DESIGN AND PERFORMANCE MEASURES FOR TWO LEVELS CONTINUOUS SAMPLING PLAN MCSP-2-T

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***ABSTRACT*: In this paper we explore the procedure for "Design of Two level Continuous sampling plan of Modified CSP-2-T". The conventional measures of performance for continuous sampling plans such as average outgoing quality, average fraction inspected, probability of acceptance etc are evaluated for this modified CSP-2-T plan by using Markov chain model. Comparisons with other continuous sampling plans are presented. Tables and figures are also presented for the selection of the modified CSP-T plan when the AQL or LQL and AOQL are specified when the AQL or LQL and AOQL are specified.**

***KEYWORDS*: Average fraction inspected, Average outgoing quality, Average Outgoing Quality Limit, Continuous production process, Markov chain, Multi-Level Continuous Sampling Plan.**

Introduction:

Continuous Sampling has been proposed by Dodge for the inspection of continuous flow of discrete products with the aim of sorting the production in order to get the fraction non-conforming upper bound (AOQL). This is achieved by alternating sequence of screening inspection and sampling inspection. The CSP plan is the simplest and most commonly used single level continuous sampling plan. It also developed a new continuous sampling plan based on the use of cumulative sum deciding when to switch between the phases of sampling and 100% inspection.

The development of three tightened multi level plans of which tightest is designated as MLP-T and proposed a tightened three level continuous sampling plan designated as CSP-T .The simplification of the Markov chain approach to continuous formulation are presented in this plan. The screening and sampling inspection with three sampling levels are alternated in CSP-T Plan. Then the order of Production and the inspection is continued until the number of consecutive conformation unit reaches some pre specific number with 100% inspection.

MCSP-T plan is the operating procedure of the modified CSP-T Plan that shows if the first consecutive units are found non conforming and discontinue 100% inspection and switch to sampling inspection at level 2,where only a pre-specified fraction f/2 of the units are inspected. Otherwise it will move to level 1 inspection, then continue until a non conforming unit is found. When the unit is identified then it revert immediately to 100% inspection and continue to the procedure in the order of production.

Two level continuous sampling plan was modified in single level continuous plan that is MCSP-2-T.A flow diagram showing the operation of this plan which shows that one cannot go from one level of sampling inspection to another without going back to 100% inspection and often be reassuring in practice it help to make a option between plans at any rate of desirability in a particular situation. Then the formulation of MCSP-2-T procedure as a markov chain is discussed. The various performance measures are specified such as AOQ,AFI will be accepted on a sampling basis or the Probability of Acceptance (p_a) are derived.Markov Chain model is used to find the certain measures of performance in various sampling plan. it also assumed that the production process is in statistical Control.

THE MCSP-2-T PROCEDURE AS A MARKOV CHAIN

Let [Xm] (m=1,2,...) denotes a discrete parameter markov chain with finite state space (S_j) (j=1,2,...3i+7). The states of the process are defined in a way similar.

S_i = A_0 = Non –Conforming unit is found on 100% inspection.

S_{i+j+1} = B_j(j=1,2...i) = j consecutives conforming units found during 100% inspection after reverting from sampling inspection at level 2.

