

Near Optimal Pilot Design Schemes in OFDM Systems with Sparse Channel Estimation

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Abstract—Pilot design schemes based on MIP requires a sufficient condition for the optimal pilot pattern generated from the CDS which depends on CIR. Since in most practical OFDM systems CIR does not exist thus optimal patterns are difficult to obtain in such conditions. To obtain a near optimal pilot pattern we propose three pilot design schemes. The first two schemes are based on stochastic search and other one is based on iterative group shrinkage. Simulation results show that the proposed schemes converge much faster, out of which SSS and SPS schemes outperform the IGS schemes. Although both SSS and SPS result in similar Q values the former converges much faster than the latter. BER and MSE are also computed for these pilot designs.

Index Terms— Compressed sensing (CS), massive multiple input– multiple-output (MIMO), pilot design, sparse channel estimation, cyclic difference set.

I. INTRODUCTION

With the increase of communications technology, the demand for higher data rate services such as multimedia, voice, and data over both wired and wireless links is also increased. New modulation schemes are required to transfer the large amount of data which existing techniques cannot support. These techniques must be able to provide high data rate, allowable Bit Error Rate (BER), and maximum delay. Orthogonal Frequency Division Multiplexing (OFDM) is one of them. OFDM has been used for Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB) in Europe and for Asymmetric Digital Subscriber Line (ADSL) high data rate wired links. OFDM has also been standardized as the physical layer for the wireless networking standard HIPERLAN2 in Europe and as the IEEE 802.11a, g standard in the US, promising raw data rates of between 6 and 54Mbps. Orthogonal Frequency Division Multiplexing (OFDM) is a digital transmission method developed to meet the increasing demand for higher data rates in communications which can be used in both wired and wireless environments wireless communication. In wireless communication, OFDM is a well known solution for overcoming the problem of multipath fading channels. ISI is avoided in OFDM by adding cyclic prefix. OFDM is used in 4G systems; hence it provides high spectrum efficiency, resistance against multipath interference and ease of filtering out noise. Channel estimation in OFDM can be done by using pilot symbols at the transmitter and receiver side. Recent advances in compressed sensing (CS) have demonstrated sparse channel estimation can be more efficient than the conventional channel estimation approaches due to the sparse nature of multipath wireless channels. The CS techniques for pilot-assisted channel estimation have been widely investigated and many sparse recovery algorithms have been applied for channel estimation. Another focus of the sparse channel estimation is the design of pilots. According to the restricted isometry property (RIP) restricted

Isometry Property (RIP), it has been shown that the measurement using random matrices guarantees a high

probability of sparse recovery, indicating that the randomly generated pilot pattern is statistically optimal. However, the implementation of the random pilot pattern is more challenging in practical systems due to its high complexity, large storage, and low efficiency.

In [1] and [5], it has been shown that the pilot pattern generated from the cyclic different set (CDS) is optimal, and in [2], proposed an iterative tree-based searching algorithm to obtain a deterministic pilot pattern. In [3] and [4], two pilot design schemes based on cross-entropy optimization and stochastic approximation, respectively, are proposed to minimize the mean square error (MSE) using the channel data. X. He, R. Song et.al proposed a pilot allocation method based on genetic algorithm (GA) and shifting mechanism is proposed for multiple-input–multiple-output (MIMO) OFDM systems. P. Cheng et.al proposed a pilot design scheme for OFDM transmission over two-way relay networks. However, it assumes the number of OFDM subcarriers to be prime. Wenbo Ding et.al has proposed compressive sensing based channel estimation for OFDM systems under long delay channels supporting high-order modulations like 256 QAM.

In this paper, the pilot design based on the mutual incoherence property (MIP) is considered, which avoids acquiring of channel data. We first analyze the impact of different lengths of channel impulse response (CIR) and provide a sufficient condition that guarantees the pilot pattern generated from the CDS to be optimal. Then, we propose three pilot design schemes to obtain a near-optimal pilot pattern. The first two schemes are based on the stochastic search, namely, stochastic sequential search (SSS) and stochastic parallel search (SPS). The third scheme called iterative group shrinkage (IGS) removes rows in a group instead of removing a single row at each time. Considering the greedy manner of the IGS-based algorithm, a tree-based searching structure is applied to keep several best

intermediate results rather than only the best result at each iteration. Later it is extended to MIMO systems patterns for multiple transmit antennas. Extensive numerical comparisons are conducted for the proposed SSS, SPS, and IGS-based pilot design schemes, as well as the extension schemes for MIMO systems. Moreover, SSS and SPS outperform IGS in terms of channel estimation performance.

