Intuitionistic Fuzzy Soft Matrix Theory in Medical Diagnosis using Min-Max Average Composition Method

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***Abstract*: In this paper, a new technique which was named as Intuitionistic fuzzy min-max average composition method is proposed to construct the decision method for medical diagnosis using different types of Intuitionistic fuzzy soft matrices and its operation.**

***Keywords*: Fuzzy soft sets, Intuitionistic fuzzy soft sets, Intuitionistic fuzzy soft matrix, Intuitionistic fuzzy Min-Max Average Composition Method.**

1. Introduction

Soft set theory was initiated by Russian researcher Molodtov [1]; he proposed soft set as a completely generic mathematical tool for modeling uncertainties. Maji et al. [2,3] applied this theory to several directions for dealing with the problems in uncertainty and imprecision. Pei and Miao [4] and Chen et al. [5] improved the work of Maji et al. Yong et al [6] initiated a matrix representation of a fuzzy soft ser and applied it in decision making problems. Borah et al [7] and in Neog et al [8] extended fuzzy soft matrix theory and its application. Chetia et al [9] propsed Intuitionistic fuzzy soft matrix theory Rajarajeswari et al [10,11,12] proposed new definitions for Intuitionistic fuzzy soft matrices and its types.

In real life most of the existing mathematical tools for formal modeling, reasoning and computing are crisp, deterministic and precise in nature. The classical crisp mathematical tools are not capable of dealing with problems in uncertainty and imprecision. There are many mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory, interval mathematics etc. Probability theory is applicable only for a stochastically stable system. Interval mathematics is not sufficiently adaptable for problems with different uncertainties. Setting the membership function value is always been a problem in fuzzy set theory. Intuitionistic Fuzzy Soft Set theory (IFSS) may be more applicable in uncertainty and imprecision. The parameterization tool of fuzzy soft set theory enhances the flexibility of its applicable.

In this paper, a new approach is proposed to construct the decision method for medical diagnosis by using Intuitionistic fuzzy soft matrices. In order to make this union, intersection and the complement of a Intuitionistic Fuzzy Soft matrices are applied. The result is obtained based on the maximum value in the score matrix. We apply Intuitionistic fuzzy soft set theory to develop a technique through Sanchez's method [13,14] to diagnose which patient is suffering from what disease.

2. Preliminaries

Definition 2.1

Suppose that U is an initial universe of discourse and E is a set of parameters, let $P(U)$ denotes the power set of U. A pair (F,E) is called a soft set over U where F is a mapping given by $F: E \to P(U)$. Clearly, a Soft set is a mapping from parameters to $P(U)$, and it is not a set, but a parameterized family of subsets of the universe.

Definition 2.2

Let U be an initial universe of discourse and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by $F: A \to I^U$, where F is a mapping given by $F: A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3

Let U be an initial universe of discourse and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called Intuitionistic fuzzy soft set over U where F is a mapping given by $F: A \to IF^U$ where IF^U denotes the collection of all Intuitionistic fuzzy subsets of U.

Definition 2.4

Let $U = \{c_1, c_2, c_3, ...c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ...e_n\}$. Let $A \subseteq E$ and (F, A) is called fuzzy soft set in the fuzzy soft class (U, E). Then fuzzy soft set (F, A) in a matrix form as $A_{m\times n} = [a_{ij}]_{m\times n}$ or $A = [a_{ij}]$, i = 1, 2, …m, j = 1,2,..,n, where

$$a_{ij} = \begin{cases} (\mu_j(c_i), \nu_j, (c_i)) & , e_j \in A \\ (0,1) & , e_j \notin A \end{cases}$$

$\mu_j(c_i)$ represents the membership of c_i in the Intuitionistic fuzzy set $F(e_j)$.

$\nu_j(c_i)$ represents the non- membership of c_i in the Intuitionistic fuzzy set $F(e_j)$.

Definition 2.5

If $A = [a_{ij}] \in IFSM_{m\times n}, B = [b_{ij}] \in IFSM_{m\times n}$, then we define the addition and subtraction of Intuitionistic fuzzy soft matrices of A and B as

$$A + B = \{\min[\mu_A(a_{ij}), \mu_B(b_{ij})], \max[\nu_A(a_{ij}), \nu_B(b_{ij})]\} \ \forall i, j$$

$$A - B = \{\max[\mu_A(a_{ij}), \mu_B(b_{ij})], \min[\nu_A(a_{ij}), \nu_B(b_{ij})]\} \ \forall i, j$$

Definition 2.6

If $A = [a_{ij}] \in IFSM_{m\times n}$, where $a_{ij} = (\mu_j(c_i), \nu_j, (c_i)) \forall i, j$. Then A^C is called a Intuitionistic Fuzzy Soft Complement Matrix if $A^C = [d_{ij}]_{m\times n}$, where $d_{ij} = (\nu_j, (c_i), \mu_j(c_i)) \forall i, j$.

