Fuzzy $\alpha_I N_1$-set, fuzzy $\alpha_I N_4$-set and decompositions of fuzzy continuity, fuzzy α-I-continuity, fuzzy semi-I-continuity via idealization

K.MALARVIZHI[1], S.KEERTHIKA[2], E.NITHYA[3]

[1]Assistant Professor, [2]Research Scholar, [3]Research Scholar,
Department of Mathematics, Sri Krishna Arts and Science College,
Coimbatore, India.

***Abstract*: In this paper, we introduce fuzzy $\alpha_I N_1$-set and fuzzy $\alpha_I N_4$ -set in fuzzy ideal topological space. The concepts of fuzzy $\alpha_I N_3$-set, fuzzy $\alpha_I N_5$-set are used via idealization. New decompositions of fuzzy continuity, fuzzy α-I-continuity, fuzzy semi-I continuity are obtained by using the above sets.**
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1. INTRODUCTION AND PRELIMINARIES

Fuzziness is one of the most important and useful concepts in the modern scientific studies. In 1965,Zadeh[2] first introduced the notion of fuzzy sets. In 1945,Vaidyanathaswamy[3] introduced the concepts of ideal topological spaces.

In 1997,Mahmoud[8] and Sarkar[1] independently presented some of the ideal concepts in the fuzzy trend and studied many other properties. Decomposition of fuzzy continuity is one of the many problems in the fuzzy topology. It becomes very interesting when decomposition is done via fuzzy topological ideals.

Throughout this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by $A \leq X$, is characterized by a membership function in the sense of Zadeh. The basic fuzzy sets are the empty set, the whole set and the class of all fuzzy subsets of X which will be denoted by 0, 1 and I^X, respectively .By (X,τ), we mean a fuzzy topological space in Chang's sense. A fuzzy point in X with support $x \in X$ and value α ($0 < \alpha \leq 1$) is denoted by x_α . For a fuzzy subset A of X , Cl(A), Int(A) and $1 - A$ will, respectively, denote the fuzzy closure, fuzzy interior and fuzzy complement of A . A nonempty collection I of fuzzy subsets of X is called a fuzzy ideal[1] if and only if

1. $B \in I$ and $A \leq B$, then $A \in I$ (heredity),
2. $A \in I$ and $B \in I$ then $A \vee B \in I$ (finite additivity).

The triple (X, τ, I) means a fuzzy topological space with a fuzzy ideal I and fuzzy topology τ. For (X, τ, I) ,the fuzzy local function $A \leq X$ with respect to τ and I is denoted by $A^*(\tau, I)$ (briefly A^*) and is defined as $A^*(\tau, I) = \vee\{x \in X : A \wedge U \notin I, \text{ for every } U \in \tau(x)\}$. While A^* is the union of the fuzzy points x such that if $U \in \tau(x)$ and $E \in I$, then there is atleast one $y \in X$ for which $U(y) + A(y) - 1 > E(y)$. Fuzzy closure operator of a fuzzy set in (X, τ, I) is defined as $Cl^*(A) = A \vee A^*$. In (X, τ, I), the collection τ*(I) means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta = \{U - E : U \in \tau, E \in I\}$ as a base[1]. This topology of fuzzy sets is considered as generalization of the ordinary one.

First, we shall recall some definitions used in the sequel.

Definition 1.1. A subset A of a fuzzy ideal topological space (X,τ,I) is said to be
(1) fuzzy α- I-open[5] if $A \leq Int(Cl^*(Int(A)))$,
(2) fuzzy semi-I-open[9] if $A \leq Cl^*(Int(A))$,
(3) fuzzy pre-I-open[10] if $A \leq Int(Cl^*(A))$,
(4) fuzzy δ-I-open[4] if $Int(Cl^*(A)) \leq Cl^*(Int(A))$,
(5) fuzzy β-I-open[6] if $A \leq Cl(Int(Cl^*(A)))$.
(6) fuzzy strong β -I-open[4] if $A \leq Cl^*(Int(Cl^*(A)))$,

The complement of fuzzy α-I-open (resp. fuzzy semi- I-open, fuzzy pre-I-open, fuzzy β-I-open, fuzzy strong β-I-open) is fuzzy α-I-closed (resp. fuzzy semi- I - closed, fuzzy pre-I-closed, fuzzy β-I-closed, fuzzy strong β-I-closed).

