STUDY OF ROAD HOLDING AND RIDE COMFORT ANALYSIS WITH THE HELP OF QUARTER CAR MODEL

[1]Akanksha Tiwari, [2]Shubham More, [3]Aditya Shahane

[1]Student-Master of Technology, [2]Associate Engineer-Design, [3]Associate Engineer-CAE
[1]Department of Mechanical Engineering-Automotive Technology,
[1]College of Engineering, Pune,[2,3]Mahle Behr India Private Limited

Abstract: **A vehicle suspension system is a complex vibration system having multiple degrees of freedom. Automobile suspension systems are designed with the objective of isolating the vibrations produced because of road disturbances from being transferred to the driver. Its purpose is to improve ride comfort and road handling of a vehicle. A quarter car model is the simplest method to analyse the dynamics of vehicle. A quarter car model considers one-fourth of the vehicle weight, one suspension system and one tyre. It is modelled with equivalent spring, damper and mass combinations. The report contains mathematical modelling and basic analysis of a two degree of freedom passive suspension system quarter car model to study for road handling and ride comfort of a vehicle for road input. It also contains cases that studies the effect of road profile over the dynamics of vehicle through a quarter car model.**

Index Terms: **Quarter car model, Ride comfort, Road holding.**
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I. INTRODUCTION

A Suspension system is the mechanism that physically separates the car body from the wheels of the car. It is the system of tires, tire air, springs, shock absorbers and linkages that connects a vehicle to its wheels and allows relative motion between the two. A typical automobile suspension system consists of spring and damper. Its main purpose is to ensure the safe, controlled movement of the car. It is reported that more than one million people per year lose their lives worldwide and approximately 50 million people suffer non-fatal injuries from road accidents. Ride safety and the handling capabilities of an automobile are mainly determined by its suspension system, which transmits the forces between the vehicle and the road. Exposure to whole body vibration associated with a prolonged seating is an important risk factor for low back pain. among drivers. Both vehicle suspension system and driver seat cushion designs have attracted significant interest over the last several decades with a significant effort being directed towards their improvements. Vibration attenuation through the suspension and seat will not only provide riding comfort but also reduce the risk of LBP due to driving. Suspension systems must support both road holding/handling and ride quality, which are at odds with each other. It is important for the suspension to keep the road wheel in contact with the road surface as much as possible, because all the road or ground forces acting on the vehicle do so through the contact patches of the tires.

Every automotive suspension has two goals: passenger comfort and vehicle control. The stiffness of the spring and the damping co-efficient of the shock absorber are the important factors to be considered while designing a suspension system. For the purpose of ride comfort soft spring and low damping characteristics are desirable whereas high damping and the stiff spring is desirable when handling is required. Hence, a tradeoff of these properties needs to be done while designing a suspension system of automobile.
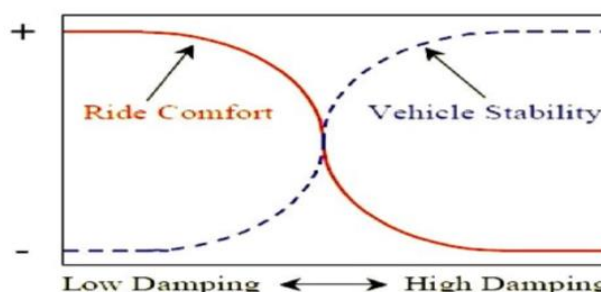

Fig.1: Damping coefficient values for ride comfort and vehicle stability [1]

There are various types of suspension systems that are currently used in the automobile suspension system. They are:

a)Passive Suspension – They are the most conventional and most widely used suspension system in automobiles today. It is an open loop control system. Passive indicates that the suspension elements cannot provide energy to the suspension system. A typical

passive suspension system consists of spring and damper whose values are fixed at the design stage itself and cannot be modified later during actual road condition driving.
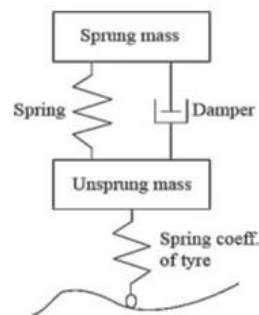
Fig.2: Passive Suspension

