EOQ FOR DETERIORATING ITEMS WITH NON-INSTANTANEOUS RECEIPT UNDER TRADE CREDITS WITH SHORTAGES

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ABSTRACT: In this paper, A permissible credit period is usually allowed to a retailer to pay back the dues without paying any interest to the supplier. That is, a retailer has the option of either paying for the goods immediately upon the receipt of the order, where interest will be charged over the delay period.

1. NOTATIONS AND ASSUMPTION

1.1. Notations

The following notations are used to develop the proposed model:

s :ordering cost
h : unit holding cost per unit time
c : purchase cost
p : selling price
Q : order quantity
θ : constant deterioration rate
$I_1(t)$: first phase
$I_2(t)$: second phase
λ : per unit rate
D : per unit time rate
T :order cycle period
T_1 :order receipt period, $0 < T_1 < T$
m :permissible delay in settling the account
I_c :interest paid per dollar per unit time
I_d :interest earned per dollar per unit time
$Z_1(T)$:total profit per cycle for case I
$Z_2(T)$:total profit per cycle for case II
T_1 : optimal order receipt period for case I
T_1 : optimal order receipt period for case II
T : optimal cycle time
$Q^*(T^*)$: optimal order quantity for case I
$Q^*(T^{**})$: optimal order quantity for case II
Z_1 (T) = T^*: optimal total profit per cycle for case I
Z_2 (T) = T^{**}:optimal total profit per cycle for case II
OC : ordering cost per order
HC : holding cost
IP : interest payable per cycle
IE_1 : interest earned per cycle for case I
IE_2 : interest earned per cycle for case II

1.2 Assumption

1. Time horizon is infinite and lead time is zero.
2. Shortages are allowed.
3. The inventory system under consideration deals with single item.
4. The replenishment rate λ, is finite and greater than demand rate D, i.e. $\lambda > D$.
5. Supplier offers a certain fixed period, m to settle the account.
6. Retailer would not consider paying the payment until receiving all items.
7. The order cycle period $[0, T]$ is divided into two phases (i) inventory replenished period (phase 1) (ii). Inventory depleted period (phase 2).

There is no replenishment or repair for a deteriorated item

1.3Mathematical formulation

According to the assumption the order cycle [0,T] is divided into two parts (i) inventory replenished period (ii) inventory depleted period. The two difference cases are shown in the figure 3.1. The change of inventory in the above two phases can be described as follows

Phase 1.

In this phase replenishment rate is greater than the demand rate, the inventory go up to maximum level .The rate of change of inventory at time't' ,$\frac{dI_1(t)}{dt}$ is given by

$\frac{dI_1(t)}{dt}$ = -I_1(t) +(λ-d), $0 \le t \le T$ ……………..(1)

With the boundary condition $I_1(T) = 0$

Phase 2.

Replenishment is stopped and the inventory decreases due to demand and deterioration .The rate of change of inventory at a time't', $\frac{dI_2(t)}{dt}$ can be described by,

$\frac{dI_2(t)}{dt}$ = -I_2(t) +(λ-d), $T_1 \le t \le T$ …. ………(2)

With the boundary condition $I_2(T) = 0$

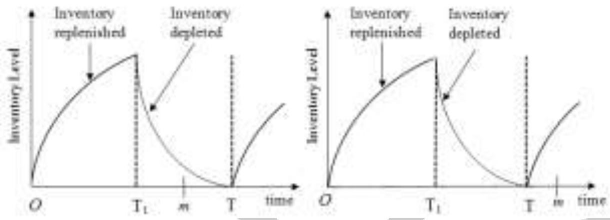

Figure 1.1 inventory level verses time

The solution of (1) and (2) are respectively given by,

$$I_1(T) = \frac{\lambda - D}{t}(1 - e^{-\theta t}), \quad 0 \le t \le T \qquad \text{.........(3)}$$

$$I_2(T) = \frac{D}{t}(e^{\theta(T-t)} - 1), \quad T_1 \le t \le T \qquad \text{.........(4)}$$

But the order quantity $Q=I_1(T)=I_2(T)$, from (3) and (4) we obtain

$T_1 = \frac{1}{t}\log\left(1 + \frac{D}{\lambda}\left[e^{\theta T} - 1\right]\right)$ ….……………..(5)

We can obtain the total profit per unit time following two cases (i) $T_1 \le m \le T$ and (ii)$T \le m$

Case I: $T_1 \le m \le T$

In this case shown figure 3.1 In this case, the total profit per cycle consists of sales revenue, ordering cost, holding cost, interest payable, interest earned and shortages. The components are calculated as follows:

(a) Sales revenue

SR= p $\{\int_0^{T_1}(\lambda - D)dt + \int_{T_1}^{T} D dt\}$ …………(6)

(b) The ordering cost per order = s
(c) The holding cost during [0,T] is given by
HC = h$\left\{\int_0^{T_1} I_1(t)dt \ \int_{T_1}^{T} I_2(t)dt\right\}$ ……..(7)
(d) The interest payable per cycle is given by
IP = $cI_c\int_m^T I_2(t)dt$ ………(8)