The family of all fuzzy α - I-open (resp. fuzzy semi-I-open, fuzzy pre-I-open, fuzzy β-I-open, fuzzy strong β - I-open, fuzzy δ-I-open) sets in (X, τ, I) will be denoted by FαIO(X) (resp. F SIO(X), F PIO(X), F βIO(X), F SβIO(X), F δIO(X)).

Definition 1.2.[7] A subset A of a fuzzy ideal topological space (X,τ,I) is called fuzzy α^*-I-set if $Int(A) = Int(Cl^*(Int(A)))$.

Definition 1.3.[7] A subset A of a fuzzy ideal topological space (X,τ,I) is said to be
(a) A fuzzy C_I -set if $A = U \cap V$, where U is fuzzy open and V is an α^*-I-set.
(b) A weakly fuzzy- I-locally-closed set if $A = U \cap V$, where U is fuzzy open and V is fuzzy τ^*- closed.

Remark 1.1.[4] The simplest fuzzy ideals on X are {0} and ρ(X), the set of all fuzzy sets of X . Obviously $I = \{0\} \Leftrightarrow A^*(\tau, I) = Cl(A)$, for any fuzzy set A of X and
$I = \rho(X) \Leftrightarrow A^*(\tau, I) = 0$

Definition 1.4.[4] A function f : (X, τ, I) → (Y, σ) is said to be
(1) fuzzy α-I -continuous if $f^{-1}(V)$ is fuzzy α-I -open in X for each V ∈ σ,
(2) fuzzy semi-I -continuous if $f^{-1}(V)$ is fuzzy semi- I-open in X for each V ∈ σ,
(3) fuzzy pre-I -continuous if $f^{-1}(V)$ is fuzzy pre-I-open in X for each V ∈ σ,
(4) fuzzy strong β-I -continuous if $f^{-1}(V)$ is fuzzy strong β-I -open in X for each V ∈ σ,
(5)fuzzy β-I-continuous if $f^{-1}(V)$ is fuzzy β-I-closed in X for each V ∈ σ.

Definition 1.5.[4] A subset A of an fuzzy ideal topological space (X,τ,I) is called
(a) an $\alpha_I N_3$-set if A=U∩V, where U ∈ FαIO(X) and Int(Cl*(V))=Int(V).
(b) an $\alpha_I N_5$-set if A=U∩V, where U ∈ FαIO(X) and Cl*(V)=V.

Definition 1.6.[4] A subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy τ∗ -closed if A =Cl*(A) .

2. Fuzzy $\alpha_I N_1$ -set, Fuzzy $\alpha_I N_4$ -set

Definition 2.1. A subset A of a fuzzy ideal topological space (X, τ, I) is called fuzzy $\alpha_I N_1$-set if A = U∩V, where U ∈ FαIO(X) and Cl*(Int(V)) ⊂ V.
The family of all fuzzy $\alpha_I N_1$-sets of (X, τ, I) is denoted by $F\alpha_1 N_1$(X).

Definition 2.2. A subset A of a fuzzy ideal topological space (X ,τ,I) is called fuzzy $\alpha_I N_4$-set if A = U∩V, where U ∈ FαIO(X) and Int(Cl*(Int(V))) ⊂ V.
The family of all fuzzy $\alpha_I N_4$-sets of (X,τ,I) is denoted by $F\alpha_I N_4$(X).

Proposition 2.1. Let (X, τ, I) be an fuzzy ideal topological space and A=U ∩ V a subset of X. Then the following hold:
(a) If A is an fuzzy $\alpha_I N_5$-set, then A is an fuzzy $\alpha_I N_1$-set.
(b) If A is an fuzzy $\alpha_I N_1$-set, then A is an fuzzy $\alpha_I N_4$-set.
(c) If A is an fuzzy $\alpha_I N_4$-set, then A is an fuzzy δ-I-open.

Proof: (a) Let A=U∩V ∈ $F\alpha_I N_5$(X) where U ∈ FαIO(X) and Cl*(V)=V. Since, Cl*(Int(V))=V. Then, Cl*(Int(V)) ⊂ V and hence A ∈ $F\alpha_I N_1$(X).
(b) obvious
(c) Let A be a fuzzy $\alpha_I N_4$-set. Then we have A=U∩V, where U ∈ FαIO(X) and Int(Cl*(Int(V)))⊂V. Since every fuzzy α-I-open set is fuzzy δ-I-open and Int(Cl*(Int(V)))⊂V, Int(Cl*(V))≤Cl*(V), Int(Cl*(V))≤Cl*(Int(V)), therefore we obtain that A ∈ FδIO(X).