b) Semi- Active Suspension system – This suspension contains a feedback circuit in addition to the conventional passive suspension system. The semi-active suspension system can only vary the viscous damping co-efficient of shock absorbers but do not add energy to the suspension system. They require less energy to function and are less expensive. One of the popular semi- active suspension is the use of Magneto-Rheological(MR) dampers.
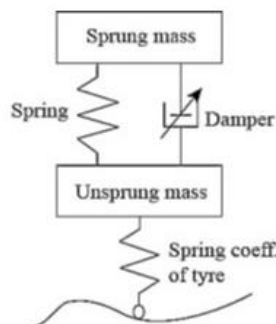
Fig.3: Semi-active suspension

c) Active Suspension system – Active suspension is a type of automotive suspension that controls the vertical movement of the wheels relative to the chassis or vehicle body with an onboard system. It has the ability to store, dissipate and to introduce energy to the system. Fully active suspension systems use electronic monitoring of vehicle conditions, coupled with the means to impact vehicle suspension and behavior in real time to directly control the motion of the car. They use separate actuators which can exert an independent force on the suspension to improve the riding characteristics. The drawbacks of this design are high cost, added complication and mass of the apparatus, and the need for frequent maintenance on some implementations.
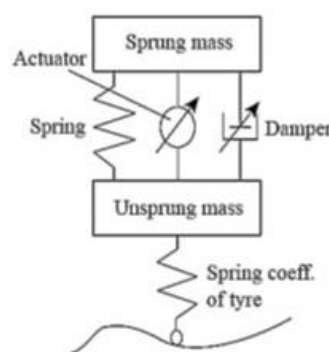
Fig.4: Active suspension

A vehicle suspension is a non-linear system that poses significant challenges to the designer. Its main purpose is to ensure the safe, controlled movement of the car. This requires different responses depending on the characteristics of the road surface encountered. In addition, it must cushion the driver from the more severe shocks, while transmitting enough information to give confidence that the vehicle will perform as expected to driver inputs.

Normally car is divided into three type of categories: full car model, half-car-model and quarter car model. In order to study the behaviour response of a car when under motion to various road irregularities, considering the entire vehicle for study becomes difficult and tedious. Quarter-car models are extensively used in automotive engineering due to their simplicity and provide the qualitatively correct information, at least in the initial design stages of vehicle dynamics. The quarter car model of 2 DOF consists

of wheel and its attachments the tire, the suspension elements, quarter portion of the chassis and rigidly connected part. A simple measure for ride comfort is the root mean square (rms) value of the vertical chassis acceleration

II. LITERATURE SURVEY

In the paper "Modelling of road vehicle suspension system for performance evaluation" by Damien Maher and Paul Young, an experimental setup of MacPherson Strut suspension system is done. The input provided to it by means of cams with variable speed motor drivers. The input signal of sinusoidal and step type is given. A frequency measuring optical sensor is also used. This model is then simulated using quarter car model in MATLAB with the frequency input from the optical sensor. The experimental and simulated models are compared for sprung and unsprung mass. The general outputs observed was that the quarter car model over predicts the sprung mass acceleration and under predicts the unsprung mass acceleration for a sine road profile input. The quarter car model accurately predicts the high frequency values. The peak points however where observed to be at the same points. Thus the paper predicts that there is variation in actual and simulated values but the peak trends are same. [2]

In the paper "Car Dynamics using Quarter Model and Passive Suspension, Part I: Effect of Suspension Damping and Car Speed" by Galal Ali Hassaan, the author aims at examining the dynamics of a car passing a circular hump for maintaining ride comfort for the passengers. The analysis is done on a passive suspension system with the suspension damping coefficient in the range of 1 to 15 kNs/m and car speed in the range of 5 to 25 km/hr. Quarter car model is been used and it is simulated using MATLAB software. The input values given are Tire stiffness= 135KN/m, Tire damping coefficient = 1.4 kNs/m, Unsprung mass= 49.8 kg and Sprung mass = 466.5 kg. A circular (parabolic) hump of height 100mm and length 5.2mm is provided as an input and sprung mass displacement for various combination of damping coefficient and vehicle velocity is studied. The paper concludes that the maximum and minimum displacement of the sprung-mass depend on both suspension damping and car speed. For suspension damping coefficient ≤ 5 kNs/m, minimum sprung-mass displacement (undershoot) existed. The undershoot value increased with increasing the car speed for suspension damping coefficient <5 kNs/m. The undershoot decreased as the suspension damping coefficient increased. The maximum sprung-mass displacement decreased with increased suspension damping coefficient. For a minimum ride comfort acceleration level of 0.8 m/s^2(minimum), the maximum car speed when passing the circular hump ranged from 5.4 km/h at 1 kNs/m suspension damping coefficient to 6.75 km/h at 15 kNs/m coefficient. [3]

