cycle $C_n$ is

$$\sigma^2{}_\chi(C_n)=\begin{cases}\frac{1}{4};if\ n\ is\ even\\ \frac{n^2+8n-9}{4n^2};if\ n\ is\ odd\end{cases}$$

the $\chi$ – chromatic median of a cycle $C_n$ is $\frac{n}{2}$,

the $\chi$ – chromatic standard deviation of a cycle $C_n$ is,

$$\sigma_\chi(C_n)=\begin{cases}\frac{1}{4};if\ n\ is\ even\\ \sqrt{\frac{n^2+8n-9}{4n^2}};if\ n\ is\ odd\end{cases}$$

The $\chi$ – chromatic skewness of a cycle $C_n$ is

$$\gamma_{\chi}(C_n) = \begin{cases} 3(3-n); if\ n\ is\ even \\ 3(\frac{-n^2+3n+3}{\sqrt{n^2+8n-9}}); if\ n\ is\ odd \end{cases}$$

The $\chi$ – chromatic kurtosis of $C_n$ is,

$$\beta_{2\chi}(C_n) = \begin{cases} 1; if\ n\ is\ even \\ \frac{n^4+80n^3-270n^2+432n-243}{n^4+16n^3+46n^2-144n+81}; if\ n\ is\ odd \end{cases}$$

**Proof:**. Consider a cycle $C_n$ on n vertices. Then, we have the following cases.

(1) If n is even and is 2-colourable then $C_n$ has exactly $\frac{n}{2}$ vertices having colour $C_1$ and $C_2$ each. Then,

(i) the $\chi$ - chromatic mean of cycle $C_n$ is, $\mu_{\chi}(C_n) = \frac{3}{2}$

(ii) the $\chi$ - chromatic variance of cycle $C_n$ is, $\sigma^2{}_{\chi}(C_n) = \frac{1}{4}$

(iii) the $\chi$ - chromatic median of cycle $C_n$ is $\frac{n}{2}$

(iv) the $\chi$ - chromatic standard deviation of $C_n$ is , $\sigma_{\chi}(C_n) = \frac{1}{2}$

(v) the $\chi$ - chromatic skewness of $C_n$ is $\gamma_{\chi}(C_n) = 3(3-n)$

we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3}{2}, \mu'_2 = \frac{5}{2}, \mu'_3 = \frac{9}{2}, \mu'_4 = \frac{17}{2}, \quad \mu_3 = 0, \mu_4 = \frac{1}{16},$$

And the $\chi$ - chromatic kurtosis of $C_n$ is $\beta_{2\chi}(C_n) = 1$

If $\beta_{2x}(C_n) < 3$ then it is known as PLATYKURTIC Curve.

(2) If n is odd, then is $C_n$ 3-colourable.Let C={ $c_1, c_2, c_3$ }be the minimal proper colouring of $C_n$. Then, the p.m.f of the corresponding (r.v.X) is given by $f(i) = \begin{cases} \frac{n-1}{2n}; i = 1,2 \\ \frac{1}{n}; i = 3 \\ 0; elsewere \end{cases}$ then,

(i) the $\chi$ - chromatic mean of cycle $C_n$ is $\mu_{\chi}(C_n) = \frac{3n+3}{2n}$

(ii) the $\chi$ - chromatic variance of cycle $C_n$ is, $\sigma^2{}_{\chi}(C_n) = \frac{n^2+8n-9}{4n^2}$

(iii)the $\chi$ - chromatic median of cycle $C_n$ is $\frac{n}{2}$

(iv) the $\chi$ - chromatic standard deviation of cycle $C_n$ is, $\sigma_{\chi}(C_n) = \sqrt{\frac{n^2+8n-9}{4n^2}}$

(v) the $\chi$ - chromatic skewness of cycle $C_n$ is $\gamma_\chi(C_n) = 3(\frac{-n^2+3n+3}{\sqrt{n^2+8n-9}})$

(vi) we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3n+3}{2n}, \mu'_2 = \frac{5n+13}{2n}, \mu'_3 = \frac{9n+45}{2n}, \mu'_4 = \frac{17n+145}{2n},$$

$$\mu_3 = \frac{9n^2-36n+27}{4n^3}, \mu_4 = \frac{n^4+80n^3-270n^2+432n-243}{16n^4},$$

And the $\chi$ - chromatic kurtosis of $\beta_{2\chi}(C_n) = \frac{n^4+80n^3-270n^2+432n-243}{n^4+16n^3+46n^2-144n+81}$,

If $\beta_{2\chi}(C_n) < 3$ then it is known as PLATYKURTIC Curve.

