DETERMINISTIC DESIGN OF JOINT PILOT POWER AND PILOT PATTERN IN OFDM SYSTEMS

[1]Priya.G, [2]Mr.ajith Joseph, [3]Mrs. Mariya Vincent

[1]PG scholar, [2]Associate Professor, [3]Assistant Professor

***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) is a digital transmission method developed to meet the increasing demand for higher data rates in communications. In OFDM, to obtain channel state information (CSI) channel estimation (CE) is performed with the aid of pilot symbols. In this paper, different pilot design schemes for sparse channel estimation based on minimizing coherence is considered. Initially condition for optimum pilot pattern with respect to channel length is derived. Since generated pilot pattern is optimum only for those which satisfies the condition L ≥N/2. A greedy algorithm namely stochastic sequential search is proposed for those cases pilot pattern generated from CDS is not optimum. Later, joint design of pilot power and pattern is done. Finally, simulation results using matlab shows that SSS converge much faster than exhaustive search. With joint design both pilot power and pilot position are updated in optimum manner. The result is a fast greedy technique that performs almost same as the existing competitors with significantly lower computational complexity.**

***Index Terms*—OFDM, Compressive Sensing, Cyclic Difference Set, MIP, sparse channel, cyclic difference set.**

I. INTRODUCTION

With the increase of communications technology, the demand for higher data rate services such as multimedia, voice, and data over both wired and wireless links is also increased. The goal of next generation of mobile wireless communication system is to achieve ubiquitous, high-quality, high-speed mobile multimedia transmission. To achieve this goal, various new technologies are constantly being applied to mobile communication systems. Academia and industry have reached a consensus that OFDM is one of the most promising core technologies in new generation of wireless mobile communication system. Orthogonal Frequency Division Multiplexing is most commonly used in modern mobile broadband wireless communication systems such as wireless interoperatability for microwave access and long term extension (LTE) systems. Channel State Information (CSI) is a crucial consideration for improving the performance of OFDM systems. Channel estimation is a method used to obtain CSI which contains information like how a signal propagates from the transmitter to receiver and represents the combined effect like fading, scattering and power decay with distance. In real scenario wireless channel is considered as sparse multipath channel. Using conventional channel estimation techniques is not effective for sparse multipath channel, since it results in overutilization of resources. With the aid of compressive sensing, this is regarded as efficient signal acquisition framework for those signals which are sparse in nature. Recent studies shows that incorporating channel sparsity in estimation of certain OFDM channels results in both higher estimation quality and lower pilot overhead[1].The CS techniques for pilot-assisted channel estimation have been widely investigated and many sparse recovery algorithms have been applied for channel estimation. Another focus of the sparse channel estimation is the design of pilots. According to the Restricted Isometry Property (RIP), it has been shown that the measurement using random matrices guarantees a high probability of sparse recovery, indicating that the randomly generated pilot pattern is statistically optimal. However, the implementation of the random pilot pattern is more challenging in practical systems due to its high complexity, large storage, and low efficiency. Since no method exists for checking RIP, alternative approach is used in which pilot design is based on mutual incoherence property (MIP).

In [2] it has been shown that pilot locations corresponding to cyclic difference sets (CDS) are optimal in terms of the coherence measure. But, such locations exist only for specific number of pilots and subcarriers. In settings where no CDS exists (L<N/2), [3] and [4] propose suboptimal and greedy methods. Besides pilot locations pilot powers are also optimized in [5].

In this paper, the pilot design based on the mutual incoherence property (MIP) is considered, which avoids acquiring of channel data. The impact of different lengths of channel impulse response (CIR) is first analyzed and provides a sufficient condition that guarantees the pilot pattern generated from the CDS to be optimal. Then, we propose a greedy algorithm for pilot design to obtain a near-optimal pilot pattern when CDS does not exist. Later joint design of pilot pattern and pilot power is performed.

The rest of the paper is organized as follows. A sufficient condition with respect to CIR length for pilot pattern generated from the CDS being optimal is shown in Section II. In section III the proposed model is given. Section IV shows the simulation results. Section V represents the conclusion.

