# V-Super(a,d) Vertex AntiMagic Labeling 

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#### Abstract

The proposal of this paper to familarize the concept of V-Super(a,d) Vertex AntiMagic Labeling of some graphs such as Path $P_{s}$, Cycle $C_{s}$, where $s$ is odd and even are secured .And also we prove that the theorems and examples for Petersen graph ,Hararay graphs.

Index Terms: Antimagic labeling,(a,d)-vertex Antimagic Labeling, Super (a,d)-vertex Antimagic Labeling,V-Super (a,d)vertex Antimagic Labeling Petersen graph ,Hararay graphs.


## I. INTRODUCTION:

Leonard Euler was developed a Graph theory in 1736. He solve the Konigsberg problem of pass over the seven bridges on the river in Russia and he check either it is available to cross all the bridges. It is honest to say that the prescribed and graphical representation in Graph theory. Graph theory has many real world application like mobile phone network, data base design, data mining and operating system. Antimagic graph was first introduced by G.Ringel. (a,d) -antimagic labeling is known as the weight of all vertices in graph $G$ is in arithmetic progression for all $a>0$ and $d \geq 0$ for fixed integers. Bodendiek and Walther was introduced a concept of (a,d)-antimagic labeling.Super(a,d)-vertex antimagic labeling (V$\operatorname{Super}(\mathrm{a}, \mathrm{d})$-vertex antimagic labeling) is called the vertices receives(vertex weight) the smallest possible values $\{1,2,3, \ldots ., \mathrm{v}\}$. It was introduced by Macdougall and Martin Baca.

## II. PRELIMINARIES:

Definition 1.1: A graph whose edges are labeled in distinct elements such that the sum of the edge-labels at every vertex in a graph is different is known as Antimagic graph.

Definition 1.2: A mapping $1-1$ and onto $\mathrm{g}: \mathrm{VUE} \rightarrow\{1,2,3, \ldots \ldots \mathrm{v}+\mathrm{e}\}$ of a graph G if the set of vertex weight of all vertices in $G$ is in arithemetic progression $\{a, a+d, a+2 d, \ldots \ldots a+(n-1) d\}$, where $a>0$ and $d \geq 0$ are two fixed integers is known as (a,d)-vertex antimagic total labeling or (a,d)-VAML.

Definition 1.3: An (a,d) -Vertex Antimagic total labeling if the vertices receives starts from 1 onwards (i.e) $\mathrm{g}(\mathrm{V})=\{1,2,3, \ldots \ldots, \mathrm{~V}\}$ is known as $\operatorname{Super}(\mathbf{a}, \mathrm{d})-V A M L$.

Definition 1.4: A (a,d)-vertex antimagic total labeling in which the vertices are labeled in smallest possible integers are known as V-Super (a,d)- Vertex Antimagic Labeling or (V-SVAML).

## III. V-SUPER(A,D) VERTEX ANTIMAGIC LABELING:

Theorem 2.1: If path $P_{s}$ is V-SVAML.
Proof:Define the labeling $g: V \cup E \rightarrow\{1,2,3, \ldots .2 s-1\}$.The vertex labeling are $g\left(b_{1}\right)=s, g\left(b_{r}\right)=r-1, r=2,3, \ldots .$. .The edge labeling are $g\left(e_{r}\right)= \begin{cases}\frac{3 s+r}{2}, & r=\text { odd } \\ \frac{r}{2}+s, & r=\text { even }\end{cases}$

The arithemetic progression $\mathrm{a}, \mathrm{a}+\mathrm{d}, \ldots \ldots, \mathrm{a}+(\mathrm{s}-1) \mathrm{d}$. put $\mathrm{a}=\frac{5 s-3}{2}, \mathrm{a}+\mathrm{d}=\frac{5 s+1}{2}, \ldots$ with difference $\mathrm{d}=2$ when s is odd.Hence, path $P_{s}$ is $\quad \mathrm{V}-\operatorname{Super}(\mathrm{a}, \mathrm{d})-\mathrm{VAML}$.