Remark 2.1. The converse of the above proposition need not be true as shown in the following example.

Example 2.1. Let x={a,b,c} and A,B,C be a fuzzy subsets of X defined as follows:
A(a)=0.8, A(b)=0.8, A(c)=0.4,
B(a)=0.3, B(b)=0.2, B(c)=0.4,
C(a)=0.3, C(b)=0.2, C(c)=0.7.
We put τ={ 0, A, B, C, A∪C, 1}. If we take I={0}, then B = B∩A is a fuzzy $\alpha_I N_4$-set, but B is not fuzzy $\alpha_I N_1$-set.
We put τ={ 0, A, B, C, A∪C, 1}. If we take I={0}, then B is fuzzy δ-I-open but B = A ∩ C is not fuzzy $\alpha_I N_4$-set.

Proposition 2.2. Let (X,τ,I) be an fuzzy ideal topological space and A=U∩V a subset of X. Then the following hold:
(a) If A is an fuzzy $A_{I\text{-}R}$-set, then A is an fuzzy $\alpha_I N_4$-set.
(b) If A is a weakly fuzzy α-I-locally closed set, then A is an fuzzy $\alpha_I N_4$-set.
(c) If A is a fuzzy C_I-set, then A is an fuzzy $\alpha_I N_4$-set.

Proof: (a) Let A be a fuzzy $A_{I\text{-}R}$-set. Then A=U∩V, where U is an fuzzy open set, V is an fuzzy I-R-closed set. Since V=Cl*(Int(V)), V=Int(Cl*(Int(V))). Therefore, Int(Cl*(Int(V)))⊂V and hence A ∈ $F\alpha_I N_4$(X).
(b) Let A be a fuzzy weakly fuzzy α-I-locally closed set. Then A=U∩V, where U is an fuzzy open set, V is an fuzzy τ*-closed set. Since V=Cl*(V), Cl*(V)=Cl*(Int(V))=Int(Cl*(Int(V))). Therefore, Int(Cl*(Int(V)))⊂V and hence A ∈ $F\alpha_I N_4$(X).
(c) A be an fuzzy C_1-set. Then A=U∩V, where U is an fuzzy open set, V is an fuzzy α*-I-set. Since Int(A)=Int(Cl*(Int(A))), Int(A)=A. Therefore, Int(Cl*(Int(V))) ⊂ A and hence A ∈ $F\alpha_I N_4$(X).

Theorem 2.1. For an ideal topological space (X, τ, I)

FIO(X) = FαIO(X) ∩ $F\alpha_I N_4$(X).

Proof: Let A be an fuzzy-I-open set. If we take V=A and U=X ∈ τ, then A=U∩V where U∈FαIO(X) and Int(Cl*(Int(V)))⊂V. Since A is fuzzy α-I-open, FIO(X) ⊂ FαIO(X) ∩ $F\alpha_I N_4$(X).

Let A be an fuzzy α-I-open set and an fuzzy $\alpha_I N_4$-set. Then, Since A is a fuzzy α-I-open set, we have A ⊂ Int(Cl*(Int(A))). Furthermore, because A is an fuzzy $\alpha_I N_4$-set, we have A = U∩V, where U is fuzzy α-I-open and Int(Cl*(Int(V))) ⊂ V. Since Cl*(A) is a kuratowski closure operation,

A ⊂ Int(Cl*(Int(A))) = Int(Cl*(Int(U∩V))) ⊂ Int(Cl*(Int(U) ∩ Int(V)))
= Int(Cl*(Int(U)) ∩ Cl*(Int(V))) = Int(Cl*(U)) ∩ Int(Cl*(V)) = Int(U) ∩ Int(V)

And hence

A ⊂ Int(U ∩ V) ⊂ Int(A)

Thus, A ∈ FIO(X).

Proposition 2.3. For a subset A of a fuzzy ideal topological space (X, τ, I) the following properties are equivalent:
A is fuzzy-I-open,
A is fuzzy α-I-open and fuzzy $\alpha_I N_4$-set.

