Fuzzy performance measures for M/M/C with infinite capacity queueing model

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***Abstract*— In this paper we show that queueing theory can accurately model the flow of in-patient in hospital. In which arrival rate, service rate and number of parallel servers are all considered as fuzzy numbers. Further Robust ranking technique is used to find the expected mean queue length and waiting time in queue. Further numerical illustration is also given to justify the validity of the model. In this model capacity of the system is infinite.**

***IndexTerms*— fuzzy number, Mean queue length, Robust ranking technique, waiting time in queue.**

I. INTRODUCTION

In real life in many situations the parameter may only be characterized subjectively (i.e) the system parameters are both possibilistic and parabolistic. Li and Lee(1989) investigated the analytical results for two typical fuzzy queues FM/FM/1/∞, M/F/1 ∞ where F represents fuzzy time and FM represents fuzzified exponential distribution. Nagi and Lee (1992) proposed a procedure using the α cut and two variables simulation to analyze fuzzy queues. Using parametric programming Kao et al(1999) constructed the membership functions of the system characteristics for fuzzy queues and applied them to four simple fuzzy queues namely F/M/1/∞, M/F/1/∞ and FM/FM/1/∞ successfully. Ritha and Lilly Robert (2012) developed the profit analysis of FM/FE$_k$/1 queues system in which they convert the fuzzy queue into a family of conventional crisp queues and construct the membership function successfully by using the approaches of parametric NLP techniques. Later the same techniqueues were utilized by Jeeva and Rathinakumari(2012) to analyze a batch arrival single server, Bernoulli feedback queue with fuzzy vacations and fuzzy parameters. Ramesh and Kumara ghuru (2013) analyzed batch arrival queue with fuzzy parameters. Jeeva and Rathinakumari (2013) had also introduced the fuzzy cost computations of M/M/1 and M/G/1 queueing models also. Chen (2005) analyzed bulk service fuzzy queueing system with fuzzy parameters. Barak, S. and Fallahnezhad, M.S. (2012) analyzed the cost of fuzzy queueing systems. J. Pavithra and K. Julia Rose Mary (2016) studied the FM/M(a,b)/1 with multiple working vacations queuing model in Robust Ranking Technique. Based on that the above literature available Rajalakshmi .R and Julia Rose Mary .K considered Fuzzy Analysis for M/M/C finite capacity queueing model. In this paper we discuss the performance measure of FM/M/C queueing system with infinite capacity.

II. MODEL DESCRIPTION

An M/M/c queue is a stochastic process whose state space is the set {0, 1, 2, 3,...} where the value corresponds to the number of customers in the system, including any currently in service.

- Arrivals occur at rate λ according to a Poisson process and move the process from state i to i+1.
- Service times have an exponential distribution with parameter μ. If there are less than c jobs, some of the servers will be idle. If there are more than c jobs, the jobs queue in a buffer.
- The buffer is of infinite size, so there is no limit on the number of customers it can contain.

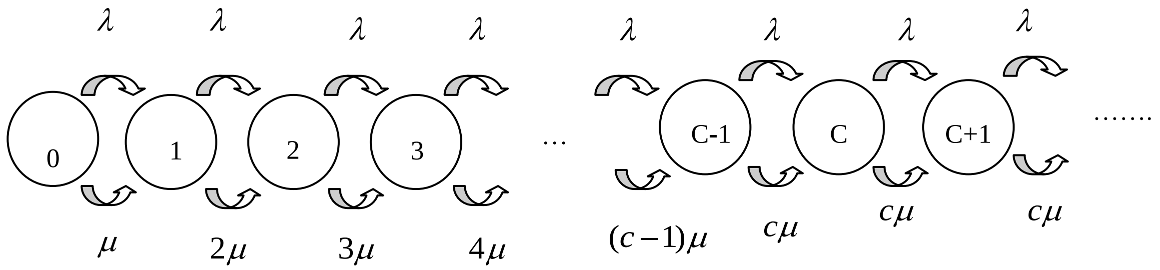
The model can be described as a continuous time Markov chain with transition rate matrix.

$$Q=\begin{pmatrix} -\lambda & \lambda & & & & & & & \\ \mu & -(\mu+\lambda) & \lambda & & & & & & \\ & 2\mu & -(2\mu+\lambda) & \lambda & & & & & \\ & & 3\mu & -(3\mu+\lambda) & \lambda & & & & \\ & & & \ddots & & & & & \\ & & & C\mu & -(C\mu+\lambda) & \lambda & & & \\ & & & & \ddots & & & & \\ & & & & C\mu & -(C\mu+\lambda) & \lambda & & \\ & & & & & \ddots & & & \end{pmatrix}$$

On the state space {0, 1, 2, 3, ...}. The model is a type of birth–death process.

