Students' Trajectory Learning in Using Odometer Strategy and Grasping Multiplication Concept by Tree-Diagram Model

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***Abstract*—The present study was a part of one-cycle design research aimed at supporting students to grasp multiplication concept in two-dimentional problem (2D) of the multiplication principle topic. To obtain the goal, a sequence of learning activities was designed in the form of Hypothetical Learning Trajectory (HLT). For that reason, design research was decided to use as research approach involving twelve 10-12 years old students. The finding suggests that the designed learning helps students use odometer strategy and particularly the tree-diagram model supports multiplication concept grasping of students.**

***IndexTerms*—HLT, Design Research**

I. INTRODUCTION

The progression of combinatorics science is very inevitable since it affects the development of some other disciplines ([1]; [2]; [3]). It is then considered that the development of the educational studies of combinatorics also develop. There are several studies of mathematics education related to combinatorics ([4]; [5]; [6]; [7]; and [8]). Especially, the three latest studies, they concerned combinatorics learning in elementary school which suggests several strategies of children and mathematics concept understanding of them.

The present study, i.e. the third cycle, sets the objective to provoke students using certain strategies as found in [6] and [7] as well as lead them to understanding of multiplication concept lying in the two-dimentional multiplication priciple topic. The strategy refers to what is so called *odometer strategy* which is identical to the odometer of vehicle of which an object in one group is changed to another in the same group if it has been paired to all objects in the other group.

Some previous attempts in two cycles of learnings have been actually undergone to reach the objectives. However, it remains one main problem which hampers the attainment of the goals. The students in the previous cycle were skillfull in applying the odometer strategy. Neverthelles, they didn't grasp the concept of multiplication although they applied multiplication. They used such an operation since they reflect from the results of discussed problems which were appropriately obtained using multiplication. It also means that several goals were attained by some of the whole activities.

It is then become a challenge for the researchers to create learning activities to solve the remaining problem. As a result, several literatures were reviewed and the discussions with some colleagues were done to reach the learning objective completely. Several discrete mathematics books and pedagogical mathematics books, e.g. ([9]; [2]; [10]) use and suggest the tree-diagram model to connect to the multiplication. Then the research question was posed, how the designed learning, especially tree-diagram, model support students to apply efficient strategies in solving the problem as well as to reach the understanding of multiplication principle concept?. It then implies to the refinement of the hypothetical learning trajectory (HLT) which is described in the research method section.

II. RESEARCH METHODS

Design research was set as the method for the present research as it fits to use to answer the posed research question. [11] stated that design research is a research method aimed at developing local instructional theory (LIT), i.e. the final form of sequence of students' activity including of the conjecture of their thinkings and workings. The activity sequence is clearly addressed to reach the learning goals. To create the LIT, an HLT should be developed and tested. HLT consists of learning goals, students' activity, and the conjecture of students' thinking and learning [12]. The HLT set in this study mostly adopt the HLT in the previous study with the tree-diagram model introduction as the part of the refinement. It is also completed with teacher guide for teacher to give the students pedagogical treatment if the students do not follow the learning trajectory smoothly. The covered topic is two-dimentional multiplication problem, i.e. two kinds of objects. Besides the previous literature reviewed, the establishment of the students' activities are in the form of problem solving is also motivated by the theory of [13].