II. PILOT PATTERN FROM CDS

(i). Pilot Design Based On MIP

To analyze the channel estimation consider an OFDM system with N subcarriers and N_p pilot patterns whose position is denoted by $P_1 \le P_2 \le \cdots . \le P_{N_p}$.Let transmitted pilot symbols are denoted by x$(P_1), x(P_2), \ldots, x(P_{N_p})$.Here the power transmitted by all the OFDM symbols are equal. Then the received signals on pilot subcarriers are given by

$$\begin{bmatrix} y(p_1) \\ y(p_2) \\ \vdots \\ y(p_{N_p}) \end{bmatrix} = \begin{bmatrix} x(p_1) & 0 & 0 & 0 \\ 0 & x(p_2) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & x(p_{N_p}) \end{bmatrix} * F_{N_p XL} \begin{bmatrix} h(1) \\ h(2) \\ \vdots \\ h(L) \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2) \\ \vdots \\ n(N_p) \end{bmatrix} \quad ..(1)$$

DFT sub matrix $F_{N_p XL} = (1/\sqrt{N}) * \begin{bmatrix} 1 & w^{P_1} & \ldots & w^{P_1(L-1)} \\ 1 & w^{P_2} & \ldots & w^{P_2(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{P_{N_p}} & \ldots & w^{P_{N_p}(L-1)} \end{bmatrix}$ ..(2)

$$W = e^{-j2\pi/N} \quad ..(3)$$

Let **A** be the observation matrix given by **A**=X F_{N_pXL}, where **X** is diagonal matrix of pilot symbols and **F** is the DFT matrix. Then received signals on pilot can be written as **y=Ah + n**. Since h is a sparse vector, less number of pilots can be used. But channel under noiseless condition can be recovered properly only if observation matrix satisfies RIP. Since no method is existing to check for RIP condition. An alternative to RIP condition is mutual incoherence property .For obtaining MIP, absolute correlation between columns of A has to be taken and finding its maximum value. Coherence of A is given by

$$g(p) = \max_{0 \le m < n \le L-1} |A(m).A(n)| \quad \ldots. (4)$$

$$= \max_{0 \le m < n \le L-1} \left| \sum_{i=1}^{N_p} |x(P_i)|^2 w^{p_i(n-m)} \right| \quad \ldots (5)$$

For pilot design on the basis of MIP, coherence has to be minimized and optimum pilot pattern is given by

Q=$\min_p g(p)$ & $p_{opt} = \arg(Q)$ … (6)

Pilot pattern generated is optimum only if it satisfies CDS property. Pilot pattern generated from CDS is optimum for (N, N_p) combination only if L≧N/2, then only it will able to achieve Welch bound .Welch bound is considered as the lower bound of coherence .According to CDS based pilot design those patterns achieving Welch bound are optimal and those crossing Welch bound are not optimal. But by analyzing the results obtained with CDS, it is seen that CDS is optimal only if length of the channel is large enough i.e. it should be greater than N/2.For low values of L there are another methods to obtain optimal pilot pattern. So for low values of L CDS cannot guarantee that the pilot pattern generated is optimal. Iteration needed to attain pilot pattern is also large. For those cases where CDS does not exist or not able to provide optimal pilot pattern an algorithm has been proposed which will provide optimum pilot pattern in less number of iterations.

III.PROPOSED PILOT DESIGN SCHEMES

With exhaustive search method it is able to obtain optimum pilot pattern with the minimum MIP. But the exhaustive search method is not easy method since it requires large computations and huge search space. So it is not applicable for power constrained mobile devices working in cognitive radio channel. Hence for reducing complexity and for those cases where CDS couldn't find an optimal pilot pattern, a greedy algorithm has been proposed namely stochastic sequential search. Exhaustive search method is able to find optimum pilot pattern with the minimum MIP. But the exhaustive search method is not easy method since it requires large computations due to huge search space. Hence, it's not practical for power constrained mobile devices working in cognitive radio channel. Thus, for reducing complexity in exhaustive search and for those cases where CDS is not an optimal pilot pattern, a greedy algorithm proposed [1], namely Stochastic Sequential Search (SSS), has been verified.

i. Stochastic Sequential Search

SSS is a greedy algorithm, which is based on minimizing MIP. It contains two loops of iterations .Outer loop is used to randomly create pilot patterns and in inner loop pilot patterns are updated in a greedy manner according to equation 7.