Definition 2.7

If $A = [a_{ij}] \in IFSM_{m\times n}, B = [b_{jk}] \in IFSM_{n\times p}$, then the min-max composition fuzzy soft matrix relation of A and B is defined as $A * B = [c_{ik}]_{m\times p}$ where

$$c_{ik} = \{\min\{\max_i[\mu_A(a_{ij}), \mu_B(b_{jk})]\}, \max\{\min_j[\nu_A(a_{ij}), \nu_B(b_{jk})]\}\}.$$

Definition 2.8

If $A = [a_{ij}] \in IFSM_{m\times n}, B = [b_{jk}] \in IFSM_{n\times p}$, then a new operation named as Intuitionistic fuzzy min-max composition fuzzy soft matrix relation is defined as

$$A^{\phi} B = \{Min\{\frac{\mu_A(a_{ij}) + \mu_B(b_{jk})}{2}\}, Max\{\frac{\nu_A(a_{ij}), \nu_B(b_{jk})}{2}\}\} \ \forall i, j$$

Example

Consider

$$A = \begin{pmatrix} (0.7,0.2) & (0.4,0.3) \\ (0.5,0.1) & (0.6,0.4) \end{pmatrix} \text{ and } B = \begin{pmatrix} (0.6,0.2) & (0.1,0.3) \\ (0.4,0.4) & (0.5,0.4) \end{pmatrix}$$

be the two Intuitionistic fuzzy soft matrices, then the addition, subtraction, complement, Min-Max Composition and Min-Max Average Composition of Fuzzy soft matrix relations are

$$A+B=\begin{pmatrix}(0.6,0.2) & (0.1,0.3)\\(0.4,0.4) & (0.5,0.4)\end{pmatrix}$$

$$A-B=\begin{pmatrix}(0.7,0.2) & (0.4,0.3)\\(0.5,0.1) & (0.6,0.4)\end{pmatrix}$$

$$A*B=\begin{pmatrix}(0.6,0.2) & (0.4,0.3)\\(0.5,0.2) & (0.5,0.3)\end{pmatrix}$$

$$A^C=\begin{pmatrix}(0.2,0.7) & (0.3,0.4)\\(0.1,0.5) & (0.4,0.6)\end{pmatrix}$$

Definition 2.9

If $A=[a_{ij}]\in IFSM_{m\times n}$, $B=[b_{jk}]\in IFSM_{m\times n}$, and A^C, B^C are the complement then the score matrix of A and B is defined as $S(A,B)=[V-W]$ where V is the matrix defined as $V=[\mu(A^{\phi}B)-\nu(A^{\phi}B)]$ and where W is the matrix defined as $W=[\mu(A^{c^{\phi}}B^c)-\nu(A^{c^{\phi}}B^c)]$.

3. Intuitionistic Fuzzy Min-Max Average Composition Method for Decision Making in Medical Diagnosis

In this section an application of intuitionistic fuzzy set theory using Min-Max average composition method for decision making is presented. In a given set of system, Let $P=\{P_1,P_2,....P_m\}$ be the set of m patients and $S=\{S_1,S_2,....S_m\}$ be the set of n symptoms and $D=\{D_1,D_2,....D_m\}$ be the set of k diseases.

Construct an Intuitionistic fuzzy soft set relation matrix A called patient symptoms matrix (F,S) over P where F is a mapping $F:S\to IF^P$, IF^P is the collection of all Intuitionistic fuzzy subsets of P. Then construct another Intuitionistic fuzzy soft set relation matrix (weighted matrix) B, called symptom-disease matrix, which is a collection of an approximate description of patient symptoms in the hospital (G,D) over S, where G is a mapping $G:D\to IF^S$, IF^S is the collection of all intuitionistic fuzzy subsets of S. In which each element denotes the weight of the symptoms for a certain disease.