Proof: (1) ⇔ (2) The proof is immediately followed from Theorem 2.1.

Remark 2.2. Fuzzy α-I-open and fuzzy $\alpha_I N_4$-set are independent of each other as shown in the following example.

Example 2.2. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:
A(a) = 0.2, A(b) = 0.3, A(c) = 0.7,
B(a) = 0.1, B(b) = 0.2, B(c) = 0.2. We put τ = {0, B, 1}. If we take I = {0}, then A = A ∩ B is fuzzy α-I-open, but A is not fuzzy $\alpha_I N_4$-set.
Example 2.3. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:
A(a) = 0.2, A(b) = 0.8, A(c) = 0.7, B(a) = 0.3, B(b) = 0.6, B(c) = 0.4, C(a) = 0.2, C(b) = 0.6, C(c) = 0.4. We put τ = {0, A, 1}. If we take I = ρ{X}, then C = A ∩ B is fuzzy $\alpha_I N_4$-set., but A is not fuzzy α-I-open.
Theorem 2.2. For an ideal topological space (X, τ, I)

$$FIO(X) = F\alpha IO(X) \cap F\alpha_I N_1(X).$$

Proof: Let A be an fuzzy-I-open set. If we take U=A, then A=U∩V where U∈FαIO(X) and Cl*(Int(V)) ⊂ V. Since A is fuzzy α-I-open, FIO(X) ⊂ FαIO(X) ∩ $F\alpha_I N_4(X)$.

Let A be an fuzzy α-I-open set and an fuzzy $\alpha_I N_4$-set. Then, Since A is a fuzzy α-I-open set, we have A ⊂ Int(Cl*(Int(A))). Furthermore, because A is an fuzzy $\alpha_I N_1$-set, we have A = U∩V, where U is fuzzy α-I-open and Cl*(Int(V) ⊂ V. Since Cl*(A) is a kuratowski closure operation,

$$A \subset Int(Cl^*(Int(A))) = Int(Cl^*(Int(U \cap V))) \subset Int(Cl^*(Int(U) \cap Int(V)))$$
$$= Int(Cl^*(Int(U)) \cap Cl^*(Int(V))) = Int(Cl^*(U)) \cap Int(Cl^*(V)) = Int(U) \cap Int(V)$$

And hence

$$A \subset Int(U \cap V) \subset Int(A)$$

Thus, A ∈ FIO(X).
Proposition 2.4. For a subset A of a fuzzy ideal topological space (X, τ, I) the following properties are equivalent:
A is fuzzy-I-open,
A is fuzzy α-I-open and fuzzy $\alpha_I N_1$-set.
Proof: (1) ⇔ (2) The proof is immediately followed from Theorem 2.1.
Remark 2.3. Fuzzy α-I-open and fuzzy $\alpha_I N_1$-set are independent of each other as shown in the following example.
Example 2.4. Let x={a,b,c} and A,B,C be a fuzzy subsets of X defined as follows:
A(a) = 0.6, A(b) = 0.8, A(c) = 0.4, B(a) = 0.2, B(b) = 0.3, B(c) = 0.1, C(a) 0.5, C(b) = 0.6, C(c) = 0.3. We put τ = {0, C, 1}. If we take I = {0}, then B = A ∩ B is fuzzy α-I-open, but A is not fuzzy $\alpha_I N_1$-set.
Example 2.5. Let X = {a,b,c} and A,B,C be a fuzzy subsets of X defined as follows:
A(a) = 0.2, A(b) = 0.6, A(c) = 0.4, B(a) = 0.2, B(b) = 0.8, B(c) = 0.7, C(a) = 0.3, C(b) = 0.6, C(c)=0.4. If we take I = ρ{X}, then A = B ∩ C is fuzzy $\alpha_I N_1$-set., but A is not fuzzy α-I-open.
Proposition 2.5. (Acikgoz et al.[1]). Let (X,τ,I) be an fuzzy ideal topological space.
(a) If V ∈ SIO(X) and A ∈ FαIO(X), then V∩A ∈ SIO(X).
(b) if V ∈ PIO(X) and A ∈ FαIO(X), then V∩A ∈ SIO(X).
Preposition 2.6. (a) Every fuzzy $\alpha_I N_4$-set of an fuzzy ideal topological space is fuzzy semi-I-open.
(b) Every fuzzy $\alpha_I N_4$-set of an fuzzy ideal topological space is fuzzy pre-I-open.
Proof: (a) Let A=U∩V ∈ $F\alpha_I N_4(X)$, where U ∈ FαIO(X) and Int(Cl*(Int(V)))⊂V. Since Cl*(Int(V))=Cl*(V), Int(Cl*(V))≤V. Therefore, Int(V)⊂Cl*(V), Thus V⊂Cl*(V) then V⊂Cl*(Int(V)) and hence V is fuzzy semi-I-open. Using Proposition 2.4 we have that A=U∩V is a fuzzy semi-I-open.
(b) obvious.
Remark 2.4. The converse of the above proposition need not be true as shown in the following example.
Example 2.6. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:
A(a) = 0.3, A(b) = 0.7, A(c) = 0.2, B(a) = 0.2, B(b) = 0.2, B(c) = 0.1. We put τ = {0, B, 1}. If we take I = {0}, then B is fuzzy semi-I-open, but B = A ∩ B is not fuzzy $\alpha_I N_4$-set. We put τ={0,B,1}. If we take I = {0}, then B is fuzzy pre-I-open, but B = A ∩ B is not fuzzy $\alpha_I N_4$-set.
Theorem 2.3. For an fuzzy ideal topological space (X,τ,I)