Table 1 The Set HLT

<table>
<tr><th>Goal</th><th>Problem</th><th>Conjecture of students' thinking and learning</th></tr>
<tr><td rowspan="3">Students can list and determine the number of all possible two-dimensional pair combinations using odometer pattern approach (hands-on activity)</td><td>1 kind of snack - 2 kinds of drinks (1-2)</td><td>The students will get 2 possible combinations by pairing the snack to each of the drinks.</td></tr>
<tr><td>1 kind of snack - 3 kinds of drinks (1-3)</td><td>The students will get 3 possible combinations by pairing the snack to each of the drinks</td></tr>
<tr><td>2 kinds of snacks – 3 kinds of snacks (2-3)</td><td>• Some students will get six possible combinations. They keep the combinations they get in the previous problem and then pairing the other snack to the other drinks (odometer-listing method). They just make an addition in determining the number of the combinations.
• The other students start pairing the snacks and the drinks from the beginning using trial and error strategy. In determining the number of the combination, the students count the couples made one by one</td></tr>
<tr><td colspan="3">Teacher shows the comparison which strategy between the trial and error or the odometer-listing better. Next, the teacher introduces tree diagram model in bridging the conception of students from listing method to multiplication concept. Tree diagram model is used to solve the above problem</td></tr>
<tr><td rowspan="4">Students can list and determine the number of possible two-dimensional combinations using multiplication.</td><td>2 different shirts – 2 different trousers</td><td>Some students will use odometer strategy with tree diagram model. In determining the number of the combinations, the students answer 4 by counting the combination one by one
The other students still use listing method and get 4 as the answer by counting the combination one by one</td></tr>
<tr><td colspan="2">Teacher shows students to compare which method more effective to encourage them in using tree diagram model.</td></tr>
<tr><td>2 different shirts – 3 different trousers</td><td>Using their inductive reasoning, most of the students have the assumption that the number of the combination can be obtained by multiplying the number of shirts and the number of trousers. They answer the number of the combination, i.e. 6, first before listing the combinations of the objects.
Most students use tree diagram model in listing the combinations.</td></tr>
<tr><td>Teacher asks the students how many combinations without listing the combination of 5 different shirts – 3 different trousers</td><td>Some of them answer 15 by their inductive reasoning
Some of them answer 15 since for each shirt can be paired to three trousers and since there are five shirts, there are fifteen combinations</td></tr>
</table>

There were twelve 10-12 year old students actively participating in the learning and working in group of three. They all are in the same school, Athirah, an elementary school in Makassar, Indonesia. The data in this study were gathered from the preparation of the experiment including the HLT and the experiment. Interview, observation, and collection of written documentation are the

methods in collecting the data. The interview and the observations were recorded by using field note and video to collect information e.g. the grade and the mathematics ability of students. Documents which were mainly collected in the experiment phases were student's written works. The written works of the students were analyzed based on the HLT and discussed with some colleagues to draw conclusion how the designed HLT support the students.

III. Result and Discussion

As the conjecture suggests, all students paired the snack to all of the drinks availabe for both 1-2 and 1-3 problem in the hands-on activity. However, some groups previously had considered that there were only one possibility of snack-drink a child can bring from the problem 1-2. Only after the teacher asked them whether the other pairs possible, the students understand that the total is not one but two instead No combination was missing also for 2-3 problem by the students.. In addition, The answers of the students for the problem were variative as the hypothesis suggests. Some groups applied odometer-listing strategy and the others use that of trial and error. The groups that understand the problem well starting from the first problem tended to use odometer-listing strategy. In determining the number of the combination, all groups did a counting.