In the paper "Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile" by Abdolvahab Agharkakli, a quarter car modelling is done of passive as well as active suspension system. The main objective is to observe the performance of active by using LQR controller and passive suspension only. For input condition of sprung mass= 290kg, unsprung mass= 59kg, spring stiffness of sprung mass = 16812 N/m, spring stiffness of tire= 190000(N/m), damping coefficient of sprung mass = 1000(Ns/m) and road input of a single road bump and multi road bump profiles, the MATLAB simulation is carried out and the results are observed. It was observed that that active suspension can give lower amplitude and faster settling time as compared to passive suspension system. It was observed that, by including an active element in the suspension, it is possible to reach a better compromise than is possible using purely passive elements. [4]

In the paper "Analysis of Ride comfort and Road holding of a Quarter car model by SIMULINK" by Trupti P. Phalke, modelling of a quarter car model of passive suspension is done. The quarter car model is then simulated using MATLAB. The purpose of this paper is to study the value of road holding and ride comfort on changing the sprung mass damping co-efficient, vehicle velocity, sprung mass stiffness and sprung mass for a road input of 0.1m height bump. It was observed that in all cases, for ride comfort the value of RMS vertical acceleration of the sprung mass was always below .315 m/s^2 which is an under safe value of RMS acceleration for human comfort according to ISO 2631-1:1997. [5]

In the paper "Analysis of Automotive Passive Suspension System with Matlab Program generation" by Pankaj Sharma the author studies the basic quarter car model with 2 DOF and develops a mathematical model and does its simulation by using MATLAB. The analysis was done by providing a speed bump of 0.1m (step input) with sprung mass= 275 kg, unsprung mass= 27 kg, sprung mass spring stiffness = 150000 N/m, sprung mass damping co-efficient= 1120 N-s/m, Tire Stiffness= 310000 N/m and Tire damping co-efficient = 3100 N-s/m. It was observed that sprung mass displacement overshoot by 70% and an acceleration amplitude of 1.75 m/s^2 was observed, which is undesirable from comfort point of view. This also affects the suspension working and durability. Also the unsprung mass had its displacement overshoots by 30% and there was a sudden drop in its acceleration value from 4 m/s^2 to 0.7 m/s^2 which seems to be uncomfortable from driver point of view and ride quality. The paper suggests that the problems could be overcome by making use of active suspension system. [6]

In the paper "Passive suspension modelling using MATLAB, Quarter car model, input signal step type" by Andronic Florin a basic study of quarter car model is carried out. A graphical model of the same is done by making use of MATLAB, transfer function and state space equation and the displacements of sprung and unsprung mass is studied. The results obtained from all the three modes are identical. [7]

In the paper "Analytical Description and Optimization of the dynamic behavior of passively suspended road vehicles" by M. Gobbi a simple 2 DOF linear model is used to derive a number of analytical formulae describing the dynamic behavior of passively suspended vehicle running on randomly profiled roads. Two different road profiles are considered for modelling the road profile. The derived analytical formula can be used in the preliminary design stage when approximate results are acceptable. [8]

III. QUARTER CAR MODEL

The dynamic behavior of passive automotive suspension systems is primarily determined by the choice of the spring (stiffness ks) and the damper (damping cs). The quarter car model is the most widely used suspension system model. A quarter car model can be considered with two degrees of freedom each at unsprung mass and sprung mass.

1. Mathematical Modelling

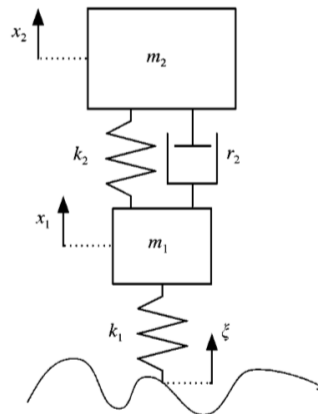Fig. shows a line diagram of a 2 DOF car quarter physical model without drive

Fig.5: Quarter Car model of Passive Suspension

$$m_1\ddot{x}_1 - r_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) + k_1(x_1 - \xi) = 0,$$

$$m_2\ddot{x}_2 + r_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0. \qquad [8]$$

Where:

m_1 – unsprung mass.

m_2 – sprung mass.

x_1 – displacement of unsprung mass.

x_2 – displacement of sprung mass.

k_1 – spring stiffness of unsprung mass (tire stiffness).

k_2 - spring stiffness of sprung mass.

