Stability Analysis of Delay Evolution and Stochastic Evolution Equations

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Abstract: **In this paper we are going to discuss about stability analysis of delay evolution equation. Here we examine the stability of evolution equation in the field of partial differential equation. Then we elongate our analysis to find the stability condition of stochastic evolution equation.**

Keywords: **Stability, Evolution Equation, Stochastic Evolution Equation, Nonlinear Partial Differential Equation.**

1. INTRODUCTION

Stability is the most important concept in the study of control systems. It plays a predominant role in control theory. The concept of stability was proposed in the nineteenth century by the Russian analyst Alexander Mikhailovich Lyapunov. The concepts of stability given by Lyapunov are a key points for solving the stability problems till now.

Lyapunov concepts can be applied to find the stability conditions of various types of differential equations. The equations include dynamical systems, ordinary differential equations, partial differential equations, functional differential equations, dynamical systems and stochastic differential equations.

Lyapunov stability gives the accurate solution of the given system. This concept gives solution which originate and terminates at the same point. Lyapunov stability which determines the actual solution of the given system.

Lyapunov stability method can also be used to solve more difficult problems of dynamical systems. The concepts like bifurcation, turbulence, hopf bifurcation and chaos are very difficult to solve using normal methods. This concepts can also be solved using the Lyapunov stability techniques.

The differential equations also involves time delay systems. Time delay systems belong to the family of functional differential equation (i.e. partial differential equations). It is also known as hereditary system. Delay differential equation is a predominant class of dynamical systems.

We discuss about some preliminaries and some stability analysis of delay evolution equation. And also we find the stability of stochastic evolution equation.

2. PRELIMINARIES

DEFINITION: 1

"The zero solution of equation

$$\left.\begin{aligned} &\frac{dv(t)}{dt} = B\big(t, v(t)\big) + g_1(t, v_t) + g_2(t, v_t), && t \geq 0 \\ &v(s) = \varphi(s), && s \in [j, 0] \qquad (i) \end{aligned}\right\}$$

is called as stable if $\forall\ \varepsilon > 0\ \exists\ \delta > 0 \ni$

$$|v(t{:}\,\varphi)| < \varepsilon \quad \forall \quad t \geq 0,$$

If $\quad |\,\varphi|_{C_J} = Max_{s\in[-j,0]}\,|\varphi(s)| < \delta$ "

DEFINITION: 2

The zero solution of equation (i) is known as exponentially stable if it is stable and here occurs a non-negative constant $\mu \ni \forall\ \ \varphi\, \epsilon\, C(-j, 0, V)\ \forall\, C$ (it based on φ) $\ni$

$$|v(t{:}\,\varphi)| \leq Ce^{-\mu(t)} \qquad \forall \quad t > 0\,.$$

DEFINITION: 3

The zero solution of equation

$$\left.\begin{aligned} dv(t) &= B\big(t, v(t)\big) + g(t, v_t) + c(t, v_t)\, dw(t), && t \geq 0 \\ v(t) &= \varphi(t), && t \in [j, 0] \qquad (ii) \end{aligned}\right\}$$

is express to be mean square stable if $\forall\, \varepsilon > 0\ \exists\ \delta > 0 \ni$

$$E|v(t{:}\,\varphi)|^2 < \varepsilon \text{ for all } t \geq 0 \text{ if}$$

$$\|\varphi\|_{C_J}^2 = sup_{s\in(-J,0]}\ E|\varphi(S)|^2 < \delta$$

DEFINITION: 4

The zero solution of equation (ii) is express to be exponentially mean square stable and there exist a positive constant μ being that for some $\varphi \in C\ (-j, 0, v)$ there exists C (that is based on φ) being that, $E|v(t;\varphi)|^2 \leq ce^{-\mu t}$ for $t > 0$.