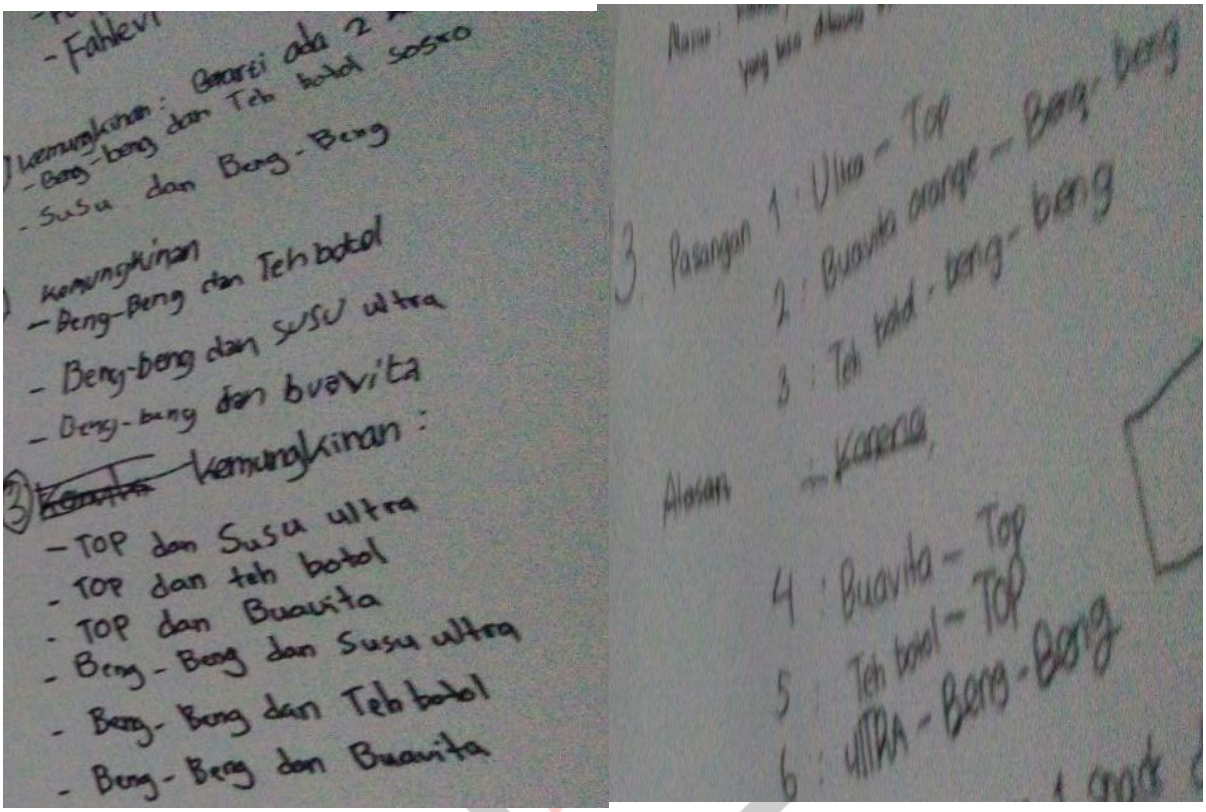

Fig 1: The works of the students: with odometer approach (left) and trial and error method (right)

Based on the guide from the HLT, the teacher showed the comparison of the two strategies they used and introduced them tree diagram model for solving the latest problem. In this step, the teacher hasn't yet introduced the concept of multiplication. Furthermore, in the problem 2-2, it was found the variety of strategies the students used. Some groups used the tree diagram model and the other used odometer-listing method as the hypothesis indicates. All of them keep using counting to identify the number of the combination. After that, based on the guide, the teacher explained the addition concept lying in the answer using tree diagram model of which for each shirt, it can be paired to two trousers, so there are four in total. Then the teacher posed new problem, i.e. five different shirts and three different trousers without the colors and most students answered fifteen. To know the reason behind the answer, the teacher made conversation with one of the groups, namely as shown in the following recorded conversation fragment:

Teacher : why is it fifteen?
Student : because five times three
Teacher : why do you multiply five by three?
Student : since each shirt can be paired to three trousers and there are five shirts so it means five time three

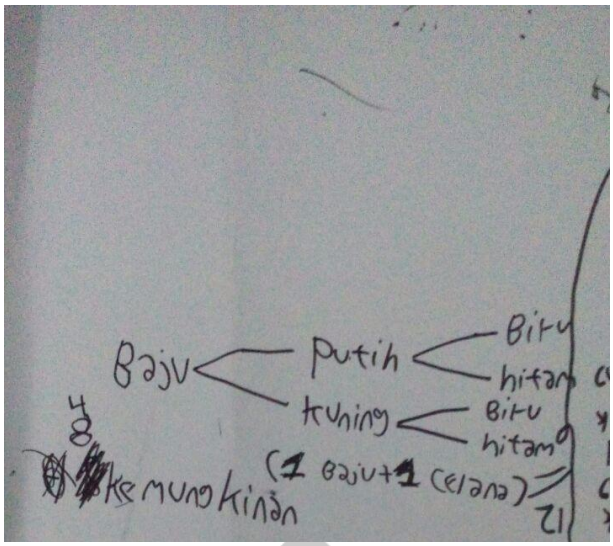

Fig 2: The work of the students with tree-diagram model

IV. CONCLUSION

This study was initiated with the question how can elementary school children be supported to apply efficient strategies in solving the problem as well as to reach the understanding of multiplication principle concept. From the learning activities, several conclusions can be suggested. Firstly, reflecting on the first activity in both HLTs, the hands-on activities quite help the students not miss each of possible combination. Secondly, the students are quite assisted to use the odometer strategy by the set of activities which starts from one object for one of the items. Thirdly, the tree-diagram model is quite helpful for the students in bridging the context to the multiplication concept.

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