$$g(p) = \max_{0\le m<n\le L-1} |A(m).A(n)| = \max_{0\le m<n\le L-1} \left|\sum_{i=1}^{N_p} |x(P_i)|^2 w^{p_i(n-m)}\right| \quad \ldots\ldots \quad (7)$$

$$\hat{P}_{P_K} = \underset{\substack{p\cdot(i)=p(i),i=1,2,..Np,i\neq k \\ p\cdot(k)\epsilon N[p(i),i=1,2,..Np,i\neq k]}}{\operatorname{argmin}} \; g(p^*) \quad \ldots\ldots \quad (8)$$

Updating p in the inner loop continues until it converges in such a way that no further update can perform. After inner loop iteration this algorithm guarantees best results.SSS can provide optimum pilot pattern within short span of time, with lesser number of iterations.

(i). SSS Based Pilot Design Algorithm

1. Initializations :
   Set inner and outer loop iterations
   Create D as a null matrix with size of M_1xN_p, r as 1xM_1 null matrix.
2. Within outer loop iteration l=1,2,.. M_1 generate random pilot pattern with a size of N_p.
3. Set $\tilde{P}=0^{N_p}$
4. Within inner loop iteration n=1, 2,.., M_2
5. if p=$\tilde{p}$
6. break
7. end for if loop
8. $\tilde{P} \leftarrow p$
9. for k=1, 2,..N_p
10. Find $\hat{P}_{P_K}$ and set p ← $\hat{P}_{P_K}$
11. end for k loop
12. end for inner loop
13. D (l) ←P and r(l) ←g(p)
14. end for outer loop
15. t = $\operatorname{argmin}_{i=1,2,..M1} r(i)$
16. Output D (t)

In SSS, optimum pilot pattern is obtained by considering equal transmitted power for all OFDM pilot symbols and concentrates only on pilot location. Next goal is to minimize coherence by selecting the pilot locations and pilot symbols. In equation (8) it is observed that coherence measure is through the magnitude of pilot symbols, so pilot design can be simplified as

$$\omega_{opt} = arg\,\min_{\omega} \max_{r\epsilon L} \left|\sum_{i=1}^{N_p} v(i) e^{-j\frac{2\pi r p_i}{N}}\right| \quad ..(9)$$

Pilot design is a preprocessing block and implemented only once, its computational complexity becomes significant when N and Np is large and in some particular applications require regular redesign of pilot pattern. So a suboptimal design with low cost is preferred which provides joint design of pilot pattern and pilot power by dividing it into separate problems for setting V and P_{N_p} and sequentially iterate between them.

ii. Joint Pilot Pattern and Power Design

A joint pilot pattern and power design methods [18] used for minimizing coherence. Initially assign pilot powers to a given set of pilot locations $P_m = \{P_1, P_2, \ldots, P_m\}$, where m starts from m=1 and gradually moves towards m=N_p.The optimal power assignment is given based on following equations.

Let $a(r) = \sum_{i=1}^{m} v(i)\, e^{-j2\frac{\pi r p_i}{N}}, r \in$ L … (11)

$$V = [v(1), \ldots\ldots, v(N_p)]^T \quad \ldots.. (12)$$

With the constraint v (i) ≥v_{min}, finally a set of pilot powers with less variance is obtained i.e.

$$f_{p_m}(v) = \sum_{r \in l} \left| \sum_{i=1}^{m} v(i) e^{\frac{-j2\pi r p_i}{N}} \right|^q \quad \ldots\ldots (13)$$

$$V_{opt} = \underset{v \geq v_{min}}{argmin} f_{p_m}(v) \quad \ldots\ldots (14)$$

Here $f_{p_m}(v)$ is both convex and differentiable for q>1.Hence its gradient can be written as

$$\nabla f_{p_m}(v) = q \sum_{r=1}^{L} |a(r)|^{q-2} \operatorname{Re}\left\{ a(r) \left[e^{\frac{j2\pi r p_1}{N}}, \ldots\ldots, e^{\frac{j2\pi r p_m}{N}} \right]^H \right\}$$

In the below algorithm, we have two parts in which position is found out using a variant of greedy algorithm followed by an algorithm for finding optimal symbols to be loaded to these positions. After the initialization steps a candidate U is used for the minimizing the cost in each iteration. The estimate is obtained on previous estimates and current gradient of cost .By setting a threshold U will be in a feasible set. In power allocation a pilot pattern position is getting fixed and only considered about power. But in joint design both pilot position and power is considered as shown in algorithm (iii).