Example: Put $m=9$. The vertex labeling are $g\left(b_{1}\right)=9, g\left(b_{2}\right)=1, g\left(b_{3}\right)=2, g\left(b_{4}\right)=3, g\left(b_{5}\right)=4, g\left(b_{6}\right)=5, g\left(b_{7}\right)=6, g\left(b_{8}\right)=7$, $\mathrm{g}\left(\mathrm{b}_{9}\right)=8$. The edge labeling are $\mathrm{g}\left(\mathrm{e}_{1}\right)=14$,
$g\left(e_{2}\right)=10, g\left(e_{3}\right)=15, g\left(e_{4}\right)=11, g\left(e_{5}\right)=16, g\left(e_{6}\right)=12, g\left(e_{7}\right)=17, g\left(e_{8}\right)=13 . a=21, d=2$


Example:Put $m=9$. The vertex labeling are $g\left(b_{1}\right)=9, g\left(b_{2}\right)=8, g\left(b_{3}\right)=7, g\left(b_{4}\right)=6, g\left(b_{5}\right)=5, g\left(b_{6}\right)=4, g\left(b_{7}\right)=3, g\left(b_{8}\right)=2$, $g\left(b_{9}\right)=9$.The edge labeling are $g\left(e_{1}\right)=16, g\left(e_{2}\right)=17, g\left(e_{3}\right)=14, g\left(e_{4}\right)=15, g\left(e_{5}\right)=12,\left(e_{6}\right)=13, g\left(e_{7}\right)=10, g\left(e_{8}\right)=11 . a=17, d=3$


Theorem 2.2: A cycle $c_{s}$ is $V-\operatorname{Super}(\mathrm{a}, \mathrm{d})$ VAML for all $s$.
Proof:Define the labeling $\mathrm{g}: \mathrm{V} \cup \mathrm{E} \rightarrow\{1,2,3, \ldots .2 \mathrm{~s}\}$. The vertex labeling are, $\mathrm{g}\left(\mathrm{b}_{\mathrm{r}}\right)=\mathrm{r}, \mathrm{r}=1,2,3, \ldots . \mathrm{s}$. The edge labeling are, $\mathrm{g}\left(\mathrm{e}_{\mathrm{r}}\right)=2 \mathrm{~s}-\mathrm{r}+1, \mathrm{r}=1,2,3, \ldots$. sThe arithemetic progression $\mathrm{a}, \mathrm{a}+\mathrm{d}, \ldots \ldots, \mathrm{a}+(\mathrm{s}-1) \mathrm{d}$. put $\mathrm{a}=3 \mathrm{~s}+2, \mathrm{a}+\mathrm{d}=3 \mathrm{~s}+3, \ldots$ with difference $\mathrm{d}=1$. Hence, cycle $C_{s}$ is V - $\operatorname{Super}(\mathrm{a}, \mathrm{d})$-VAML.

Example: Case:1: Put $s=8$. The vertex labeling are $g\left(b_{1}\right)=1, g\left(b_{2}\right)=2, g\left(b_{3}\right)=3, g\left(b_{4}\right)=4, g\left(b_{5}\right)=5, g\left(b_{6}\right)=6, g\left(b_{7}\right)=7, g\left(b_{8}\right)=8$. The edge labeling are $g\left(e_{1}\right)=16, g\left(e_{2}\right)=15, g\left(e_{3}\right)=14, g\left(e_{4}\right)=13, g\left(e_{5}\right)=12, g\left(e_{6}\right)=11, g\left(e_{7}\right)=10, g\left(e_{8}\right)=9$ The vertex weight with A.P are $\mathrm{a}=26$ with difference $\mathrm{d}=1$. Hence, cycle $C_{8}$ is V - Super $(26,1)$-VAML when $s$ is even.

Case: 2: Put $s=9$ The vertex labeling are $g\left(b_{1}\right)=1, g\left(b_{2}\right)=2, g\left(b_{3}\right)=3, g\left(b_{4}\right)=4, g\left(b_{5}\right)=5, g\left(b_{6}\right)=6, g\left(b_{7}\right)=7, g\left(b_{8}\right)=8, g\left(b_{9}\right)=9$. The edge labeling are $g\left(e_{1}\right)=18, g\left(e_{2}\right)=17, g\left(e_{3}\right)=16, g\left(e_{4}\right)=15, g\left(e_{5}\right)=14, g\left(e_{6}\right)=13, g\left(e_{7}\right)=12, g\left(e_{8}\right)=11, g\left(e_{9}\right)=10$.The vertex weight with A.P are $\mathrm{a}=29$ with difference $\mathrm{d}=1$ Hence, cycle $C_{9}$ is $\mathrm{V}-\operatorname{Super}(29,1)$-VAML when s is odd.

