NECESSITY AND POSSIBILITY OPERATORS ON INTUITIONISTIC FUZZY SETS

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Abstract: **The purpose of the dissertation is to study fuzzy and Intuitionistic fuzzy sets and also give basic relations of Intuitionistic fuzzy sets. The main results of this paper are to discuss Necessity and Possibility operator of Intuitionistic fuzzy sets and using such operators, some of the theorems are discussed.**

Keywords: **Intuitionistic Fuzzy Set (IFS) and Intuitionistic Fuzzy Set operators, Necessity and Possibility operators.**

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [5]. The concept of intuitionistic fuzzy set was introduced by Atanassov [1] as the generalization of the notation of fuzzy set. In this paper, we discuss further properties of intuitionistic fuzzy sets of necessity and possibility operators are discussed.

2. PRELIMINARIES

Definition 2.1- Fuzzy Set:

Let S be a Semigroup and F be a "fuzzy" and let f be a subsemigroup. A function f from S to the unit interval $[0,1]$ is called a fuzzy set of S. Let F(S) denote the set of all fuzzy sets in S.

(or)

Non crisp sets are called fuzzy set. In Fuzzy set the membership function takes the value [0,1]

Definition 2.2- Intuitionistic Fuzzy sets:

Intuitionistic fuzzy sets are sets whose elements have degrees of membership and non-membership function. An Intuitionistic fuzzy set A is a non empty set X is an object having the form $A = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \mid x \in E \right\rangle \right\}$ and satisfies $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.3- Basic Operations and Relations over Intuitionistic Fuzzy Sets:

$A \subseteq B \ iff \ (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x))$ (2.1)

$A \supseteq B \ iff \ B \subseteq A$ (2.2)

$A = B \ iff \ (\forall x \in E)(\mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x))$ (2.3)

$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \epsilon E \}$ (2.4)

$A \cup B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \epsilon E \}$ (2.5)

$A+B = \{ \langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x) \rangle | x \epsilon E \}$ (2.6)

$A.B = \{ \langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x) \rangle | x \epsilon E \}$ (2.7)

$A@B = \left\{ \langle x, \frac{\mu_{A(x)} + \mu_{B(x)}}{2}, \frac{\nu_{A(x)} + \nu_{B(x)}}{2} \rangle | x \in E \right\}$ (2.8)

Definition 2.4- Necessity operator of Intuitionistic Fuzzy:

For every IF Set A is called the necessity operator, then it defined as follows

$\Box A = \left\{ \left\langle x, \mu_A(x), 1 - \mu_A(x) \right\rangle / x \in E \right\}$

Definition 2.4- Possibility operator of Intuitionistic Fuzzy:

For every IF Set A is called the possibility operator, then it defined as follows

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$$\Diamond A = \left\{ \left\langle x, 1 - \nu_A(x), \nu_A(x) \right\rangle / x \in E \right\}$$

3. PROPERTIES BASED ON NECESSITY AND POSSIBILITY OPERATORS

Theorem 3.1

For every two Intuitionistic fuzzy sets A and B :

Prove that $\Box(A \cup B) = \Box A \cup \Box B$

Proof:

L.H.S $\Box(A \cup B) = \Box\left(\left\{\left\langle x, \max\left(\mu_A(x), \mu_B(x)\right), \min\left(\nu_A(x), \nu_B(x)\right)\right\rangle / x \in E\right\}\right)$

$$= \left\{\left\langle x, \max\left(\mu_A(x), \mu_B(x)\right), 1 - \max\left(\mu_A(x), \mu_B(x)\right)\right\rangle / x \in E\right\}$$

$$= \left\{\left\langle x, \mu_A(x), \min\left(1 - \mu_A(x), 1 - \mu_B(x)\right)\right\rangle / x \in E\right\}$$

$$= \left\{\left\langle x, \mu_A(x), 1 - \mu_B(x)\right\rangle / x \in E\right\} \rightarrow \qquad (1)$$

R.H.S $\Box A = \left\{\left\langle x, \mu_A(x), 1 - \mu_A(x)\right\rangle / x \in E\right\}$

$\Box B = \left\{\left\langle x, \mu_B(x), 1 - \mu_B(x)\right\rangle / x \in E\right\}$

$\Box A \cup \Box B = \left\{\left\langle x, \max\left(\mu_A(x), \mu_B(x)\right), \min\left(1 - \mu_A(x), 1 - \mu_B(x)\right)\right\rangle / x \in E\right\}$

$$= \left\{\left\langle x, \mu_A(x), 1 - \mu_B(x)\right\rangle / x \in E\right\} \rightarrow \qquad (2)$$

From (1) and (2) we have $\Box(A \cup B) = \Box A \cup \Box B$

Hence it completes the proof.

