IDEMPOTENT & COMPLEMENT LAW IN INTUITIONISTIC FUZZY SETS

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***Abstract*: In this paper, various operations in Intuitionistic Fuzzy Sets are discussed. Some theorems are proved for establishing the properties of intuitionistic fuzzy operators with respect to different intuitionistic fuzzy sets.**

***Keywords*: Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy set Operators.**

1. INTRODUCTION

L.A. Zadeh [5] introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. After the introduction of Fuzzy sets by L.A.Zadeh [5], the Fuzzy concept has been introduced in almost all branches of Mathematics. Then the concept of Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [1] as a generalization of the notation of a Fuzzy set.

2. PRELIMINARIES

Definition 2.1 - Crisp Sets:

Either the element belongs to the set or it does not is known as Crisp set.

Definition 2.2 - Fuzzy set:

A fuzzy set is built from a reference set called universe of discourse. Assume that U = $\{x_1, x_2,..., x_n\}$ is the universe of discourse, then a fuzzy set A in U (A ⊂ U) is defined as a set of ordered pairs

$$\{(x_i, \mu_A(\text{xi}))\}$$

Where $x_i \in$ U, μ_A : U → [0, 1] is the membership function of A and, μ_A(x) ∈ [0, 1] is the degree of membership of x in A.

Definition 2.3 - Intuitionistic Fuzzy Set:

An Intuitionistic Fuzzy Set A in a non empty set X is an object having the form

A= { $\langle x, \mu_A(x), \nu_A(x)\rangle | x \epsilon E\rangle$ } where the functions μ_A : X→ [0.1] and ν_A : X→ [0.1] denote the degrees of membership and non membership of the element xϵX to A respectively and satisfy 0≤$\mu_A(x) + \nu_A$ (x) ≤ 1 for all xϵX. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

Definition 2.4 - Intuitionistic Fuzzy Set Operations:

For every two IFSs A and B we define the following relations and operations:

$$A \subset B \quad \text{iff} \quad (\forall x \in E)(\mu_A(x) \le \mu_B(x) \,\&\, \nu_A(x) \ge \nu_B(x))$$

$$A \supset B \quad \text{iff} \quad (B \supset A)$$

$$A=B \quad \text{iff} \quad (\forall x \in E)(\mu_A(x) = \mu_B(x) \,\&\, \nu_A(x) = \nu_B(x))$$

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\}$$

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle / x \in E\}$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_A(x))\rangle / x \in E\}$$

$$A+B = \{\langle x, \mu_A(x)+\mu_B(x)-\mu_A(x)\mu_A(x), \nu_A(x)\nu_A(x)\rangle / x \in E\}$$

$$A \cdot B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x)\nu_B(x)\rangle / x \in E\}$$

$$A@B = \left\{\left\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2}\right\rangle / x \in E\right\}$$

$$A\$B = \{\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)}\rangle / x \in E\}$$

$$A\infty B = \left\{\left\langle x, 2.\frac{\mu_A(x)\mu_B(x)}{\mu_A(x)+\mu_B(x)}, 2.\frac{\nu_A(x)\nu_B(x)}{\nu_A(x)+\nu_B(x)}\right\rangle / x \in E\right\}$$

3. IDEMPOTENT & COMPLEMENT LAW IN INTUITIONISTIC FUZZY SETS

THEOREM 3.1:

Prove that $A \cap A = A$

PROOF:

$$A \cap A = \{\langle x, \min(\mu_A(x), \mu_A(x)), \max(\nu_A(x), \nu_A(x))\rangle / x \in E\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$$

=A

Hence it completes the proof.

THEOREM 3.2:

Prove that $A@A = A$

PROOF:

$$A@A = \left\{\left\langle x, \frac{(\mu_A(x)+\mu_A(x))}{2}, \frac{(\nu_A(x)+\nu_A(x))}{2}\right\rangle / x \in E\right\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$$

=A

Hence it completes the proof.