$C_{8}:(26,1)$

$C_{9}:(29,1)$

Corollary: 2.3 A cycle $c_{5}$ is $\mathrm{V}-\operatorname{Super}(15,2)$ VAML.


Definition 2.4: A graph has 2 n -vertices and 3 n -edges is known as Petersen graph.
Theorem 2.5:If $s$ is odd then the Petersen graph is V-Super (a,d)VAML.

Proof: Define the labeling $\mathrm{g}: \mathrm{V} \cup \mathrm{E} \rightarrow\{1,2,3, \ldots .5 \mathrm{r}\}$. Take two cycles one is interior and another cycle is exterior. The vertex labeling of interior are , $\mathrm{g}\left(\mathrm{b}_{\mathrm{r}}\right)=\mathrm{r}+1, \mathrm{r}=1,2,3, \ldots \mathrm{~s}-1$. The edge labeling of interior are , $g\left(e_{r}\right)=\left\{\begin{array}{cc}4 s-\frac{r}{2} & , r=\text { even } \\ \frac{7 s-r}{2}, & r=\text { odd }\end{array}\right.$

The vertex labeling of exterior are, $\mathrm{g}\left(\mathrm{c}_{\mathrm{r}}\right)=\mathrm{r}+1, \mathrm{r}=1,2,3, \ldots . \mathrm{s}-1$.
The edge labeling of exterior are,

$$
\mathrm{g}\left(\mathrm{e}_{\mathrm{r}}\right)=\left\{\begin{array}{lc}
3 \mathrm{~s}-\frac{\mathrm{r}}{2} & , \mathrm{r}=\text { even } \\
\frac{5 s-\mathrm{r}}{2}, & \mathrm{r}=\text { odd }
\end{array}\right.
$$

The labeling of spokes are $g\left(b_{r} c_{r}\right)=4 s+r+1$. The vertex weight of the exterior and interior cycle $a=\frac{21 s+5}{2}$, $a=\frac{23 s+5}{2}$ with $d=1$.Hence, The Petersen graph is V-Super ( $\mathrm{a}, 1$ )VAML.

Example: Put $\mathrm{s}=7$. Define the labeling $\mathrm{g}: \mathrm{V} \cup \mathrm{E} \rightarrow\{1,2,3, \ldots . .35\}$. Take two cycles one is interior and another cycle is exterior. The vertex labeling of interior are, $g\left(b_{0}\right)=1, g\left(b_{1}\right)=2, g\left(b_{2}\right)=3, g\left(b_{3}\right)=4, g\left(b_{4}\right)=5, g\left(b_{5}\right)=6, g\left(b_{6}\right)=7$. The edge labeling of interior are, $\mathrm{g}\left(\mathrm{e}_{1}\right)=24, \mathrm{~g}\left(\mathrm{e}_{2}\right)=27, \mathrm{~g}\left(\mathrm{e}_{3}\right)=23, \mathrm{~g}\left(\mathrm{e}_{4}\right)=26, \mathrm{~g}\left(\mathrm{e}_{5}\right)=22, \mathrm{~g}\left(\mathrm{e}_{6}\right)=25, \mathrm{~g}\left(\mathrm{e}_{7}\right)=22$. The vertex labeling of interior are, $g\left(c_{0}\right)=8, g\left(c_{1}\right)=9, g\left(c_{2}\right)=10, g\left(c_{3}\right)=11, g\left(c_{4}\right)=12, g\left(c_{5}\right)=13, g\left(c_{6}\right)=14$. The edge labeling of interior are, $g\left(e_{1}\right)=17, g\left(e_{2}\right)=20$, $\mathrm{g}\left(\mathrm{e}_{3}\right)=16, \mathrm{~g}\left(\mathrm{e}_{4}\right)=19, \mathrm{~g}\left(\mathrm{e}_{5}\right)=15, \mathrm{~g}\left(\mathrm{e}_{6}\right)=18, \mathrm{~g}\left(\mathrm{e}_{7}\right)=21$. The spokes are, $\mathrm{g}\left(\mathrm{b}_{0} \mathrm{c}_{0}\right)=29, \mathrm{~g}\left(\mathrm{~b}_{1} \mathrm{c}_{1}\right)=30, \mathrm{~g}\left(\mathrm{~b}_{2} \mathrm{c}_{2}\right)=31, \mathrm{~g}\left(\mathrm{~b}_{3} \mathrm{c}_{3}\right)=32$, $g\left(b_{4} c_{4}\right)=33, g\left(b_{5} c_{5}\right)=34, g\left(b_{6} c_{6}\right)=35$. The vertex weight of $a=76$ with difference $d=1$. The Petersen graph V-Super $(76,1)$ VAML.