From the matrices A and B corresponding to the Intuitionistic fuzzy soft sets (F,E) and (G,E) and compute the complements $(F,E)^c$ and $(G,E)^c$ and their matrices A^c and B^c corresponding to $(F,E)^c$ and $(G,E)^c$ respectively.

Compute $A^{\phi}B$ and $A^{c^{\phi}}B^c$ which is the minimum membership and maximum non membership of symptoms of the diseases using definition (2.8).

Compute $A^{\phi}B$, $A^{c^{\phi}}B^c$ and the Score matrix $S(A^{\phi}B, A^{c^{\phi}}B^c)$ using the definition (2.9). Finally find the maximum score for each patient P_i in the score matrix, and then conclude that the patient P_i is suffering from disease D_j.

3.1. Algorithm

Step 1: Input the Intuitionistic Fuzzy Soft Set (F,S) and (G,D) and obtain the intuitionistic matrices A and B corresponding to (F,S) and (G,D) respectively.

Step 2: Using Intuitionistic Fuzzy Soft Complement matrices obtain the matrices A^c and B^c.

Step 3: Using the definition of Intuitionistic Fuzzy Min-Max average composition, compute the values of $A^{\phi}B$ and $A^{c^{\phi}}B^c$.

Step 4: Compute the matrices V,W and obtain the score matrix $S(A^{\phi}B, A^{c^{\phi}}B^c)$ using the definition of score matrix.

Step 5: Identify the maximum score S_{ij}, for each patient P_i. Then we conclude that the P_i is suffering from disease D_j.

4. Case Study

Suppose $P=\{P_1,P_2,P_3,P_4\}$ be the four patients which was taken as universal set where P_1,P_2,P_3, and P_4 represents patients names such as George, Abdul, Karthick, Shiva. Let $S=\{S_1,S_2,S_3,S_4,S_5\}$ as the set of symptoms where S_1,S_2,S_3,S_4, and S_5 are represents Indigestion, Skin change, Weight loss, Night Sweats and Abdominal pain. Let $D=\{D_1,D_2,D_3\}$ are the possible diseases relating to the above symptoms where D_1,D_2, and D_3 represents Lung cancer, Leukemia, and Lymphoma.

Suppose that Intuitionistic Fuzzy Soft Set (F,S) over P, where F is a mapping $F:S\to IF^P$, gives a collection of an approximate description of patient symptoms in the hospital.

$$(F,S)=\begin{Bmatrix} F(S_1)=\{(P_1,0.6,0.3),(P_2,0.7,0.1),(P_3,0.5,0.4),(P_4,0.2,0.8)\} \\ F(S_2)=\{(P_1,0.8,0.2),(P_2,0.3,0.4),(P_3,0.5,0.1),(P_4,0.3,0.7)\} \\ F(S_3)=\{(P_1,0.9,0.1),(P_2,0.2,0.5),(P_3,0.5,0.3),(P_4,0.2,0.5)\} \\ F(S_4)=\{(P_1,0.1,0.8),(P_2,0.7,0.2),(P_3,0.4,0.4),(P_4,0.5,0.2)\} \\ F(S_5)=\{(P_1,0.7,0.1),(P_2,0.3,0.3),(P_3,0.6,0.2),(P_4,0.8,0.1)\} \end{Bmatrix}$$

This Intuitionistic Fuzzy Soft Set is represented by following Intuitionistic Fuzzy Soft Matrix.

$$A=\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} (0.6,0.3) & (0.8,0.2) & (0.9,0.1) & (0.1,0.8) & (0.7,0.1) \\ (0.7,0.1) & (0.3,0.4) & (0.2,0.5) & (0.7,0.2) & (0.3,0.3) \\ (0.5,0.4) & (0.5,0.1) & (0.5,0.3) & (0.4,0.4) & (0.6,0.2) \\ (0.2,0.8) & (0.3,0.7) & (0.6,0.2) & (0.5,0.2) & (0.8,0.1) \end{bmatrix}$$

This Intuitionistic fuzzy soft set complement matrix

$$A^C=\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} (0.3,0.6) & (0.2,0.8) & (0.1,0.9) & (0.8,0.1) & (0.1,0.7) \\ (0.1,0.7) & (0.4,0.3) & (0.5,0.2) & (0.2,0.7) & (0.3,0.3) \\ (0.4,0.5) & (0.1,0.5) & (0.3,0.5) & (0.4,0.4) & (0.2,0.6) \\ (0.8,0.2) & (0.7,0.3) & (0.2,0.6) & (0.2,0.5) & (0.1,0.8) \end{bmatrix}$$