**Proposition 4:** The $\chi$ - chromatic mean of a wheel graph $(W_n)$ is

$$\mu_\chi(W_n) = \begin{cases} \frac{3n+8}{2n}; if\ n\ is\ even \\ \frac{3n+3}{2n}; if\ n\ is\ odd \end{cases}$$

And the $\chi$ – chromatic variance of a wheel graph $(W_n)$ is

$$\sigma^2{}_\chi(W_n) = \begin{cases} \frac{n^2+32n-64}{4n^2}; if\ n\ is\ even \\ \frac{n^2+8n-9}{4n^2}; if\ n\ is\ odd \end{cases}$$

the $\chi$ – chromatic median of a Wheel Graph $(W_n)$ is $\frac{n}{2}$

the $\chi$ – chromatic standard deviation of a Wheel graph $(W_n)$ is

$$\sigma_\chi(W_n) = \begin{cases} \sqrt{\frac{n^2+32n-64}{4n^2}}; if\ n\ is\ even \\ \sqrt{\frac{n^2+8n-9}{4n^2}}; if\ n\ is\ odd \end{cases}$$

the $\chi$ – chromatic skewness of a wheel graph $(W_n)$ is

$$\gamma_\chi(W_n) = \begin{cases} 3(\frac{-n^2+3n+8}{\sqrt{n^2+32n-64}}); if\ n\ is\ even \\ 3(\frac{-n^2+3n+3}{\sqrt{n^2+8n-9}}); if\ n\ is\ odd \end{cases}$$

the $\chi$ – chromatic kurtosis of a Wheel graph $(W_n)$ is

$$\beta_{2\chi}(W_n) \begin{cases} \frac{n^4+704n^3-4480n^2+12288n+122888}{n^4-64n^2+1024}; even \\ \frac{n^4+1456n^3+1026n^2+432n-243}{n^4+16n^3+46n^2-144n+81}; odd \end{cases}$$

**Proof:** A wheel graph Wn is 4-colourable, when n is even and 3-colourable when n is odd. Then, we have the following cases.

(1) First assume that n is an even integer. Then, the outer cycle $C_{n-1}$ of Wn is an odd cycle. Hence, $\frac{n-2}{2}$ vertices of Cn−1 have colour $c_1$ and $c_2$ , one vertex of $C_{n-1}$ have colour $c_3$ and the central vertex of Wn has colour $c_4$. Hence the corresponding p.m.f of the corresponding Wn is given by, $f(i)=\begin{cases}\frac{n-2}{2n};i=1,2\\ \frac{1}{n};i=3,4\\ 0;elsewhere\end{cases}$

(i) the $\chi$ - chromatic mean of $(W_n)$ is $\mu_\chi(W_n)=\frac{3n+8}{2n}$

(ii) the $\chi$ - chromatic variance of $(W_n)$ is $\sigma^2{}_\chi(W_n)=\frac{n^2+32n-64}{4n^2}$

(iii)the $\chi$ - chromatic median of $(W_n)$ is $\frac{n}{2}$

(iv) the $\chi$ - chromatic standard deviation of $(W_n)$ is $\sigma_\chi(W_n)=\sqrt{\frac{n^2+32n+64}{4n^2}}$

(v) the $\chi$ - chromatic skewness of $(W_n)$ is , $\gamma_\chi(W_n)=3(\sqrt{\frac{-n^2+3n+8}{n^2+32n-64}})$

(vi) we can obtain kurtosis by various moments,

$$\mu'_1=\frac{3n+8}{2n},\mu'_2=\frac{5n+40}{2n},\mu'_3=\frac{9n+164}{2n},\mu'_4=\frac{17n+640}{2n},$$

$$\mu_3=0,\mu_4=\frac{n^4+704n^3-4480n^2+12288n-12288}{(2n)^4},$$

And the $\chi$ - chromatic kurtosis of $\beta_{2\chi}(W_n)=\frac{n^4+704n^3-4480n^2+12288n+122888}{n^4-64n^2+1024}$

If $\beta_{2\chi}(W_n)<3$ then it is known as PLATYKURTIC Curve.