The algorithms for power allocation and joint pilot power and pilot pattern are discussed; where joint design will provide a low cost suboptimal design with reduced run time about 99 percentage while coherence measures are similar.

(i). Power Allocation Algorithm

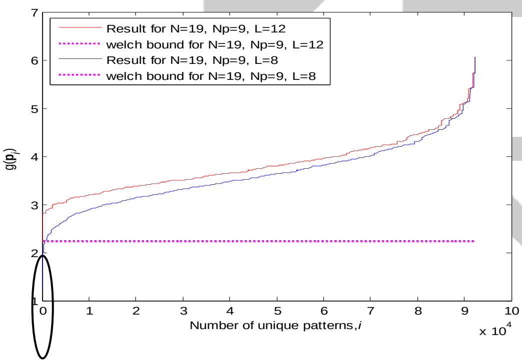
1. Inputs:
   Step size $\alpha_0 = 6.4 * 10^{-3}$, ε=stopping measure=10^{-6}
   Minimum acceptable pilot power v_{min}=10^{-3}, Maximum allowed iteration I_{max} =20
2. Initialization:
   for m=1: Np
   Set iteration count n=1, $V^{(0)} \leftarrow \frac{1}{m} 1$, $X^{(0)} \leftarrow V^{(0)}$ $V^{(1)} \leftarrow 0$, $\theta_1 \leftarrow 1$, q=20
3. Compute $f_{p_m}(v^n)$ and $f_{p_m}(v^{n-1})$ with the aid of equation (13)
4. While ($| f_{p_m}(v^n)$ - $f_{p_m}(v^{n-1}) |$) > ε
5. Set $\bar{U}$ = (1- θ_n) v^{n-1} +$\theta_n X^{n-1}$ and find $\nabla f_{p_m}(U)$
6. Compute the normalized gradient $\bar{d}$= $\nabla f_{p_m}(\bar{U})$ / $\|\nabla f_{p_m}(\overline{U})\|_2$
7. Update $\bar{U}$ =$\bar{U}$ - $\alpha_0 \bar{d}$
8. for i=1:m
9. if u(i) < v_{min}
10. u (i)= v_{min}
11. end if loop
12. end for loop
13. Set $V^{(n)} \leftarrow U$
14. Set $X^{(n)}$= v^{n-1} +$\frac{1}{\theta_n}$ $v^{(n)}$ $- v^{(n-1)}$
15. n ←n+1 and $\theta_n \leftarrow {}^{2}/_{(n+1)}$
16 end while loop

(iii). Joint Pilot Design and power allocation Algorithm

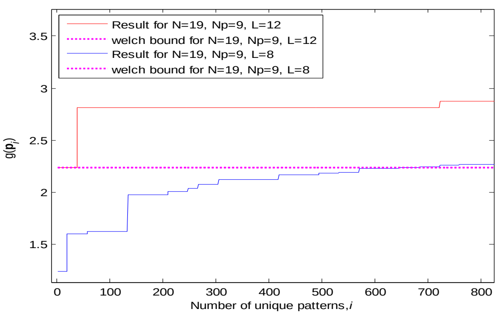
1. Set initially $P_1^* = [1]$.
2. Increase the size of P up to Np using below procedure.
3. Let current set of pilot locations are $p_{n-1}^* = \{p_1^*, \ldots, p_{n-1}^*\}$
4. Next location is chosen by considering all possible choices of p, $p \epsilon \{N \backslash p_{n-1}^*\}$ and form $p_{n|p} = p_{n-1}^* \cup p$.
5. For each choice perform the power allocation algorithm and choose the one which is able to provide minimum $f_{p_{n|p}}(v)$
6. $j(p_{n|p}) = \min_{v \geq v_{min}} f_{p_{n|p}}(v)$
7. $p_n^* = \underset{p_{n|p}}{argmin}\, j(p_{n|p})$
8. that value is updated as current position. Repeat the procedure until Np locations

IV. RESULTS AND DISCUSSIONS

Fig 1 shows the result of exhaustive search for N=19 and Np=9, represented as (19, 9), for different values of L. The pilot patterns that achieve Welch bound, is shown in 1 (a) by an elliptical area in the left portion which is zoomed in and shown in 1(b). From the simulation results we find that for L=12, 38 patterns are best patterns achieving Welch bound out of which one of them is CDS and time taken was 34.30 s. For L=8, within 33.09s we get 665 patterns which cross the welch bound out of which 19 patterns are best pattern. It is observed that g(p) drops drasticaly to the Welch bound in the vicinity area of the optimal pilot patterns which indicates that there is a huge difference between performance achieved by optimal pilot patterns and by other pilot patterns.