$$F\alpha IO(X)=FPIO(X) \cap F\alpha_I N_4(X).$$

Proof: Let A be an fuzzy α-I-open set. If we take U=A and V=X ∈ τ, then A=U∩V where U ∈ FαIO(X) and Int(Cl*(Int(V)))⊂V. Since A is fuzzy pre-I-open, FαIO(X) ⊂ FPIO(X) ∩ $F\alpha_I N_4(X)$.

Let A be an fuzzy pre-I-open set and an fuzzy $\alpha_I N_4$-set. Then, Since A is a fuzzy pre-I-open set, we have A ⊂ Int(Cl*(A)). Furthermore, because A is an fuzzy $\alpha_I N_4$-set, we have A = U∩V, where U is fuzzy α-I-open and Int(Cl*(Int(V))) ⊂ V. Since Cl* is a kuratowski closure operation,

$$A \subset Int(Cl^*(A)) = Int(Cl^*(U \cap V)) \subset Int(Cl^*(U) \cap Cl^*(V))$$
$$= Int(Cl^*(Int(U)) \cap Int(Cl^*(V)) = Int(Cl^*(Int(U)) \cap Int(V)$$

And hence

$$A \subset Int(Cl^*(Int(U)) \cap Int(Int(V)) = Int[Cl^*(Int(U)) \cap Int(V)]$$
$$\subset Int(Cl^*(Int(U \cap V))) = Int(Cl^*(Int(A))).$$

Thus, A ∈ FαIO(X).
Proposition 2.7. For a subset A of a fuzzy ideal topological space (X,τ,I) the following properties are equivalent:
A is fuzzy α-I-open,
A is fuzzy pre-I-open and fuzzy $\alpha_I N_4$-set.
Proof: (1) ⇔ (2) The proof is immediately followed from Theorem 2.4.
Remark 2.5. Fuzzy pre-I-open and fuzzy $\alpha_I N_4$-set are independent of each other as shown by the following example
Example 2.7. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.3, A(c) = 0.1.
B(a) = 0.6, B(b) = 0.8, B(c) = 0.4,
C(a) = 0.5, C(b) = 0.6, C(c) = 0.3,
We put τ = { 0, C, 1}. If we take I={0}, then C = C∩B is fuzzy pre-I-open, but C is not fuzzy $\alpha_I N_4$-set.

Example 2.8. Let X = {a, b, c} and A,B and C be fuzzy subsets of X defined as follows:
A(a) = 0.2, A(b) = 0.6, A(c) = 0.4, B(a) = 0.3, B(b) = 0.6, B(c) =0.4, C(a) = 0.2, C(b) = 0.8, C(c)= 0.7. We put τ = {0, C, 1}. If we take I = ρ{X}, then A = B ∩ C is fuzzy $\alpha_I N_4$-set, but A is not fuzzy pre-I-open.