Definition: 2.6 A graph is constructed from a cycle of $n$-vertices joined by any two vertices is known as Harary graph.

Corollary 2.7: The Hararay graph $H_{k}^{s}$ is V-Super (a,d)VAML.
Proof:The labeling of vetices and edges are denoted $\mathrm{a}_{\mathrm{r}}: 1 \leq \mathrm{r} \leq \mathrm{k}$ and $\mathrm{a}_{\mathrm{r}} \mathrm{a}_{\mathrm{r}+1} \cup \mathrm{a}_{\mathrm{r}} \mathrm{a}_{\mathrm{r}+\mathrm{s}}: 1 \leq \mathrm{r} \leq \mathrm{k}, \mathrm{g}\left(\mathrm{a}_{\mathrm{r}}\right)=\mathrm{r}, 1 \leq \mathrm{r} \leq \mathrm{k}$,
$g\left(a_{r} a_{r+1}\right)=\left\{\begin{array}{l}2 k, r=k \\ 2 k-1,1 \leq r \leq k-1\end{array} g\left(a_{r} a_{r+s}\right)=\left\{\begin{array}{c}2 k+1, r=k \\ 2 k+2+\frac{2 r-1}{2}, 1 \leq r \leq k-1\end{array}\right.\right.$
The vertex weight of A.P are $a=8 k+3$ with difference $d=1$. Hence, Hararay graph $H_{k}^{s}$ is V-Super(a,d)VAML.
Example : Put $s=4, k=9$. The vertex labeling are, $g\left(a_{1}\right)=1, g\left(a_{2}\right)=2, g\left(a_{3}\right)=3, g\left(a_{4}\right)=4, g\left(a_{5}\right)=5, g\left(a_{6}\right)=6, g\left(a_{7}\right)=7, g\left(a_{8}\right)=8$, $g\left(a_{9}\right)=9$.

The edge labeling are, $g\left(a_{1} a_{2}\right)=17$, $g\left(a_{2} a_{3}\right)=16, g\left(a_{3} a_{4}\right)=15$, $g\left(a_{4} a_{5}\right)=14, g\left(a_{5} a_{6}\right)=13, g\left(a_{6} a_{7}\right)=12, g\left(a_{7} a_{8}\right)=11, g\left(a_{8} a_{9}\right)=10$, $g\left(a_{9} a_{10}\right)=18, g\left(a_{1} a_{5}\right)=20, g\left(a_{2} a_{6}\right)=21, g\left(a_{3} a_{7}\right)=22, g\left(a_{4} a_{8}\right)=23, g\left(a_{5} a_{9}\right)=24, g\left(a_{6} a_{10}\right)=25, g\left(a_{7} a_{11}\right)=26, g\left(a_{8} a_{12}\right)=27, g\left(a_{9} a_{13}\right)=19$. The vertex weight of A.P are $a=75$ with difference $d=1$.


Hence, Hararay graph is V-Super $(75,1)$ VAML.

## CONCLUSION:

In this paper we discussed the concept of V - super (a,d)-vertex antimagic for Path $\mathrm{P}_{\mathrm{s}}$, Cycle $\mathrm{C}_{\mathrm{s}}$ when is odd and even . Also we proved the theorems and examples for Petersen graphs,Hararay graph.

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