$\dot{x}_1$ - velocity of unsprung mass.

$\dot{x}_2$ - velocity of sprung mass.

r_2 - Damping coefficient of sprung mass.

$\ddot{x}_1$ - acceleration of unsprung mass.

$\ddot{x}_2$ - acceleration of sprung mass.

ξ - road excitation.

The responses of the vehicle model are observed to be:

- The vertical vehicle body response.
- The force applied between road and wheel
- The relative displacement between wheel and vehicle body

The discomfort is evaluated by computing the standard deviation of the vertical vehicle body acceleration (RMS value of acceleration). Higher standard deviation leads to higher discomfort. The standard deviation of the relative displacement between the wheel and vehicle body gives the rattle space or working space which is related to designing and packaging constraints and wheel.

Let any displacement x be given by a complex function:

$x = ae^{j\omega t}$

Where:

a=amplitude

ω=frequency of vibration

t=time

Thus, velocity is given as

$\dot{x} = j\omega ae^{j\omega t}$

ie. $\dot{x} = j\omega x$

and acceleration is given by,
$$\ddot{x} = (j\omega)^2 a e^{j\omega t}$$
ie. $\ddot{x} = -\omega^2 x$.

Substituting the above derivations in the above equations of quarter car model for each displacement, velocity and acceleration of sprung and unsprung mass we get,

$$x_2 = \frac{(j\omega r_2 + k_2) * x_1}{(-m_2\omega^2 + j\omega r_2 + k_2)}$$

Then, substituting the value of x_2 in first equation and solving we get the results as ,

$$\frac{k_1 k_2 + j\omega k_1 r_2 - (k_2 m_1 + k_1 m_2 + k_2 m_2) * \omega - jr_2(m_1 + m_2) * \omega^3 + m_1 m_2 * \omega^4}{-m_2\omega^2 + j\omega r_2 + k_2} * x_1 = k_1 * \xi$$

Thus we get it as,

$$\frac{x_1}{\xi} = \frac{k_1 * (-m_2\omega^2 + j\omega r_2 + k_2)}{k_1 k_2 + j\omega k_1 r_2 - (k_2 m_1 + k_1 m_2 + k_2 m_2) * \omega - jr_2(m_1 + m_2) * \omega^3 + m_1 m_2 * \omega^4}$$

Thus, the transfer function between the displacement/road excitation (ξ) and unsprung mass displacement(x_1) is given by:

$$X_1(j\omega) = \frac{k_1(k_2 + jr_2\omega - m_2\omega^2)}{D(j\omega)},$$

$$D(j\omega) = k_1 k_2 + jk_1 r_2\omega - (k_2 m_1 + k_1 m_2 + k_2 m_2)\omega^2$$

$$- jr_2(m_1 + m_2)\omega^3 + m_1 m_2\omega^4. \qquad [8]$$

Now substituting the value of x_1 in terms of ξ to obtain the ratio of x_2 and ξ we get,

$$x_2 = \frac{(j\omega r_2 + k_2)}{(-m_2\omega^2 + j\omega r_2 + k_2)} * \frac{k_1 * (-m_2\omega^2 + j\omega r_2 + k_2)}{k_1 k_2 + j\omega k_1 r_2 - (k_2 m_1 + k_1 m_2 + k_2 m_2) * \omega - jr_2(m_1 + m_2) * \omega^3 + m_1 m_2 * \omega^4} * \xi$$

Thus we get it as,

$$\frac{x_2}{\xi} = \frac{k_1 * (j\omega r_2 + k_2)}{k_1 k_2 + j\omega k_1 r_2 - (k_2 m_1 + k_1 m_2 + k_2 m_2) * \omega - jr_2(m_1 + m_2) * \omega^3 + m_1 m_2 * \omega^4}$$

The transfer function between the displacement/road excitation (ξ) and sprung mass displacement(x_1) is given by:

$$X_2(j\omega) = \frac{k_1(k_2 + jr_2\omega)}{D(j\omega)}. \qquad [8]$$

Thus, the transfer function between the displacement/road excitation (ξ) and vertical body acceleration response($\ddot{x}_2$) is given as :

$$H_1(j\omega) = -\omega^2 X_2(j\omega). \qquad [8]$$

The transfer function between the displacement/road excitation (ξ) and force applied between road and wheel is given as :

$$H_2(j\omega) = k_1(1 - X_1(j\omega)). \qquad [8]$$

The transfer function between the displacement/road excitation (ξ) and relative displacement between wheel and vehicle body is given by:

$$H_3(j\omega) = X_2(j\omega) - X_1(j\omega). \qquad [8]$$

The above quarter car model considered has 2 mass elements, hence it is bound of two natural frequencies for its system, where the system will vibrate with high amplitude due to resonance condition. Hence appropriate care must be taken while designing a system so as not to design them near the natural frequency values.