Theorem 2.4. For an fuzzy ideal topological space (X,τ,I)

FαIO(X)=FPIO(X) ∩ $F\alpha_I N_1$(X).

Proof: Since FαIO(X) = FPIO(X) ∩ FSIO(X) then we have FαIO(X) ≤ FPIO(X). Also since FαIO(X) ≤ $F\alpha_I N_1$(X) we obtain that FαIO(X) ≤ FPIO(X) ∩ $F\alpha_I N_1$(X).
From Theorem2.2, we have FαIO(X) = FPIO(X) ∩ $F\alpha_I N_4$(X).
If we use $F\alpha_I N_4$(X) ≤ $F\alpha_I N_1$(X), we obtain that FPIO(X) ∩ $F\alpha_I N_4$(X) ≤ FPIO(X) ∩ $F\alpha_I N_1$(X) = FαIO(X).

Proposition 2.8. For a subset A of a fuzzy ideal topological space (X,τ,I) the following properties are equivalent:
A is fuzzy α-I-open,
A is fuzzy pre-I-open and fuzzy $\alpha_I N_1$-set.

Proof: (1) ⇔ (2) The proof is immediately followed from Theorem 2.4.

Remark 2.6. Fuzzy pre-I-open and fuzzy $\alpha_I N_1$-set are independent of each other as shown by the following example

Example 2.9. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:
A(a) = 0.5, A(b) = 0.4, A(c) = 0.6, B(a) = 0.5, B(b) = 0.7, B(c) = 0.6,
C(a) = 0.7, C(b) = 0.4, C(c) = 0.8, We put τ = { 0, C, 1}. If we take I={0}, then A = A∩C is fuzzy pre-I-open, but A is not fuzzy $\alpha_I N_1$-set.

Example 2.10. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:
A(a) = 0.6, a(b)= 0.3, A(c) = 0.5, B(a) = 0.3, B(b) = 0.1, B(c) = 0.2, C(a) = 0.8, C(b) = 0.4, C(c)=0.6. We put τ = { 0, A, 1}. If we take I = ρ{X}, then A = A∩C is fuzzy $\alpha_I N_1$-set, but A is not fuzzy pre-I-open.

Proposition 2.9 Let (X,τ,I) be an fuzzy ideal topological space and A=U∩V a subset of X. Then the following hold:
(a) If A is an fuzzy $\alpha_I N_3$-set, then A is an fuzzy $\alpha_I N_4$-set.
(b) If A is an fuzzy $\alpha_I N_2$-set, then A is an fuzzy $\alpha_I N_4$-set.

Proof: (a) Let A=U∩V ∈ $F\alpha_I N_3$(X) where U ∈ FαIO(X) and Int(Cl*(V)) = Int(V). Since Cl*(V) = Cl*(Int(V)), Int(Cl*(V)) = V. Therefore, Int(Cl*(V))⊂V and hence A ∈ $F\alpha_I N_4$(X).
(b) Proof is obvious.

Remark 2.7. The converse of the above proposition need not be true as shown in the following example.

Example 2.11. Let X = {a,b,c} and A,B and C be the fuzzy subsets of X defined as follows:
A(a) = 0.3, A(b) = 0.2, A(c) = 0.7, B(a) = 0.8, B(b) = 0.8, B(c) = 0.4, C(a) = 0.3, C(b) = 0.2, C(c)=0.4. we put τ {0, A, B, A∪B, C, 1}. If we take I= {0}, then C = C∩B is a fuzzy $\alpha_I N_4$-set, but C is not fuzzy $\alpha_I N_3$-set. we put τ {0, A, B, A∪B, C, 1}. If we take I= {0}, then C = C∩B is a fuzzy $\alpha_I N_4$-set, but C is not fuzzy $\alpha_I N_2$-set.

Proposition 2.10. Let (X,τ,I) be an ideal topological space and A= U∩V a subset of X, If A is an fuzzy β-I-closed set, then A is an fuzzy $\alpha_I N_4$-set.

Proof: Let A be an fuzzy β-I-closed set. Since every fuzzy β-I-closed set is fuzzy α-I-open set and Cl*(Int(Cl(A))) ≤ A. Since Cl(A) = A, Cl*(Int(A)) ≤ A. Therefore, Int(Cl*(Int(A))) ≤ A and hence A ∈ $F\alpha_I N_4$.

Remark 2.8. The converse of the above proposition need not be true as shown in the following example.