Suppose that Intuitionistic Fuzzy soft set (G,D) over S, where G is a mapping $G:D\to IF^S$, gives an approximate description of Intuitionistic fuzzy soft medical knowledge of the three type of cancer and their symptoms.

$$(G,D)=\begin{Bmatrix} G(D_1)=\{(S_1,0.2,0.3),(S_2,0.1,0.8),(S_3,0.4,0.5),(S_4,0.2,0.6),(S_5,0.6,0.3)\} \\ G(D_2)=\{(S_1,0.1,0.6),(S_2,0.2,0.4),(S_3,0.2,0.1),(S_4,0.5,0.1),(S_5,0.7,0.2)\} \\ G(D_3)=\{(S_1,0.9,0.1),(S_2,0.4,0.2),(S_3,0.3,0.3),(S_4,0.5,0.2),(S_5,0.8,0.2)\} \end{Bmatrix}$$

This Intuitionistic is represented by the following Intuitionistic fuzzy soft matrix.

$$B=\begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{matrix}\begin{bmatrix} (0.2,0.3) & (0.1,0.6) & (0.9,0.1) \\ (0.1,0.8) & (0.2,0.4) & (0.4,0.2) \\ (0.4,0.5) & (0.2,0.1) & (0.3,0.3) \\ (0.2,0.6) & (0.5,0.1) & (0.5,0.2) \\ (0.6,0.3) & (0.7,0.2) & (0.8,0.2) \end{bmatrix}$$

$$B^C = \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{matrix}\begin{bmatrix} (0.3,0.2) & (0.6,0.1) & (0.1,0.9) \\ (0.8,0.1) & (0.4,0.2) & (0.2,0.4) \\ (0.5,0.4) & (0.1,0.2) & (0.3,0.3) \\ (0.6,0.2) & (0.1,0.5) & (0.2,0.5) \\ (0.3,0.6) & (0.2,0.7) & (0.2,0.8) \end{bmatrix}$$

Then Min-Max Average Composition Method matrices are

$$A^{\phi}B = \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} (0.15,0.7) & (0.3,0.45) & (0.3,0.5) \\ (0.2,0.6) & (0.2,0.4) & (0.25,0.4) \\ (0.3,0.45) & (0.3,0.5) & (0.4,0.3) \\ (0.2,0.75) & (0.15,0.7) & (0.35,0.45) \end{bmatrix}$$

$$A^{c\phi}B^c = \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} (0.2,0.65) & (0.1,0.7) & (0.15,0.75) \\ (0.2,0.45) & (0.15,0.6) & (0.1,0.8) \\ (0.25,0.6) & (0.2,0.65) & (0.15,0.7) \\ (0.2,0.7) & (0.15,0.75) & (0.15,0.8) \end{bmatrix}$$

Intuitionistic fuzzy Min-Max average composition method using the definition (2.10)

$$V = \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} -0.55 & -0.15 & -0.2 \\ -0.4 & -0.2 & -0.15 \\ -0.5 & -0.2 & 0.1 \\ -0.55 & -0.55 & -0.1 \end{bmatrix}$$

$$W = \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} -0.45 & -0.6 & -0.6 \\ -0.25 & -0.45 & -0.7 \\ -0.35 & -0.45 & -0.55 \\ -0.5 & -0.6 & -0.65 \end{bmatrix}$$

$$S(A,B) = [V - W]$$

$$S(A,B) = \begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix}\begin{bmatrix} -0.1 & 0.45 & 0.4 \\ -0.15 & 0.25 & 0.55 \\ -0.15 & 0.25 & 0.65 \\ -0.05 & 0.6 & 0.55 \end{bmatrix}$$

It is clear from the above matrix that patient P_2 and P_3 is suffering from the disease Lymphoma, P_1 and P_4 is suffering from the disease Leukemia.

Conclusion

It is show that the min-max average composition method and min-max composition method [10,11] gives the same maximum score in the score matrix of the patients and the diseases. The doctors agree that Abdul and karthick are suffered from Leukemia, whereas George and shiva are suffered from Lymphoma. Compared with conventional techniques, the proposed approach in medical diagnosis effectively reduces the repetition. As a result, our approach makes it possible to introduce weights for all symptoms and reduces the confusion about the possibility of two diseases in a patient and also it is an efficient tool for decision making problem.

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