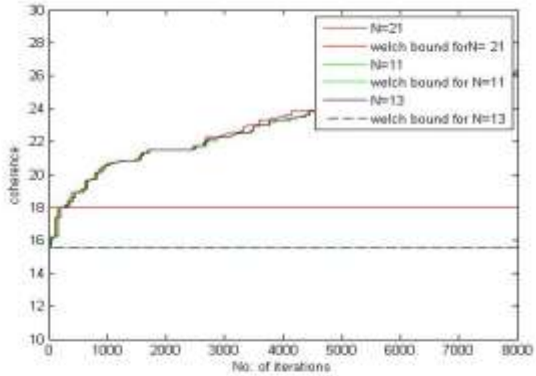
(a)



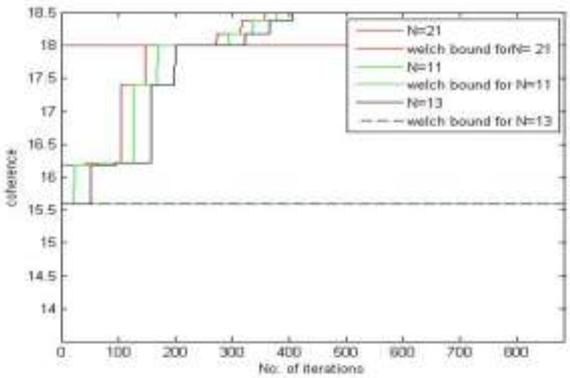
(b)

Fig 1 Results of g (p) in the increasing order over exhaustively generated p for (N = 19, Np =9) with L = 12and L = 8.

While checking for L=8, (13, 4), (11, 5) provides optimum pilot pattern whereas (21, 5) is not optimum according to CDS condition. We choose different combinations of (N, Np) as (21, 5), (11, 5) and (13, 4) and exhaustively generate all possible pilot patterns p. We then evaluate g (**p**) using (7) for every pilot pattern and for L=8. The results of g(p) sorted in the ascending order are shown in Fig. 2 with L = 8. The Welch bounds, i.e., 18 for (21, 5) and 16 for (11, 5) and (13, 4), are also indicated in figure for comparison. Since L = 8 does not satisfy the sufficient condition L$\geq$N/2 for (21, 5), thus there exist some pilot patterns with smaller g(**p**) than the Welch bound.



(a)



(b)

Fig 2 Coherence versus number of iterations for different (N, Np) with L=8

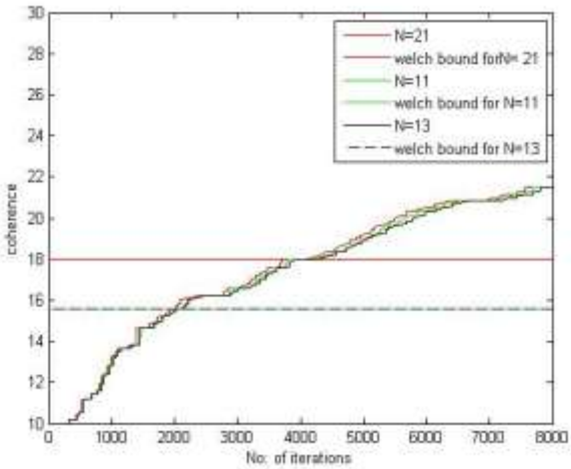


Fig.3 Coherence versus number of iterations for different (N, Np) with L=5

For the case of L =5, all the three combinations of (N, Np) are not providing optimum pilot patterns since it is not satisfying L $\geq$N/2 condition.