Example 2.12. Let X = { a, b, c} and A, B be fuzzy subsets of X defined as follows:
A(a) = 0.3, A(b) = 0.2, A(c) = 0.4.
B(a) = 0.8, B(b) = 0.8, B(c) = 0.4,
We put τ = { 0, B, 1}. If we take I = ρ{X}, then A = A∩B is a fuzzy $\alpha_I N_4$-set, but A is not fuzzy β-I-closed set.

Theorem 2.5. Let (X, τ, I) be a fuzzy ideal topological space.

FSIO(X) = FSβIO(X) ∩ $F\alpha_I N_1$(X)

Proof: Let A be an fuzzy semi-I-open set. If we take V=A, then A=U∩V where U ∈ FαIO(X) and Cl*(Int(V)) ⊂ V. Since A is strong-β-I-open, FSIO(X) ⊂ FSβIO(X) ∩ $F\alpha_I N_1$(X).

Let A be an fuzzy strong-β-I-open set and an fuzzy $\alpha_I N_1$-set. Then, Since A is a fuzzy strong-β-I-open set, we have A ⊂ Cl*(Int(Cl*(A))). Furthermore, because A is an fuzzy $\alpha_I N_1$-set, we have A = U∩V, where U is fuzzy α-I-open and Cl*(Int(V)) ⊂ V. Since Cl*(A) is a kuratowski closure operation,

A ⊂ Cl*(Int(Cl*(A))) = Cl*(Int(Cl*(U∩V))) ⊂ Cl*(Int(Cl*(U) ∩ Cl*(V)))
= Cl*(Int(Cl*(U)) ∩ Int(Cl*(V))) =Cl*(Int(U) ∩ Int(V))

And hence

A ⊂ Cl*(Int(U∩V)) = Cl*(Int(A)).

Thus, A ∈ FSIO(X).

Proposition 2.11. For a subset A of a fuzzy ideal topological space (X, τ, I) the following properties are equivalent:
A is fuzzy semi-I-open,
A is fuzzy strong-β-I-open and fuzzy $\alpha_I N_1$-set.

Proof: (1) ⇔ (2) The proof is immediately followed from Theorem 2.5.

Remark 2.9. Fuzzy strong β-I-open and fuzzy $\alpha_I N_1$-set are independent of each other as shown in the following example.

Example 2.13. Let X = { a, b, c} and A, B, C be fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.4, A(c) = 0.7,
B(a) = 0.3, B(b) = 0.4, B(c) = 0.8,
C(a) = 0.2, C(b) = 0.6, C(c) = 0.7.
We put τ = { 0, C, 1}. If we take I = ρ(X), then A = B ∩ C is fuzzy $\alpha_I N_1$-set, but A is not fuzzy strong β-I-open set.
Example 2.14. Let X = { a, b, c} and A, B be fuzzy subsets of X defined as follows:
A(a) = 0.8, A(b) = 0.8, A(c) = 0.4,
B(a) = 0.3, B(b) = 0.2, B(c) = 0.4.
We put τ = { 0, A, 1}. If we take I = {0}, then A is fuzzy strong β-I-open set but A = A ∩ B is not fuzzy $\alpha_I N_1$-set.
Theorem 2.6. For an fuzzy ideal topological space (X,τ,I)

$$FSIO(X)=FS\beta IO(X) \cap F\alpha_I N_4(X).$$

Proof: Let A be an fuzzy semi-I-open set. If we take V=A, then A=U∩V where U ∈ FαIO(X) and Cl*(Int(V)) ⊂ V. Since A is strong-β-I-open, FSIO(X) ⊂ FSβIO(X) ∩ F$\alpha_I N_4$(X).