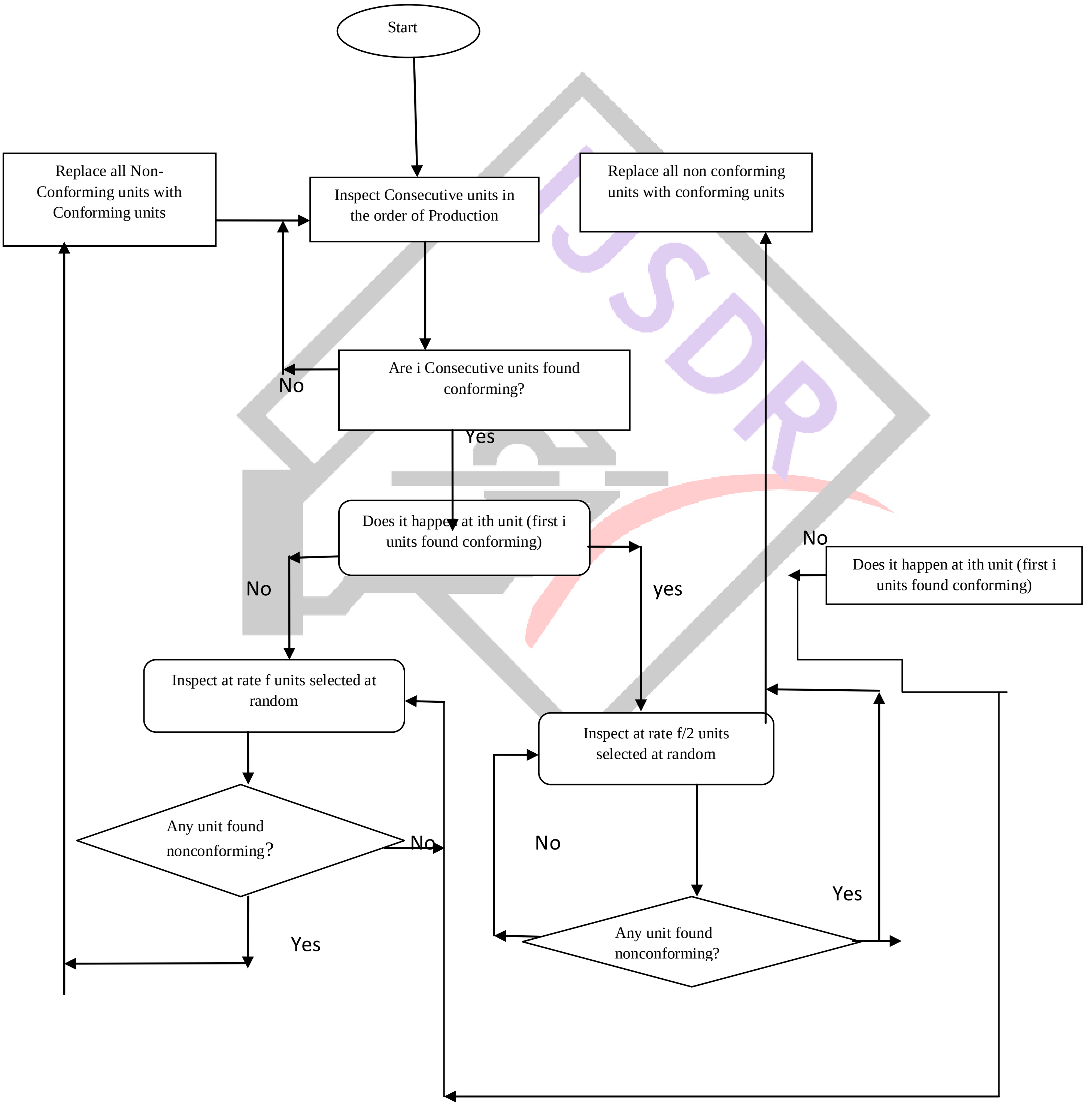
S_{2i+j+1}= A_j(j=1,2...i) = j consecutives conforming units found during 100% inspection after having a non conforming unit found on 100% inspection.

S_{3i+2}= Id_1= Sampling inspection at level 1 is in effect and the last inspected unit was found to be non conforming.
S_{3i+3}= In_1= Sampling inspection at level 1 is in effect and the last unit submitted for inspection was conforming.
S_{3i+4}=N_1=Sampling inspection at level 1 is in effect and the last unit produced was not inspected.
S_{3i+5}=Id_2=Sampling inspection at level 2 is in effect and the last inspected unit was found to be non conforming.
S_{3i+6}=In_2= Sampling inspection at level 2 is in effect and the last unit submitted for the inspection was conforming.
S_{3i+7}=N_2= Sampling inspection at level 2 is in effect and the last unit produced was not inspected.

The set of (3i+7) states defined above completely describe the mutually exclusive phased of inspection for the MCSP-2-T plan.A flow chart showing the description of the process by means of states and transition and one step transition probability matrix for the process are given.

The process also reveals the discrete parameter, finite, recurrent, irreducible, aperiodic Markov Chain.

Figure 1 Flow Diagram for the operation of MCSP-2-T plan



THE PERFORMANCE MEASURES OF MCSP-2-T PLAN

If one employs an MCSP-2-t plan for a process fraction non conforming p, the following performance measures may be obtained. Their derivatives are described in the Appendix.