(2)Next assume that n is an odd integer. Then, the outer cycle $C_{n-1}$ of Wn is an even cycle. Hence, $\frac{n-1}{2}$ vertices of the outer cycle $C_{n-1}$ have colour $c_1$, $\frac{n-1}{2}$ vertices of Cn−1 have colour $c_2$ and the central vertex of Cn−1 has colour $c_3$. Hence, the p.m.f for (Wn) is given by $f(i)=\begin{cases}\frac{n-1}{2n};i=1,2\\ \frac{1}{n};i=3,\\ 0;elsewhere\end{cases}$

(i) the $\chi$ - chromatic mean of $(W_n)$ is $\mu_\chi(W_n)=\frac{3n+3}{2n}$

(ii) the $\chi$ - chromatic variance of $(W_n)$ is $\sigma^2{}_\chi(W_n)=\frac{n^2+8n-9}{4n^2}$

(iii)the $\chi$ - chromatic median of $(W_n)$ is $\frac{n}{2}$

(iv) the $\chi$ - chromatic standard deviation of $(W_n)$ is $\sigma_\chi(W_n) = \sqrt{\frac{n^2+8n-9}{4n^2}}$

(v) the $\chi$ - chromatic skewness of $(W_n)$ is $\gamma_\chi(W_n) = 3(\sqrt{\frac{-n^2+3n+3}{n^2+8n-9}})$

(vi) we can obtain kurtosis by various moments,

$$\mu'_1 = \frac{3n+3}{2n}, \mu'_2 = \frac{5n+13}{2n}, \mu'_3 = \frac{9n+18}{2n}, \mu'_4 = \frac{17n+155}{2n},$$

$$\mu_3 = \frac{-45n^2-36n+27}{4n^3}, \mu_4 = \frac{n^4+1456n^3+1026n^2+432n-243}{16n^4},$$

$\chi$ - chromatic kurtosis of $\beta_{2\chi}(W_n) = \frac{n^4+1456n^3-1026n^2+432n+243}{n_4+16n^3+46n^2-144n+81}$,

If $\beta_{2\chi\backslash}(W_n) < 3$ then it is known as PLATYKURTIC Curve.

**CONCLUSION:**

In this paper,we extend the concept of mean,variance, standard deviation, median, skewness and kurtosis,some important statistical parameters to various graphs based on vertex coloring sum. Based on the vertex coloring sum of graphs, we have investigated these statistical inferences for vertex colourable graphs such as complete graphs, Path, Cycle and Wheel. This concept can be extended to several other operations on graphs such as Edge colouring, cartesian product, total colouring graphs, Johan colouring , lexicographic product, corona product, sum and product of graphs etc.,

**BIBILOGRAPHY:**

[1] Batsyn.M and Kalyagin.V, An analytical expression for the distribution of the sum of random variables with a mixed uniform density and mass function, In models, Algorithms, and Technologies for Network Analysis,(Editors: B. I. Goldengorin, P.M. Prdalos and V. Kalyagin),51-63, Springer, 2012.
[2] Bondy.J.A and U.S.R.Murty, Graph theory with application, NorthHolland, New York, 1982.
[3] Brandstadt.A, V. B. Le and J. P. Spinrad, Graph classes: A survey, SIAM,Philadelphia, 1999.
[4] G. Chartrand and P. Zhang, Introduction to graph theory, McGraw-Hill Inc., 2005.
[5] G. Chartrand and P. Zhang, Chromatic graph theory, CRC Press, 2009.
[6] F. Harary, Graph theory, Addison-Wesley Pub. Co. Inc., Philippines, 1969.
[7] T. R. Jensen and B. Toft, Graph coloring problems, John Wiley Sons,1995.
[8] M. Kubale, Graph colorings, American Mathematical Society, 2004.
[9] J. Kok, N. K. Sudev and K. P. Chithra, General colouring sums of graphs, Cogent Math., 3(1)(2016), 1-11, DOI:10.1080/23311835.2016.1140002.62
[10] E. Kubicka and A. J. Schwenk, An introduction to chromatic sums, Proc.ACM Computer Science Conference, Louisville (Kentucky), 3945(1989).
[11] E. Kubicka, The chromatic sum of a graph: History and recent developments, Int. J.Math. Math. Sci., 30,(2004), 1563-1573.
[12] V. K. Rohatgi, A. K. Md. E. Saleh, An introduction to probability and statistics, Wiley, New York, 2001.
[13] S. M. Ross, Introduction to probability and statistics for engineers and scientists, Academic Press, 2004.
[14] N. K. Sudev, K. P. Chithra and J. Kok, Certain chromatic sums of some cycle related graph classes, Discrete Math. Algorithms Appl., 8(3)(2016), 1-24, DOI 10.1142/S1793830916500506.
[15] E. W. Weisstein, CRC concise encyclopaedia of mathematics, CRC press, 2011.
[16] D. B. West,Introduction to graph theory, Pearson Education Inc., 2001.
[17] N.K. Sudev, S. Satheesh, K.P. Chithra and J. Kok, On certain colouring parameters of graphs.