We write $\rho = \lambda/(c\,\mu)$ for the server utilization and require $\rho < 1$ for the queue to be stable and ρ represents the average proportion of time which each of the servers is occupied (assuming jobs finding more than one vacant server choose their servers randomly).

The state space diagram for this chain is as below.



Here we have applied fuzzy analysis in the M/M/C: ∞/FIFO Queueing model Particularly to the Mean queue length and waiting time in the queue. Suppose the arrival rate λ with Poisson distribution, service rate μ for busy period, number of parallel servers c. Then we represent as fuzzy set $\bar{\lambda}, \bar{\mu}, \bar{c}$.

(i.e) $\bar{\lambda} = \left\{w, \theta(w)/\theta_{\bar{\lambda}}(w)/w \in S(\bar{\lambda})\right\}, \bar{\mu} = \left\{v, \theta_{\bar{\mu}}(v)/v \in S(\bar{\mu})\right\}, \ \bar{c} = \left\{z, \theta_{\bar{c}}(z)/z \in S(\bar{c})\right\}.$

Here $\theta_a(b)$ and S(a) denote the membership function and support of a where $a = \bar{\lambda}, \bar{\mu}, \bar{c}$ are fuzzy numbers and b=w, v, z are crisp values corresponding to arrival rate, service rate for busy period, number of parallel servers respectively.

On the basis of the concept of α-cut we develop a mathematical programming approach for deriving the α-cuts of $\bar{\lambda}, \bar{\mu}, \bar{c}$ as crisp intervals which are given by,

$$\bar{\lambda}(\alpha) = \left\{w \in W / \theta_{\bar{\lambda}}(w) \geq \alpha\right\},$$

$$\bar{\mu}(\alpha) = \left\{u \in U / \theta_{\bar{\mu}}(u) \geq \alpha\right\},$$

$$\bar{c}(\alpha) = \left\{z \in Z / \theta_{\bar{c}}(z) \geq \alpha\right\},$$

where 0<α≤1. Hence a fuzzy queue can be reduced to a family of crisp queues with difference α-cuts as, $\{\lambda(\alpha)/0<\alpha\leq 1\}$, $\{\mu(\alpha)/0<\alpha\leq 1\}$, $\{c(\alpha)/0<\alpha\leq 1\}$. Let the confidence interval of the fuzzy sets $\lambda(\alpha), \mu(\alpha), c(\alpha)$ be $[l_\lambda(\alpha) \quad u_\lambda(\alpha)]$, $[l_\mu(\alpha) \quad u_\mu(\alpha)]$, $[l_c(\alpha) \quad u_c(\alpha)]$, then the expected queue length (Lq) is given by,

$$L_q = \frac{(\lambda/\mu)^c p_0 \rho}{c!(1-\rho)^2} \qquad where \quad \rho = \frac{\lambda}{c\mu} \quad and$$

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

In the above formula by applying the fuzzy variable for arrival rate, service rate for busy, number of parallel servers then we get, The expected queue length (Lq) is given as,

$$L_q = \frac{(w/u)^z p_0 (w/zu)}{z!(1-w/zu)} \tag{i}$$

Similarly, waiting time in the queue as,

$$w = \frac{L_q}{\lambda} = \frac{(w/u)^z p_0 (w/zu)}{wz!(1-w/zu)} \tag{ii}$$

Now by applying the Robust Ranking Technique to the required equations (i) and (ii).

III.NUMERICAL EXAMPLE:

Consider fuzzy M/M/C: ∞/FIFO queueing system. The corresponding parameters such as arrival rate, service rate for busy, number of parallel servers and finite capacity are fuzzy numbers. Let us consider the parameters us, $\lambda = [0.1, 0.2, 0.3, 0.4]$, $\mu = [0.05, 0.15, 0.25, 0.35]$, whose intervals of confidence are $[0.1+\alpha \quad 0.4-\alpha]$, $[0.05+\alpha \quad 0.35-\alpha]$ respectively, Now we evaluate R(0.1,0.2,0.3,0.4) by applying Robust Ranking Method.