1. The average number inspected (ANI) under the screening inspection is

$$ANI100(p)=\frac{(1-q^{i})(q+q^{2i})}{pq^{i}} \qquad (1)$$

2. The Average number inspected (ANI) under the Sampling inspection

$$ANISAM(p)=\frac{1+q^{i}+2q^{2i}}{p} \qquad (2)$$

3. The Average fraction of total produced units inspected in the long run is

$$AFI(p)=\frac{fq+fq^{i}+fqq^{i}+fq^{3i}+2fq^{2i}}{fq(1-q^{i})+q^{2i}(1+f)+q^{3i}(2-f)+q^{i}} \qquad (3)$$

4. The average fraction of the total production accepted on a sampling basis is

$$P_a(p)=\frac{fq+fq^{i}+fqq^{i}+fq^{3i}+2fq^{2i}}{fq(1-q^{i})+q^{2i}(1+f)+q^{3i}(2-f)+q^{i}} \qquad (4)$$

5. The average outgoing quality is

$$AOQ(p)=\frac{pq^{i}[1-f+q^{i}(2-f)]}{q^{i}+2f-fq(1-q^{2i})+2q^{2i}(1-fq^{i})} \qquad (5)$$

Figure-2 Flow chart showing states and transitions of the MCSP-2-T

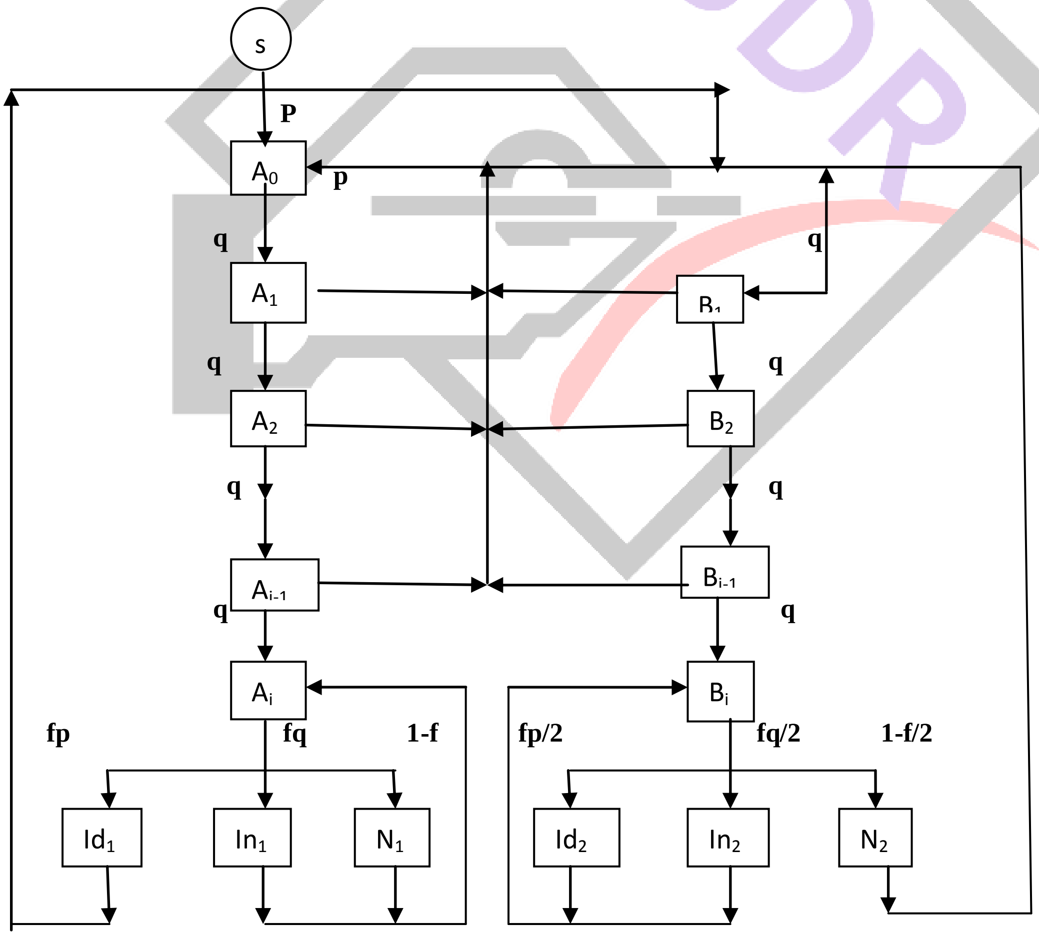


Table value for One step transition probability matrix of the MCSP-2-T plan

	A_0	B_1	B_2	......	B_I	A_1	A_2	......	A_I	Id_1	In_1	N_1	Id_2	In_2	N_2
A_0	P	0	0	......	0	q	0	......	0	0	0	0	0	0	0
B_1	P	0	q	......	0	0	q	......	0	0	0	0	0	0	0
B_2	p	0	0	......	0	0	0	......	0	0	0	0	0	0	0
.															
.															
.															
B_{i-1}	p	0	0	......	q	0	0	......	0	0	0	0	0	0	0
B_I	0	0	0	......	0	0	0	......	0	0	0	0	fp/2	fp/2	(1-f/2)
A_1	p	0	0	......	0	0	q	......	0	0	0	0	0	0	0
A_2	p	0	0	......	0	0	0	......	0	0	0	0	0	0	0
.															
.															
.															
A_{i-1}	p	0	0	......	0	0	0	......	q	0	0	0	0	0	0
A_I	0	0	0	......	0	0	0	......	0	fp	fq	(1-f)	0	0	0
Id_1	p	0	0	......	0	0	0	......	0	0	0	0	0	0	0
In_1	0	0	0	......	0	0	0	......	0	fp	fq	(1-f)	0	0	0
N_1	0	0	0	......	0	0	0	......	0	fp	fq	(1-f)	0	0	0
Id_2	p	q	0	......	0	0	0	......	0	0	0	0	0	0	0
In_2	0	0	0	......	0	0	0	......	0	0	0	0	fp/2	fp/2	(1-f/2)
N_2	0	0	0	......	0	0	0	......	0	0	0	0	fp/2	fp/2	(1-f/2)

CONCLUSION:

In this Paper, a modification to the CSP-2-T has been proposed. The basic principles of continuous sampling plan and related modified plans are included. It has been observed an important property that the Modified MCSP-2-T plan, MCSP-T Plan and CSP-T Plan have identical performance measures. The objectives and feature of the continuous plan is that one cannot go back from one level of sampling inspection to another sampling level without going back to 100% inspection. The Performance measures and table and figures are developed in this paper can be used for the selection of both CSP-T and the MCSP-2-T plans because of their nature is identical.

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Appendix A: Glossary of Symbols

p= The Probability of a unit produced by the process being non conforming.
q=1-p. i= The Clearance Number.
f= The Sampling Frequency at inspection level 1.
f/2= The sampling frequency at inspection level 2.
F= The average Fraction Inspected
$P_a(p)$= The Probability of acceptance during the sampling phase when the submitted product is of quality p.
AOQ(p)= Average outgoing quality when the submitted product is of quality p.
$S_j=j^{th}$ state of the process.
$\pi(S_j)$ = The Steady state probability for the state S_j.
P_{ij}=The Probability that the Process transits from state S_i to S_j in one step
α= The Producer's risk and β= The Consumer Risk.

Appendix B: Derivative of Performance Measures of the MCSP-2-T plan.

The Formulation of the MCSP-2-T Procedure as a markov chain with states(S_j)(j=1,2,3,...........3i+7) implies that the Markov chain is a DFRIA Markov chain .These Steady State probabilities $\pi(S_j)$ satisfy the following Conditions:

$$\pi(S_k) \geq 0, for\ k = 1,2, \dots \dots 3i + 7\pi(S_k) = \sum_{j=1}^{3i+7} \pi(S_j)P_{jk}, \text{for k} = 1,2, \dots ..3i + 7, \qquad \text{(A1)}$$

$$\sum_{j=1}^{3i+7} \pi(S_j) = 1 \qquad \text{(A2)}$$

Conditions A1 and A2 give the Following equations:

$$\pi(A_0) = p\left(\pi(A_0) + \textstyle\sum_{j=1}^{i-1}\left(\pi(A_j) + \pi(B_j)\right) + \pi(Id_1) + \pi(Id_2)]\right) \qquad \text{(A3)}$$