The natural frequency of quarter car model is given by

$$\omega_{n1}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

$$\omega_{n2}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

$$A_1 = m_s m_{us},$$

$$B_1 = m_s k_s + m_s k_{tr} + m_{us} k_s,$$

$$C_1 = k_s k_{tr}.$$

[9]

Where,

m_s – sprung mass

m_{us} – unsprung mass

k_s – spring stiffness of sprung mass

k_{tr} – spring stiffness of tire

The negative values are discarded as being of no physical significance. The natural frequency of a passenger car is found to be 1-1.5 Hz for the sprung mass and around 10 Hz for unsprung mass. For passenger cars, the damping ratio provided by shock absorbers is usually in the range of 0.2-0.4. [9]

As the equations above consists of second order differential it is difficult to solve analytically therefore, MATLAB simulations are usually used to study the effect on the quarter car model due to road irregularities and its corresponding effect on the sprung and unsprung mass.

When the vehicle system vibrates, the normal force acting between the tire and the road fluctuates. Since the cornering force, tractive effort, and braking effort developed by the tire are related to the normal load on the tire, the vibration of the tire affects the road-holding capability and influences the handling and performance of the vehicle. [9].

The road profile consists of various kinds of irregularities, which in turns affects the road holding and ride comfort. The sinusoidal kind of road input provides a similarity to the real road profiles (bumps).

CASE 1: Hence considering the below sinusoidal input (mathematical expression) for bump excitation on wheel:

$$Z_r = 0 \qquad\qquad for \quad t < \frac{d}{v}$$
$$= h \sin\left[\frac{\pi v}{L}\left(t - \frac{d}{v}\right)\right] \qquad for \quad \frac{d}{v} \leq t \leq \frac{d+L}{v}$$
$$= 0 \qquad\qquad for \quad t > \frac{d+L}{v}$$

[5]

Where

h= road bump height

d= distance of approach of the wheel to the bump.

L= road bump width.

v= vehicle velocity.

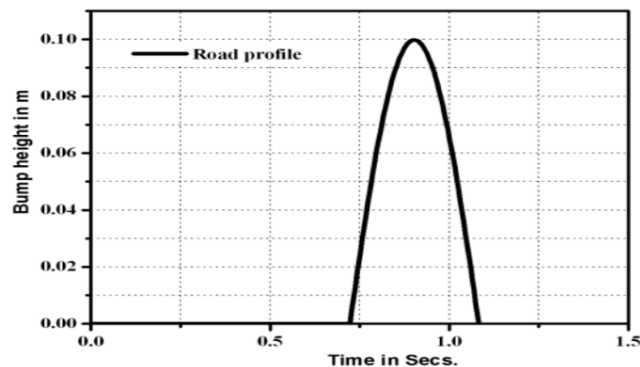Now considering the below figure shows the road profile with bump height = 0.1 m

Fig.6: Road bump with height 0.1 m[5]

Now since the road input is being decided various parameters such as spring stiffness, shocker absorber's damping values, sprung mass values and vehicle speed are varied and their effect on road holding and ride comfort is studies.

Iteration 1: Taking the input parameters as: [5]

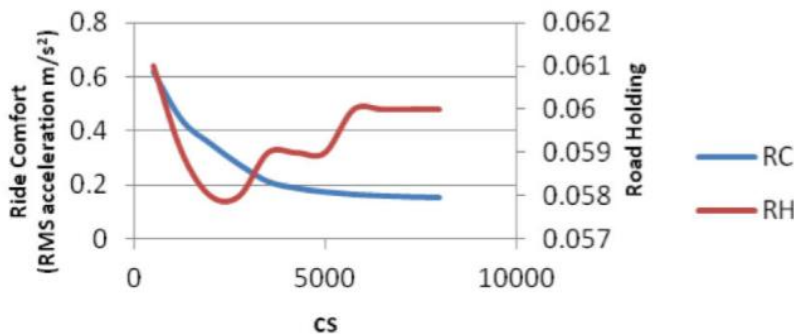| sprung mass(ms)	300kg
unsprung mass(mu)	45kg
suspension stiffness(ks)	5700 N/m
tire stiffness(kt)	57000 N/m
velocity(v)	40kmph
damper coefficient(c)	500 N-s/m to 8000N-s/m