THEOREM 3.3:

Prove that $A\$A = A$

PROOF:

$$A\$A = \{\langle x, \sqrt{\mu_A(x)\mu_A(x)}, \sqrt{\nu_A(x)\nu_A(x)}\rangle / x \in E\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$$

=A

Hence it completes the proof.

THEOREM 3.4:

Prove that $A \infty A = A$

PROOF:

$$A \infty A = \left\{ \left\langle x, 2.\frac{\mu_A(x).\mu_A(x)}{\mu_A(x)+\mu_A(x)}, 2.\frac{\nu_A(x).\nu_A(x)}{\nu_A(x)+\nu_A(x)} \right\rangle / x \in E \right\}$$

$$= \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in E\}$$

=A

Hence it completes the proof.

THEOREM 3.5:

Prove that $\overline{\overline{A} \cap \overline{B}} = A \cup B$

PROOF:

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in E\}$$

$$\overline{B} = \{\langle x, \nu_B(x), \mu_B(x) \rangle / x \in E\}$$

$$\overline{A} \cap \overline{B} = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in E\} \cap \{\langle x, \nu_B(x), \mu_B(x) \rangle / x \in E\}$$

$$= \{\langle x, \min(\nu_A(x), \nu_B(x)), \max(\mu_A(x), \mu_B(x)) \rangle / x \in E\}$$

$$\overline{\overline{A} \cap \overline{B}} = \{\langle x, \max(\mu_A(x), \mu_B(x), \min(\nu_A(x), \nu_B(x)) \rangle / x \in E\}$$

$$= A \cup B$$

Hence it completes the proof

THEOREM 3.6:

Prove that $\overline{\overline{A} \cup \overline{B}} = A \cap B$

PROOF:

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in E\}$$

$$\overline{B} = \{\langle x, \nu_B(x), \mu_B(x) \rangle / x \in E\}$$

$$\overline{A} \cup \overline{B} = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in E\} \cup \{\langle x, \nu_B(x), \mu_B(x) \rangle / x \in E\}$$

$$= \{\langle x, \max(\nu_A(x), \nu_B(x)), \min(\mu_A(x), \mu_B(x)) \rangle / x \in E\}$$

$$\overline{\overline{A} \cup \overline{B}} = \{\langle x, \min(\mu_A(x), \mu_B(x), \max(\nu_A(x), \nu_B(x)) \rangle / x \in E\}$$

$$= A \cap B$$

Hence it completes the proof

THEOREM 3.7:

Prove that $\overline{\overline{A}+\overline{B}} = A.B$

PROOF:

L.H.S:

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\}$$

$$\overline{B} = \{\langle x, \nu_B(x), \mu_B(x)\rangle / x \in E\}$$

$$\overline{A}+\overline{B} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\} + \{\langle x, \nu_B(x), \mu_B(x)\rangle / x \in E\}$$

$$= \{\langle x, \nu_A(x)+\nu_B(x)-\nu_A(x)\nu_B(x), \mu_A(x)\mu_B(x)\rangle / x \in E\}$$

$$\overline{\overline{A}+\overline{B}} = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x)\nu_B(x)\rangle / x \in E\}$$

=A.B

Hence it completes the proof.

THEOREM 3.8:

Prove that $\overline{\overline{A}@\overline{B}} = A@B$

PROOF:

L.H.S:

$$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\}$$

$$\overline{B} = \{\langle x, \nu_B(x), \mu_B(x)\rangle / x \in E\}$$

$$\overline{A}@\overline{B} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\} @ \{\langle x, \nu_B(x), \mu_B(x)\rangle / x \in E\}$$

$$= \left\{\left\langle x, \frac{\nu_A(x)+\nu_B(x)}{2}, \frac{\mu_A(x)+\mu_B(x)}{2}\right\rangle / x \in E\right\}$$

$$\overline{\overline{A}@\overline{B}} = \left\{\left\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2}\right\rangle / x \in E\right\}$$

$$= A@B$$

Hence it completes the proof.

Conclusion

We have defined different operations of Intuitionistic Fuzzy Sets. Using these, we have proved different relations between these operators in the intuitionistic fuzzy sets.

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