$$\pi(A_j) = q^j\,\pi(A_0), \qquad for\ j = 1,2 \dots .\, i, \qquad \text{(A4)}$$

$$\pi(B_1) = q[\pi(Id_2)] \qquad \text{(A5)}$$

$$\pi(B_j) = q^{j-1}\,\pi(B_1),\ \ for\ j = 1,2, \dots \dots i, \qquad \text{(A6)}$$

$$\pi(Id_1) = q^j\,\pi(A_0) \qquad \text{(A7)}$$

$$\pi(In_1) = \frac{q^{i+1}}{p}\,\pi(A_0) \qquad \text{(A8)}$$

$$\pi(N_1) = \frac{(1-f)q^i}{fp}\,\pi(A_0) \qquad \text{(A9)}$$

$$\pi(Id_2) = q^{2i}\pi(A_0) \qquad \text{(A10)}$$

$$\pi(In_2) = \frac{q^{2i+1}}{p}\,\pi(A_0) \qquad \text{(A11)}$$

$$\pi(N_2) = \frac{(2-f)q^{2i}}{fp}\,\pi(A_0) \qquad \text{(A12)}$$

By solving the above equation using the condition given below:

$$\pi(A_0) + \sum_{j=1}^{i}\left((\pi(A_j) + \pi(B_j)\right) + \sum_{j=1}^{2}[\pi(In_j) + \pi(Id_j) + \pi(N_j)] = 1$$

Then we get $\pi(A_0) = \frac{fp}{q^i + 2f - fq[1-q^{2i}] + 2q^{2i}(1-fq^i)} = \frac{fp}{D}$

By taking D=$q^i + 2f - fq[1 - q^{2i}] + 2q^{2i}(1 - fq^i)$, the steady state probabilities can now be written as follows;

$\pi(A_j) = \frac{f(1-q^{i+1})}{D}$, for j=1,2,.......,i, , $\pi(B_j) = \frac{fq^{2i}(1-q^i)}{D}$, for j=1,2,.....,i,

$\pi(Id_1) = \frac{fpq^i}{D}$, $\pi(In_1) = \frac{fq^{i+1}}{D}$, $\pi(N_1) = \frac{(1-f)q^i}{D}$

$\pi(Id_2) = \frac{fpq^{2i}}{D}$, $\pi(In_2) = \frac{fq^{2i+1}}{D}$, $\pi(N_2) = \frac{(2-f)q^{2i}}{D}$

$$\pi(100) = \left(\pi(A_0) + \sum_{j=1}^{i}\left(\pi(A_j) + \pi(B_j)\right)\right) = \frac{f(1 - q^i)(q + q^{2i})}{D}$$

$$\pi(S) = \sum_{j=1}^{2}[\pi(In_j) + \pi(Id_j) + \pi(N_j)] = \frac{fq^i[1 + q^i(1 + 2q^i)]}{D}$$

$$\pi(FN) = \sum_{j=1}^{2}[\pi(N_j)] = \frac{q^i[1 - f + 2q^i - fq^i]}{D}$$

Then, $ANI100(p) = \frac{\pi(100)}{\pi(A_i)} = \frac{f(1-q^i)(q+q^{2i})}{pq^i}$ and ANISAM(p)= $\frac{\pi(s)}{\pi(A_i)} = \frac{[1+q^i+2q^{2i}]}{p}$

$$\mathrm{Pa(p)} = \frac{\mathrm{nANISAM(p)}}{ANI100(p) + \mathrm{nANISAM(p)}} = \frac{q^i[1 + q^i + 2q^i]}{fq(1 - q^i) + q^{2i}(1 + f) + q^{3i}(2 - f) + q^i}$$

$$\mathrm{AFI(p)} = \frac{\mathrm{ANI100(p)} + \mathrm{ANISAM(p)}}{ANI100(p) + \mathrm{nANISAM(p)}} = \frac{fq + fq^i + fqq^i - fq^{3i} + 2fq^{2i}]}{fq(1 - q^i) + q^{2i}(1 + f) + q^{3i}(2 - f) + q^i}$$

$$\mathrm{AOQ(p)} = \mathrm{p}\,\pi(FN) = \frac{pq^i[1 - f + q^i(2 - f)]}{D}$$