Let A be an fuzzy strong-β-I-open set and an fuzzy $\alpha_I N_1$-set. Then, Since A is a fuzzy strong-β-I-open set, we have A ⊂ Cl*(Int(Cl*(A))). Furthermore, because A is an fuzzy $\alpha_I N_4$-set, we have A = U∩V, where U is fuzzy α-I-open and Cl*(Int(V)) ⊂ V. Since Cl*(A) is a kuratowski closure operation,

$$A \subset Cl^*(Int(Cl^*(A))) = Cl^*(Int(Cl^*(U \cap V))) \subset Cl^*(Int(Cl^*(U) \cap Cl^*(V)))$$
$$= Cl^*(Int(Cl^*(U)) \cap Int(Cl^*(V))) = Cl^*(Int(U) \cap Int(V))$$

And hence

$$A \subset Cl^*(Int(U \cap V)) = Cl^*(Int(A)).$$

Thus, A ∈ FSIO(X).
Proposition 2.12. For a subset A of a fuzzy ideal topological space (X, τ, I) the following properties are equivalent:
A is fuzzy semi-I-open,
A is fuzzy strong-β-I-open and fuzzy $\alpha_I N_4$-set.
Proof: (1) ⇔ (2) The proof is immediately followed from Theorem 2.6.
Remark 2.10. Fuzzy strong β-I-open and fuzzy $\alpha_I N_4$-set are independent of each other as shown in the following example.
Example 2.15. Let X = {a,b,c} and A,B and C be fuzzy subsets of X defined as follows:
A(a) = 0.3, A(b) = 0.6, A(c) = 0.4, B(a) = 0.2, B(b) = 0.8, B(c) = 0.7, C(a) = 0.2, C(b) = 0.6, C(c)= 0.4. we put τ = {0, B, 1}. If we take I = ρ(X), then C = B ∩ A is fuzzy $\alpha_I N_4$-set, but C is not fuzzy strong β-I-open set.
Example 2.16. Let X = {a,b,c} and A,B and C be fuzzy subsets of X defined as follows:
A(a) = 0.7, A(b) = 0.6, A(c) = 0.7, B(a) = 0.7, B(b) = 0.6, B(c) = 0.6, C(a) = 0.6, C(b) = 0.4, C(c)= 0.5, D(a) = 0, D(b) = 0, D(c) = 0.5. we put τ = {0, C, D, 1}. If we take I = {0}, then B is fuzzy strong β-I-open set, but B = B ∩ A is not fuzzy $\alpha_I N_4$-set.

3. Decomposition of fuzzy continuity, fuzzy α-I-continuity and fuzzy semi-I-continuity.

Definition 3.1. A function f : (X, τ, I) → (Y, σ) is said to be fuzzy $\alpha_I N_4$- continuous, if $f^{-1}(V)$ is fuzzy $\alpha_I N_4$-set in X for each V ∈ σ.
Definition 3.2. A function f : (X, τ, I) → (Y, σ) is said to be fuzzy $\alpha_I N_1$- continuous, if $f^{-1}(V)$ is fuzzy $\alpha_I N_1$-set in X for each V ∈ σ.
Theorem 3.1. For a function f : (X, τ, I) → (Y, σ) the following properties are equivalent:
A is a fuzzy-I-continuous;
A is a fuzzy α-I-continuous and fuzzy $\alpha_I N_4$-continuous.
Proof: This is an immediate consequence of proposition 2.3.
Theorem 3.2. For a function f : (X, τ, I) → (Y, σ) the following properties are equivalent:
A is a fuzzy-I-continuous;
A is a fuzzy α-I-continuous and fuzzy $\alpha_I N_1$-continuous.
Proof: This is immediately follows from proposition 2.4.
Theorem 3.3. For a function f : (X, τ, I) → (Y, σ) the following properties are equivalent:
A is a fuzzy α-I-continuous;
A is a fuzzy pre-I-continuous and fuzzy $\alpha_I N_4$-continuous.
Proof: This is immediately follows from proposition 2.7.
Theorem 3.4. For a function f : (X, τ, I) → (Y, σ) the following properties are equivalent:
A is a fuzzy α-I-continuous.
A is a fuzzy pre-I-continuous and fuzzy $\alpha_I N_1$-continuous.
Proof: This is immediately follows from proposition 2.8.
Theorem 3.5. For a function f : (X, τ, I) → (Y, σ) the following properties are equivalent:
f is fuzzy semi-I-continuous;
f is fuzzy strong β-I- continuous and fuzzy $\alpha_I N_1$- continuous.
Proof: This is an immediate consequence of proposition 2.11.
Theorem 3.6. For a function f : (X, τ, I) → (Y, σ) the following properties are equivalent:
f is fuzzy semi-I-continuous;
f is fuzzy strong β-I- continuous and fuzzy $\alpha_I N_4$- continuous.
Proof: This is an immediate consequence of proposition 2.12.

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