(e) The interest earned per cycle is given by

$IE_1 = pI_d \int_0^m Dt\,dt.$ ……….……..(9)

The shortage is given by

$SC = -c\int_{T_1}^{T} I_1(t)dt$

Therefore, the total profit per unit time is given by,

$Z_1(T) = \frac{1}{T}$ [SR – s – HC – IP + IE_1 – SC] ………..(10)

Case II T≤ m

In this case shown figure 3.2 In this case, the total profit per cycle consists of sales revenue , ordering cost, holding cost, interest payable and interest earned. Since cycle time is less than credit period , the retailer pays on no interest and earns the interest during the period [0,m].The interest earned in this case is given by,

$IE_1 = p\,I_d\int_0^m Dt\,dt + (m-T)\,D$ ……………(11)

$Z_2(T) = \frac{1}{T}$ [SR – s – HC – IP + IE_2] ……….…..(12)

1.4 DETERMINATION OF OPTIMAL REPLENISHMENT TIME

Since it is difficult to handle above equations for finding the exact value of T, therefore, we make use of the second order approximation for the exponential and logarithm in equation (10),(12)and(5), which follows as

$$e^{t(T-T_1)} \approx 1 + t(T-T_1) + \frac{t^2\ (T-T_1)^2}{2}$$

And

$$e^{t(T-m)} \approx 1 + (T-m) + \frac{t^2\ (T-m)^2}{2}$$

Also for low deterioration rate, we can assume

$e^{-T} \approx 1 - T + \frac{t^2\,T^2}{2}$ ……………..(13)

Hence, the total profit per unit time from (10)and (12) is approximated by

$Z_1(T,T_1) \approx \frac{1}{T}\left[p\{\lambda T_1 + D(T-2T_1)\} - s - \frac{h}{2}\left(\lambda {T_1}^2 + DT^2 - 2DTT_1\right) - \frac{cI_c\,(T-m)^2}{2} + \frac{pI_d\,Dm^2}{2}\right].$
……………(14)

$Z_2(T,T_1) \approx \frac{1}{T}\left[p\{\lambda T_1 + D(T-2T_1)\} - s - \frac{h}{2}\left(\lambda {T_1}^2 + DT^2 - 2DTT_1\right) + pI_d\,D(m-\frac{T}{2})\right]$ ……………(15)

Also from (5) we obtain

$T_1 = \frac{DT}{\lambda}\left(1 + \frac{\theta kT}{2}\right)$ …………(16)

Where $k = 1 - \frac{D}{\lambda}$. using (16) in (14) and (15), we obtain

$Z_1(T) \approx pD + p(2k-1)D\left(1 + \frac{\theta kT}{2}\right) - \frac{s}{T} - \frac{hkDT}{2}\left\{1 + \frac{k(1-k)\theta^2\,T^2}{4}\right\} - \frac{cI_c\,D}{2}\left(T - 2m + \frac{m^2}{T}\right) + \frac{pI_d\,Dm^2}{2T}$ …………(17)

$Z_2(T) \approx pD + p(2k-1)D\left(1 + \frac{\theta kT}{2}\right) - \frac{s}{T} - \frac{hkDT}{2}\left\{1 + \frac{k(1-k)\theta^2\,T^2}{4}\right\} + pI_d\,D(m-\frac{T}{2})$

……………(18)

Note that the propose of this approximation is to obtain the unique closed from solution for the optimal value of T. By taking first and second order derivatives of $Z_1(T)$ and $Z_2(T)$ from (17) and (18), with respect to T, we obtain

$$\frac{dZ_1(T)}{dT} = \frac{2s+Dm^2(cI_c - pI_d)}{2T^2} - \frac{3k^2(1-k)\theta^2 T^2}{8} - \frac{hkD}{2} - \frac{cI_c D}{2} + \frac{pk(2k-1)D}{2} \quad \ldots\ldots\ldots\ldots\ldots(19)$$

$$\frac{dZ_2(T)}{dT} = \frac{pk(2k-1)D}{2} - \frac{s}{T^2} - \frac{3k^2(1-k)\theta^2 T^2}{8} - \frac{hkD}{2} - \frac{cI_c D}{2} - \frac{pI_d D}{2} \quad \ldots\ldots\ldots\ldots(20)$$

$$\frac{d^2Z_1(T)}{dT^2} = -\left[\frac{2s+Dm^2(cI_c - pI_d)}{T^3} + \frac{3k^2(1-k)\theta^2 T^2}{4}\right] < 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

$$\frac{d^2Z_2(T)}{dT^2} = -\left[\frac{2s}{T^3} + \frac{3k^2(1-k)\theta^2 T^2}{4}\right] < 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots..(22)$$

From (21) and (22) it is clear that $Z_1(T) and\ Z_2(T)$ both are concave function of T. It can be seen from the following graph:

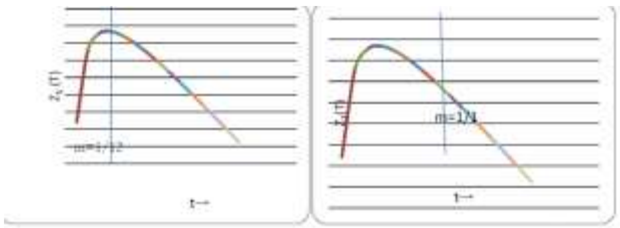

Fig. 2(a) Case 1: m ≤ T Fig. 2(b) Case 2: T ≤ m

The objective is to determine the optimal value of T = T^* for case I which maximum the total profit per unit time $Z_1(T_1^*)$. The necessary condition for $Z_1(T) to\ be\ maximum$ at a point T = T^* is that $\frac{dZ_1(T)}{dT}$ = 0 .solving $\frac{dZ_1(T)}{dT}$ = 0, (after neglecting $\theta^2 T^4$, $since\ \theta^2 T^4 \lll\lll 1$) , we obtain

$$T = T^* = \sqrt{\frac{2s+D(cI_c - pI_d)m^2}{D\{hk+cI_c - pk(2k-1)\}}} \quad \ldots\ldots\ldots\ldots(23)$$

The optimal value T = T^{**} is obtain by solving , $\frac{dZ_2(T)}{dT}$ = 0. (after neglecting $\theta^2 T^4$, $since\ \theta^2 T^4 \lll\lll 1$) , we obtain

$$T = T^{**} = \sqrt{\frac{2s}{D\{hk+pI_d - pk(2k-1)\}}} \quad \ldots\ldots\ldots\ldots\ldots(24)$$

The optimal economic order quantity for each case is given by

$$Q(T) = Q^*(T^*) = DT^* = \sqrt{\frac{2s+(cI_c - pI_d)m^2}{\{hk+cI_c - pk(2k-1)\}}} \quad \ldots\ldots\ldots\ldots\ldots(28)$$

$$Q(T) = Q^*(T^{**}) = DT^{**} = \sqrt{\frac{2sD}{\{hk+pI_d - pk(2k-1)\}}} \quad \ldots\ldots\ldots\ldots..(29)$$

In classical EOQ model with non-instantaneous receipt, the retailer must pay the payment at the beginning of each cycle. Hence the classical optimal economic order

$$Q^* = \sqrt{\frac{2sD}{\{hk+cI_c - pk(2k-1)\}}} \quad \ldots\ldots\ldots\ldots\ldots(30)$$

4.1 Numerical examples:

The effect of changing the parameters s, b, c, p, , I_d and on the optimal replenishment policy are studied by assuming the values for s, b, c, p, and are all 400,10, 200, 100,0.15,0.10,1/12, and 0.05 for case I and 200, 10, 200, 100, 0.15,0.1, 1/12, and 0.05 for case II . The results are summarized in tables 1-8.

The change in the values of parameters may happen due to variation or uncertainties in any decision – making situation. The sensitivity analysis will be very useful in decision making on order to examine the effect and variation of these changes. Using the above data, the sensitivity analysis of various parameters has been done.

The following inferences can be made from the result obtained.

(a) When ordering cost per order ‘s’ increases, the optimal receipt T_1, optimal cycle time T, and optimal order quantity Q increases while total profit per cycle decreases. That is, the change in‘s’ will cause the positive change in optimal receipt period, optimal cycle time, and optimal quantity while negative change in optimal total profit per cycle.
(b) When purchase cost ‘c’ increases, the optimal receipt T_1, optimal cycle time T, and optimal order quantity Q increases while total profit per cycle decreases. That is, the change in ‘c’ will cause the negative change in optimal receipt period, optimal cycle time, and optimal quantity while negative change in optimal total profit.
(c) When the selling price ‘p’ increases, the optimal receipt T_1, optimal cycle time T, and optimal order quantity Q decreases while total profit per cycle decreases. That is, the change in ‘p’ will cause the negative change in optimal receipt period, optimal cycle time, and optimal quantity while positive change in optimal total profit.
(d) When the unit holding cost ‘b’ increases, the optimal receipt T_1, optimal cycle time T, and optimal order quantity Q and optimal total profit decreases. That is, the change in ‘b’ will cause the positive change in optimal receipt period, optimal cycle time, and optimal quantity while negative change in optimal total profit.

Conclusion

A permissible credit period is usually allowed to a retailer to pay back the dues without paying any interest to the supplier. The retailer can pay the supplier either at the end of the credit period or later incurring interest charges on the unpaid balance for the overdue period. The retailer is expected to settle the account at a time before the end of the inventory cycle time because the payable interest rate is generally higher than the earned interest rate. A model for optimal cycle and payment times is developed here for a retailer in a deteriorating-item inventory situation where a supplier allows a specified credit period to the retailer for payment without penalty. Under these conditions, this supplier-and-retailer system is modeled as a cost minimization problem to determine the optimal payment time under various system parameters. An iterative search procedure is applied to solve the problem, and the overall findings indicate that the retailer always has an option to pay after the permissible credit period depending on unit purchase and selling price, the deterioration rate of the products and the interest rate.

The model can be extended in several ways. For instance, we may extend the model for stock-dependent demand rate. Also, we could consider the demand as a function of inflation or selling price as well as time varying. Finally, we could generalize the model to allow quantity discount, time value of money and others.

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