3. STABILITY ANALYSIS OF DELAY EVOLUTION EQUATION AND LYAPUNOV FUNCTIONAL DEVELOPMENT WITH TIME-VARYING DELAY

Notation

Consider the two actual separable Hilbert Spaces V&J. Obivously

$$J \supset V \equiv V^* \supset J^*$$

Where the one-to-one functions are dense and continuous. That is $\|.\|_*$ is the norm in V^*, $\|.\|$is the norm in V & $|\,.\,|$ is the norm in J consequently, $((.,.))$ be the scalar multiplication in V also $(.,.)$ be the scalar multiplication in J & $\langle .,. \rangle$ be the dual multiplication among $V\ and\ V^*$.

Estimate that , $|v| \leq \alpha\ \|v\|,\ \ v \in V$ (a^*)

Consider $C(-j, 0, J)$ is a complete normed linear space of all continuous function follows from $[-j, 0]$ to J, let $y_t \epsilon C(-j, 0, J)\ for\ every\ t \in [0, \infty\infty]$ is the function described by $y_t(s) = y(t+s)$ for complete $s \in [-j, 0]$. The space $C(-j, 0, V)$ is likely described.

Assume that , $B(t, .)\colon V \to V^*$,

$g_1(t, .)\colon C(-j, 0, J) \to V^*$ &

$g_2(t, .)\colon C(-j, 0, V) \to V^*$

Are the three groups of non-linear operators assigned for $t > 0$, and we have $B(t, 0) = 0$, $g_1(t, 0) = 0$ $and\ also$ $g_2(t, 0) = 0$.

Now we examine the equation,

$$\left.\begin{aligned} \frac{dv(t)}{dt} &= B(t, v(t)) + \ g_1(t, v_t)) + g_2(t, v_t), && t > 0 \\ v(s) &= \varphi(s), && s \in [-j, 0] \qquad (a) \end{aligned}\right\}$$

We indicate that $v(.;\varphi)$ is the solution of equation (a) which is equivalent to the primary condition φ.

Theorem: 1 Suppose here occurs a functional $U(t, v_t)$ corresponding to the succeeding conditions contains for any non-negative numbers K_1, K_2, & μ:

$$U(t, v_t) \geq K_1 e^{\mu(t)} |v(t)|^2, \qquad t \geq 0 \qquad (1)$$

$$U(0, v_0) \leq\ K_2 |\varphi|_{C_J}^2, \qquad (2)$$

$$\frac{d}{dt} U(t, v_t) \leq 0, \qquad t \geq 0 \qquad (3)$$

Thus the zero solution of equation (a) is said to be exponentially stable.

Proof:

From the given conditions, we integrate the equation (3) we attain ,

$U(t, v_t) \leq U(0, v_0)$.

Then it follows from the above equation and from equations (1)&(2) we get,

i.e) $U(t, v_t) \leq U(0, v_0)$

$\because U(t, v_t) \geq K_1 e^{\mu(t)} |v(t)|^2$

i.e) $K_1 e^{\mu(t)} |v(t)|^2 \leq U(0, v_0)$

$\because \ U(0, v_0) \leq \ K_2 |\varphi|_{C_J}^2$

$K_1 |v(t)|^2 \leq e^{-\mu(t)} K_2 |\varphi|_{C_J}^2$

$\leq e^{-\mu(0)} K_2 |\varphi| \qquad \because e^{(0)} = 1$

$K_1 |v(t)|^2 \leq \ K_2 |\varphi|_{C_J}^2$

This gives that the zero solution of our differential equation (a) is stable. It is stable by the definition(1). Then it is said to be exponentially stable by the definition (2).