Fig. 7: Effect of cs on road holding and ride comfort. [5]

Now from the above graph it can be concluded that the road holding values decreases initially and as the damper coefficient is increased its value goes on increasing. The RMS value of acceleration is seen to be decreasing, thus giving the maximum value of ride comfort at a damper co-efficient of 8000 N-s/m.

Iteration 2: The Input Conditions are as follows: [5]

sprung mass(ms)	300kg
unsprung mass(mu)	45kg
damper coefficient(cs)	3500N-s/m
tire stiffness(kt)	57000 N/m
velocity(v)	40kmph
Suspension stiffness (ks)	2.5 to 12kN/m

Here all the other parameters are kept constant but the spring stiffness of the spring attached to the sprung mass is varied and its corresponding effect on road holding and ride comfort is studied.
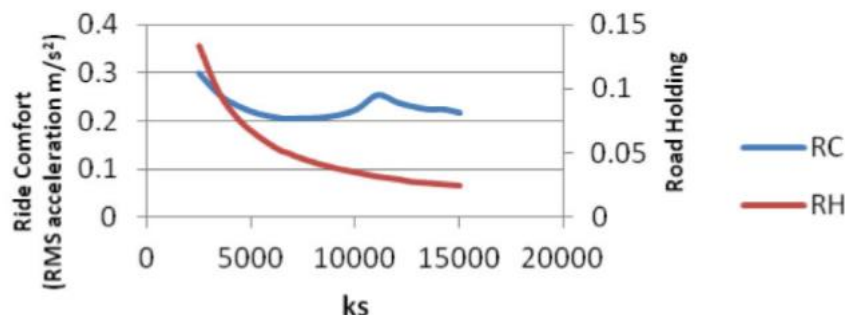

Fig. 8: Effect of ks on road holding and ride comfort [5]

As observed from the above graph, as the value of spring stiffness is increased the road holding of the vehicle keeps on decreasing, thus affecting stability. On the other hand, the value of road holding initially increases, then decreases and then again increases. The maximum value of ride comfort is obtained at 7000 N-m.

Iteration 3: The input conditions are as follows: [5]

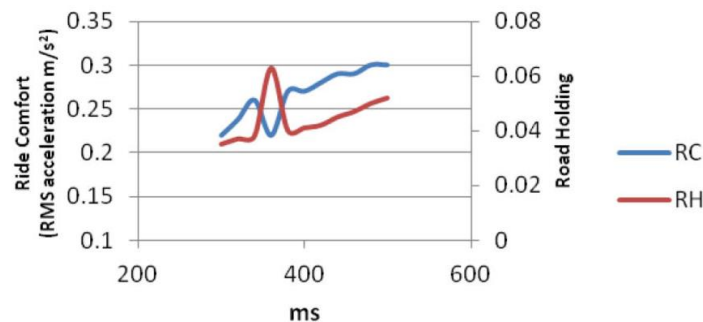| unsprung mass(mu)	45kg
damper coefficient(cs)	3500N-s/m
sprung stiffness(ks)	10,000N/m
velocity(v)	40kmph
Tire Stiffness(kt)	100,000 N/m
Sprung mass(ms)	300 to 500 kg

Fig. 9: Effect of ms on road holding and ride comfort [5]

From the above graph, as the sprung mass increases the value of road holding initially increases, suddenly decreases and then gradually increases. The maximum road holding value is obtained when the sprung mass value is around 370. The RMS value of acceleration first increases then decreases then again increases. Thus, the ride comfort first decreases then increase then again decreases. The maximum ride comfort is obtained at the value of sprung mass as 370 kg.