$$R(\bar{\lambda}) = R(0.1, 0.2, 0.3, 0.4)$$

$$= \int_0^1 0.5(0.1+0.4)d\alpha = \int_0^1 0.5(0.5)d\alpha = 0.25$$

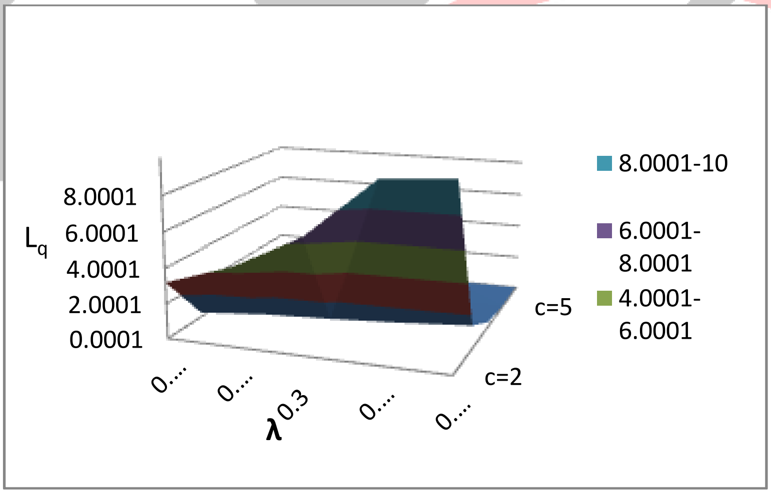
$$R(\bar{\mu}) = R(0.05, 0.15, 0.25, 0.35)$$

$$= \int_0^1 0.5(0.05+0.35)d\alpha = \int_0^1 0.5(0.4)d\alpha = 0.2$$

By proceeding similarly the Robust Ranking indices for the fuzzy numbers $\bar{\lambda}, \bar{\mu}, \bar{c}$ are calculated and tabulated for $R(\bar{\lambda}) = 0.25, R(\bar{\mu}) = 0.2$, based on that Mean queue length for M/M/C: ∞/FIFO model are tabulated in table (1) & (2) and they are represented graphically in graph (1) & (2) respectively.

Table 1: Mean queue length for M/M/C: ∞/FIFO model.

λ / C	0.25	0.3	0.35	0.4	0.45
2	0.8010	1.2327	1.9284	3.1597	5.7211
3	0.0647	0.1641	0.2368	0.3350	0.4670
4	0.0191	0.0298	0.0447	0.0649	0.0919
5	0.0032	0.0053	0.0085	0.0131	0.0195
6	0.0004	0.0008	0.0015	0.0029	0.0040

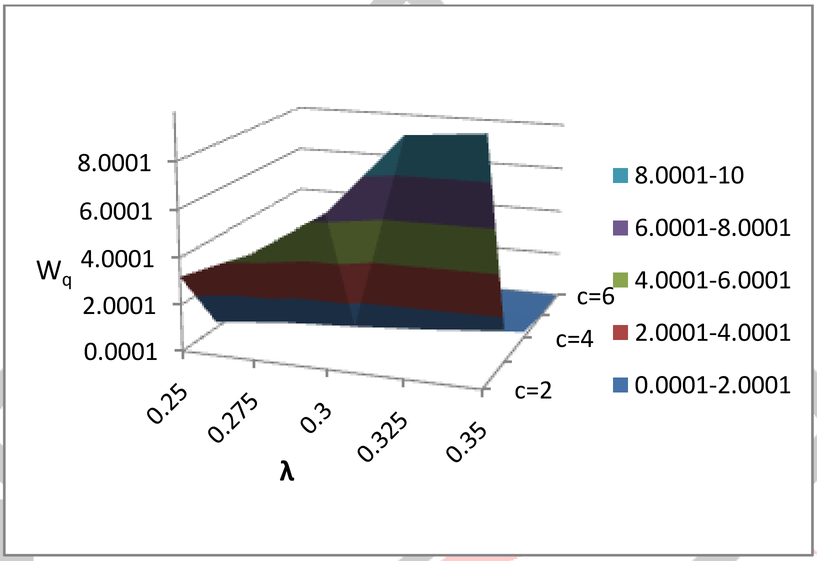


Graph(1) Lq versus (c , λ)

From the table and graph(1), we conclude that, the mean queue length L_q increases when the arrival rate increases. Also, we find that the mean queue length (L_q) decreases when the number of beds increases.

Table 2: Waiting time in queue for M/M/C: ∞/FIFO model.

λ / C	0.25	0.3	0.35	0.4	0.45
2	3.204	4.4825	6.428	9.7221	16.346
3	0.2588	0.5967	0.7893	1.0307	1.3342
4	0.0764	0.1083	0.149	0.1996	0.2625
5	0.0128	0.0192	0.0283	0.0403	0.0557
6	0.0016	0.0029	0.005	0.0089	0.0114



Graph(2) Wq versus (c, λ)

From the table(2) and graph(2) we observe that w_q increases for increasing the arrival rate and decreases for increasing number of beds.

IV.CONCLUSION:

Many of the queuing systems have been analyzed by using the fuzzy set theory which provides wider application in many fields. Thus in this paper by applying the techniques of α-cut and Zadeh's extension principle to the arrival rate, service rate, number of parallel numbers and finite capacity as fuzzy number which are more realistic and general in nature. Further numerical results which are calculated by using Robust Ranking Technique, we observe that the solution of fuzzy problem becomes more efficient.

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