4. STABILITY ANALYSIS OF STOCHASTIC EVOLUTION EQUATION

Notations: Consider the actual separable Hilbert spaces V, J, L

$J \supset V \equiv V^* \supset J^*$

Here V^* is the bifold of V and the one-to-one function are dense and continuous. Next we specify the constant α satisfying the condition $|v| \leq \alpha \|v\|$, $v \in V$. Now we consider both V and V^* are both invariantly convex. Let $\|.\|$ be the norms in V, $|.|$ be the norms in J, $\|.\|_*$ be the norms in V^*. Consequently, $\langle .,. \rangle$ is the dual multiplication among V^*, V and the scalar multiplication in H is $(.,.)$. Now certify $W(t)$ be a p-valued Wiener process (continuous time stochastic process or Brownian motion process) on a few complete probability space(Ω, ψ, Q) that gets their values in the separable Hilbert spaces L, where $P \epsilon \psi(L, L)$ is a balanced non-negative operators and

$$EW(t) = 0$$

$$Cov(W(t) = tp$$

Let $(\psi_t)_{t \geq 0}$ be the $\Omega - algebras$ developed by $\{W(s), 0 \leq s \leq t\}$. Then W(t) harness corresponding to $(\psi_t)_{t \geq 0}$ and we represent $W(t)$ by

$$W(t) = \sum_{i=1}^{\infty} \alpha_i (t) e_i$$

i.e) The Eigen vectors of P has an orthogonal set $\{e_i\}_{i \geq 1}$. $\alpha_i(t)$ is jointly separable absolute Wiener processes with additional covariance $\mu_i > 0$, $Pe_i = \mu_i e_i$, and

$$TrP = \sum_{i=1}^{\infty} \mu_i < \infty$$

Now we consider the non-linear stochastic partial differential equations

$$\left.\begin{aligned} dv(t) &= B(t, v(t)) + g(t, v_t) + c(t, v_t)\, dw(t), && t \in [0, T] \\ v(t) &= \varphi(t), && t \in [j, 0] \end{aligned}\right\} \qquad (4)$$

Now we investigate the stability of our non-linear stochastic partial differential equation.

Theorem (2.2): suppose here occurs a functional $U(t, v_t)$ corresponding to the succeeding conditions contains for any positive numbers K_1, K_2 and μ:

$$EU(t, v_t) \geq k_1 e^{\mu t} t |v(t)|^2, t \geq 0 \qquad (5)$$

$$EU(0, v_0) \geq k_2 \|\varphi\|_{C_J}^2 \qquad (6)$$

$$EMU(t, v_t) \leq 0, \qquad t \geq 0 \qquad (7)$$

Where M is the operator of equation, $V(t) = \varphi(t),\ t \in [-j, 0]$. Later that the zero solution of our equation is mean square and exponentially mean square stable.

Proof: From the given condition, we integrate the equations (7) we attain

$$EU(t, v_t) \leq EU(0, v_0)$$

Then it follows from the above equation and from (5) & (6) we get,

$$EU(t, v_t) \leq EU(0, v_0)$$

$$K_1 e^{\mu t} |v(t)|^2 \leq EU(0, v_0)$$

$$K_1 |v(t)|^2 \leq e^{-\mu t} EU(0, v_0)$$

$$\leq e^{-\mu t} K_2 |\varphi|^2_{c_J}$$

$$\leq K_2 |\varphi|^2_{c_J}$$

$$\therefore\ K_1 E|v(t)|^2 \leq e^{-\mu t}\ EU(0, v_0)$$

$$\leq K_2 \|\varphi\|^2_{C_J}.$$

Then the inequality, $E|v(t)|^2 \leq K_2 \|\varphi\|^2_{C_J}$.

Greedy that the trivial zero solution of our stochastic differential equation of (4) is stable. We say it is mean square stable by the definition (3). Then it is said to be exponentially mean square stable by the definition (4).

Then the inequality,

$$K_1 E|v(t)|^2 \leq e^{-\mu t}\ EU(0, v_0)$$

is being that the trivial zero solution of equation of (4) is exponentially mean square stable. Therefore we proved the stability analysis for our stochastic partial differential equations.

CONCLUSION

In this paper we discussed about stability analysis of delay evolution equation and we diminished this evolution equation by using various stages of Lyapunov functional development with time-varying delay. Finally, we extended our analysis by finding the stability condition for stochastic evolution equation.

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