Iteration 4: The input conditions are as follows:

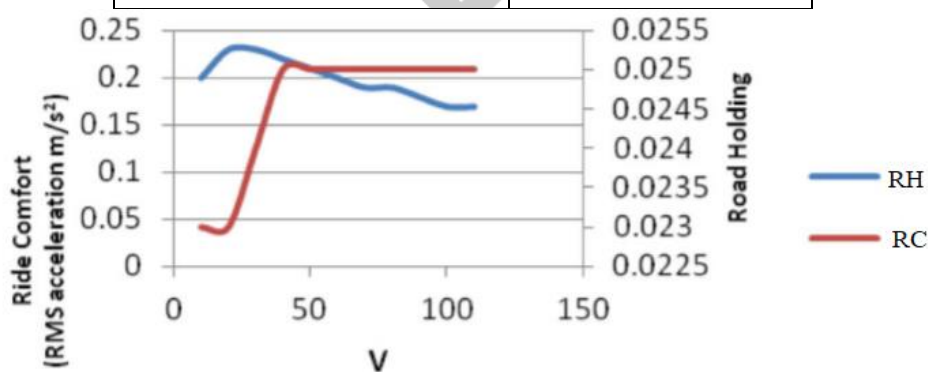| unsprung mass(mu)	45kg
damper coefficient(cs)	3500N-s/m
sprung stiffness(ks)	10,000N/m
Sprung mass(ms)	300 kg
Tire Stiffness(kt)	100,000 N/m
velocity(v)	10 to 100 kmph

Fig. 10: Effect of v on road holding and ride comfort [5]

From the above figure it is evident that the road holding of the vehicle increases initially for a small window and takes a maximum value at 20kmph and after that the value of road holding is seen to be decreasing. After attaining the speed of 45kmph the effective reduction of road holding value becomes very slow. The ride comfort of the vehicle is observed to be good at low speed with a

maximum value at 25kmph and then the ride comfort is seen to decrease. After 45kmph, the value of ride comfort is approximately constant. The maximum rms acceleration is 0.21 m/s^2 which is below 0.315 m/s^2, hence within human comfort zone.

CASE 2: Earlier we had considered a road input with single bump, now let us consider the case where there are multiple bumps: The sinusoidal bump with frequency of 8 HZ, 4 HZ and 8 HZ has been characterized by,

$$r(t) = \begin{cases} \dfrac{a(1-\cos 8\pi t)}{2}, & \begin{array}{l} 0.5 \le t \le 0.75, \\ 3 \le t \le 3.25, \\ 5 \le t \le 5.25 \end{array} \\ 0, & \text{otherwisw} \end{cases} \quad [4]$$

Where, a= 0.05 (road bump height is 10cm, 5 cm and 10 cm).
The below figure shows the multiple bump road input profile that is fed as input to the simulation of quarter car model.
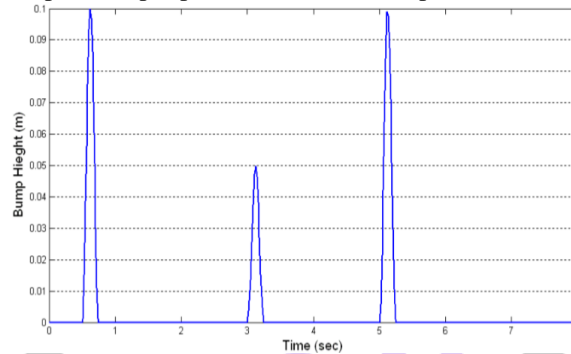

Fig. 11: Road profile with multiple bumps.[4]

The various other input parameters considered for the model are:

Body mass	290 kg
Suspension mass	59 kg
Spring constant of suspension system	16182 N/m
Spring constant of wheel and tire	190000 N/m
damping constant of suspension system	1000 Ns/m

The comparisons are done both for passive as well as active suspension in the below graphs but we will consider the results of passive suspension for our study:
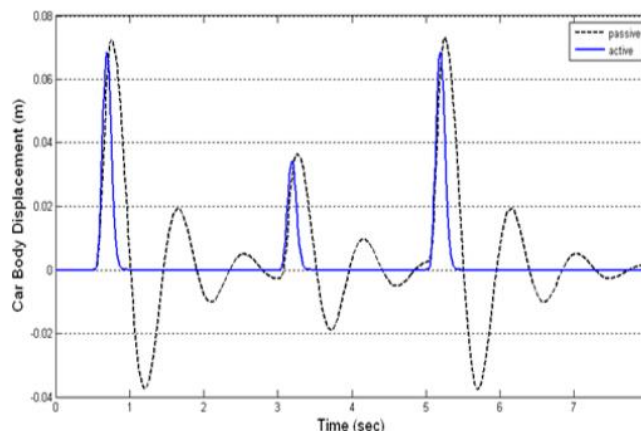