Theorem 3.2

Prove that $\overline{\Box(\overline{A} + \overline{B})} = \Diamond A \Diamond B$

Proof:

$$\overline{A} = \left\{\left\langle x, \nu_A(x), \mu_A(x)\right\rangle / x \in E\right\}$$

$$\overline{B} = \left\{\left\langle x, \nu_B(x), \mu_B(x)\right\rangle / x \in E\right\}$$

$$\overline{A} + \overline{B} = \left\{\left\langle x, \nu_A(x), \mu_A(x)\right\rangle / x \in E\right\} + \left\{\left\langle x, \nu_B(x), \mu_B(x)\right\rangle / x \in E\right\}$$

$\Box(\overline{A} + \overline{B}) = \{\langle x, \nu_A(x) - \nu_B(x) - \nu_A(x)\nu_B(x), 1 - \nu_A(x) - \nu_B(x) + \nu_A(x)\nu_B(x)\rangle | x \in E\}$

$\overline{\Box(\overline{A} + \overline{B})} = \{\langle x, 1 - \nu_A(x) - \nu_B(x) + \nu_A(x)\nu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle | x \in E\}$ (1)

$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}$

$\Diamond B = \{\langle x, 1 - \nu_B(x), \nu_B(x)\rangle | x \in E\}$

$\Diamond A \Diamond B = \{\langle x, (1 - \nu_A(x))(1 - \nu_B(x)), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle | x \in E\}$

$\Diamond A \Diamond B = \{\langle x, 1 - \nu_A(x) - \nu_B(x) + \nu_A(x)\nu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle | x \in E\}$ (2)

From (1) and (2) we have

$\overline{\Box(\overline{A} + \overline{B})} = \Diamond A \Diamond B$

Hence it completes the proof.

Lemma 3.3

Prove that $\Box(A@B) = \Box A @ \Box B$

Proof:

Let $A@B = \left\{ \langle x, \frac{\mu_{A(x)} + \mu_{B(x)}}{2}, \frac{\nu_{A(x)} + \nu_{B(x)}}{2} \rangle \mid x \in E \right\}$

$\Box(A@B) = \left\{ \langle x, \frac{\mu_{A(x)} + \mu_{B(x)}}{2}, 1 - \left(\frac{\mu_{A(x)} + \mu_{B(x)}}{2} \right) \rangle \mid x \in E \right\}$

$\qquad = \left\{ \langle x, \frac{\mu_{A(x)} + \mu_{B(x)}}{2}, \frac{2 - \mu_{A(x)} - \mu_{B(x)}}{2} \rangle \mid x \in E \right\} \qquad (1)$

$\qquad \Box A = \left\{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle / x \in E \right\}$

$\qquad \Box B = \left\{ \langle x, \mu_B(x), 1 - \mu_B(x) \rangle / x \in E \right\}$

$\Box A @ \Box B = \left\{ \langle x, \frac{\mu_{A(x)} + \mu_{B(x)}}{2}, \frac{1 - \mu_{A(x)} + 1 - \mu_{B(x)}}{2} \rangle \mid x \in E \right\}$

$\qquad = \left\{ \langle x, \frac{\mu_{A(x)} + \mu_{B(x)}}{2}, \frac{2 - \mu_{A(x)} - \mu_{B(x)}}{2} \rangle \mid x \in E \right\} \qquad (2)$

From (1) and (2) we have

$\Box(A@B) = \Box A @ \Box B$

Hence it completes the proof

CONCLUSION

In this work, we have introduced two types of operators over intuitionistic fuzzy sets and some theorems are proved by using the proposed operators.

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