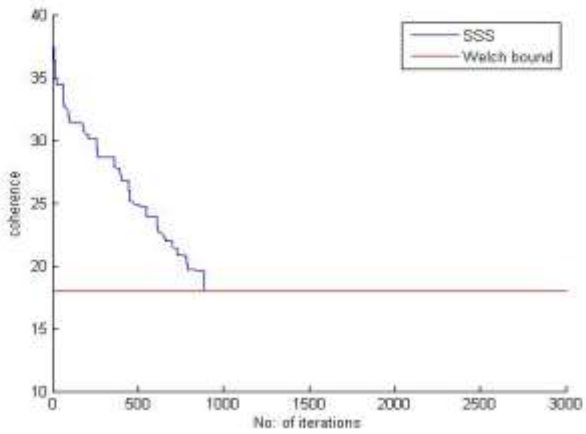


Fig 4.coherence versus number of iterations for (21, 5) with L=8, M1=10 and M2=5

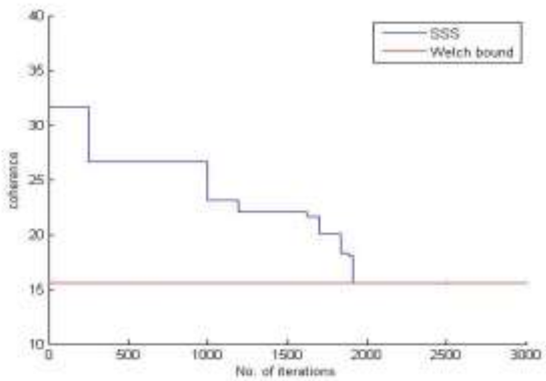


Fig 5.coherence versus number of iterations for (21, 5) with L=5, M1=20 and M2=10

We first consider the case of N = 21, Np = 5, and L = 8, which does not satisfy the sufficient condition for the optimal pilot pattern. Here since N is large it is almost impossible to perform the exhaustive search to obtain the optimal solution due to the large candidate set of (N, Np) pilot patterns. Therefore, we applied the low-complexity design scheme namely SSS. In fig. 5, we observe SSS can find the optimal pilot pattern that achieves the Welch bound, indicating that the verified stochastic search schemes is capable of finding an optimal pattern much easier than the exhaustive search. Within less number of iterations SSS generated pattern is achieving Welch bound, so SSS converges much faster. In both cases SSS can provide optimum pilot pattern with less number of iterations and it is converging faster as compared to exhaustive search.

The results including pilot position, optimized power and run time are shown in table 1.The experiment is conducted for different values of N. Experiment results indicate that joint design method results in about 99% reduction in run time compared to that of the method in [19] under the considered setup, while the coherence measures are similar. All simulations are done using MATLAB (R 2013 a). Here exhaustive pilot design for different combinations of (N, Np) are compared in fig (1-3).Then it is observed that for the cases where L<N/2, pilot pattern generated from exhaustive search is not optimum as in fig (1-3), since it is not satisfying L ≥N/2 condition. Then SSS is implemented for the cases where CDS does not exist. Simulation results demonstrate that SSS can achieve Welch bound within short time. Then joint power and pattern is designed and the results are shown in table 1.

N	Np	Run Time	Optimized pilot position	Optimized power
64	5	6.638534	1 4 8 58 62	0.1698 0.1665 0.2513 0.3010 0.1112
50	5	4.952857	1 8 10 41 48	0.2241 0.2162 0.1566 0.29832 0.1045
41	5	3.893881	1 10 27 31 40	0.2692 0.1861 0.1975 0.2678 0.0795
21	5	2.129210	1 5 10 14 18	0.2259 0.2516 0.1695 0.2447 0.1081
15	5	1.559047	1 5 10 11 12	0.2567 0.2666 0.1334 0.2424 0.1008

Table 1: Optimized power and pilot position

V.CONCLUSION

Two deterministic pilot design schemes for sparse channel estimation in OFDM systems is verified. MIP-based pilot design schemes for sparse channel estimation in OFDM systems are considered. With respect to the CIR length, a sufficient condition to guarantee that the pilot pattern following CDS is optimal has been shown. Then, we verified a greedy algorithm (SSS) which provides optimum pattern when CDS based pilot pattern is not optimum. Then later joint design of both pilot position and pilot symbol is considered. Simulation results demonstrate that proposed algorithm SSS are converging faster than CDS and able to achieve Welch bound within short span of time. The Joint design scheme works even better than SSS and reduces time of calculation by 99%.

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