Fig. 12: Car body displacement vs time [4]

The above figure signifies the car body displacement to the above road input which reaches a maximum value of approximately 0.068m. It is also observed that the displacement amplitude increases at the point where it again encounters a bump. The damping of the displacement is relatively slow as the displacement amplitude fades down gradually.
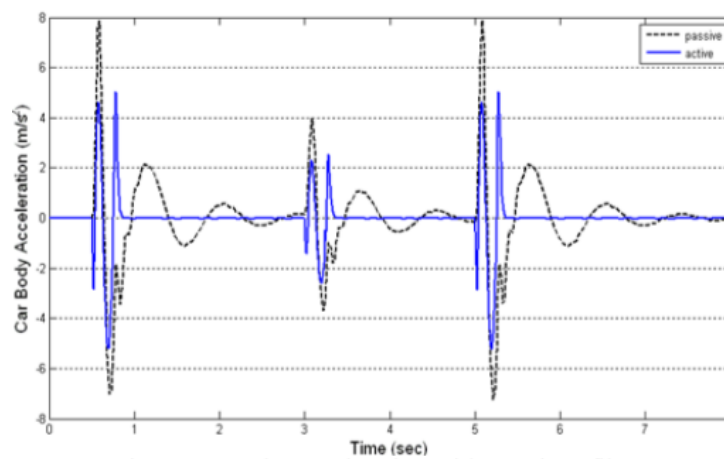
Fig. 13: Car body acceleration vs time [4]

Now, comparing the car body acceleration, it can be seen that the value shoots up par the human comfort acceleration range, which needs to be damped further if a good ride comfort is desired. The maximum value shoots up to as high as $8 m/s^2$ which is highly undesirable. The peak values here again are obtained at the same points where there are bumps.
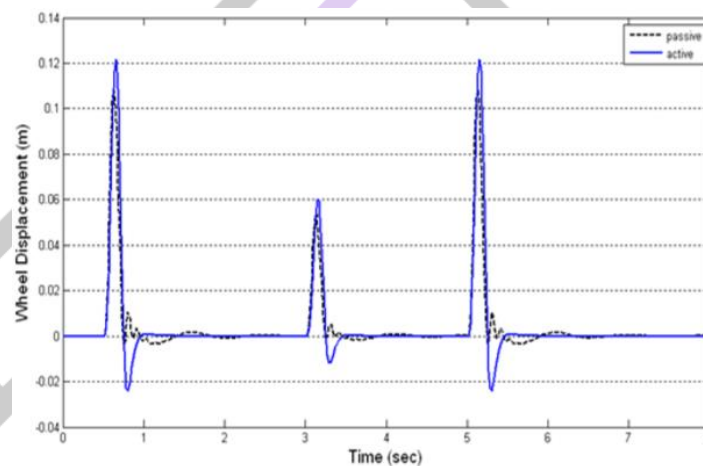

Fig. 14: Wheel displacement vs time. [4]

As seen from the graph as the wheel displacement reaches a peak value of .12 m for a bump input of 0.1m, thus affecting the road holding of the vehicle. The wheel displacement should be as minimum as possible to ensure better road holding.

IV. CONCLUSION

According to ISO 2631-1: 1997, the range of RMS value of vertical acceleration of the vehicle for human comfort is defined as follows:

Less than 0.315 m/s^2	Not uncomfortable
0.315 m/s^2 to 0.63 m/s^2	A little uncomfortable
0.5 m/s^2 to 1 m/s^2	Fairly uncomfortable
0.8 m/s^2 to 1.6 m/s^2	Uncomfortable
1.25 m/s^2 to 2.5 m/s^2	Very uncomfortable
Greater than 2 m/s^2	Extremely uncomfortable

From the above case study, it is observed that the rms value of acceleration is within the human comfort range of 0.315 m/s^2. Hence, it provides appropriate ride comfort. The value of road holding changes for all the cases by showing increasing trend for increasing damping co-efficient value, decrease on increase spring stiffness, fluctuations with sprung mass values and decrease with velocity increase. Hence, the above observations help in understanding ride comfort and road holding behavior of a vehicle under dynamic

conditions and varying parameters. The case study 2 gives the graph of car body displacement, acceleration and wheel displacement. A lower car body acceleration and wheel displacement is thus desirable for good ride comfort and road holding.

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