The rest of the paper is organized as follows. In section II provides the proposed model. A sufficient condition with respect to CIR length for pilot pattern generated from the CDS being optimal is shown in Section III. Section IV provide s the near optimal pilot design schemes. Section V shows the simulation results. section VI represents the conclusion.

II. PROPOSED METHOD

For pilot-assisted channel estimation in OFDM systems, we usually employ a comb-type pilot pattern on the time–frequency 2-D grids, as shown in the left portion of Fig. 1. Each column of the grids represents an OFDM symbol transmitted at a different time slot, and each row represents a subcarrier. Then, each subcarrier in an OFDM symbol forms the minimum resource unit, which is used to transmit either a pilot symbol or a data symbol. Once the subcarrier positions for the pilot symbols, marked in black in Fig. 1, are determined for some specified OFDM symbols, we use interpolations and channel tracking schemes to obtain channel estimates for the other OFDM symbols. It is well known that, for the channel estimations based on least squares (LS) and minimum MSE methods, the optimal pilot pattern in OFDM systems is equally spaced pilot subcarriers.

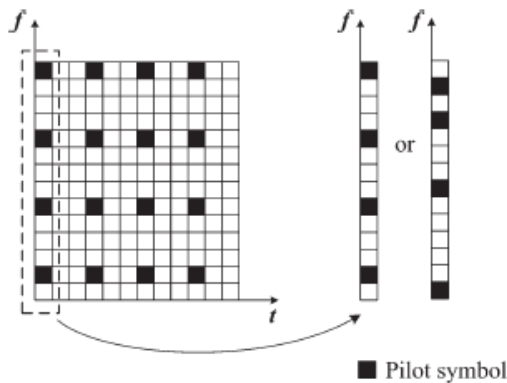


Fig.1 Pilot pattern in OFDM systems.

However, with the case of $N_p < L$ where N_p is number of pilot subcarriers and L denotes number of samples, we can save more pilots and therefore improve the data rate. In practice, since the sampling period is usually much smaller than the channel delay spread, most components of h are either zero or nearly zero, meaning that h is a sparse vector. With this a priori condition, we can use less pilots than the unknown channel coefficients, i.e., $N_p < L$, and apply CS algorithms to estimate h . In our paper the system model represents the OFDM system. After constructing an OFDM system, we need to obtain the MIP for pilot design and then obtain near optimal pilot patterns based on stochastic search schemes.

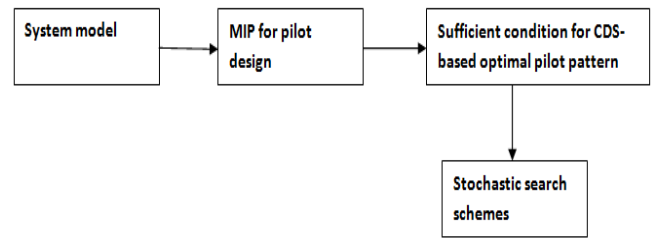


Fig.2 proposed pilot design scheme

III. ANALYSIS OF OPTIMAL PILOT PATTERN

With advance in CS show that, under noiseless condition, h can be reconstructed from the measurement y with a high probability when the dictionary matrix A satisfies the RIP. However, there is no existing method having polynomial complexity to check whether a given matrix satisfies the RIP.

Alternatively, we can minimize the coherence of A , which is known as the MIP. MIP is more intuitive and more practical than RIP. Therefore, in this paper, we consider the MIP as the pilot design rule. For a given pilot pattern

$$P = \{p_1, p_2, \dots, p_{N_p}\} \quad (1)$$

Where $1 \leq p_1 < p_2 < \dots < p_{N_p}$. The coherence of A is defined as the maximum absolute correlation between any two different columns of A . So that the objective of pilot design is to minimize the coherence of A given by

$$Q = \min g(p) \quad (2)$$

$$g(p) = \max | (A(m) \cdot A(n)) |, \quad 0 \leq m < n \leq L-1 \quad (3)$$

For an N -subcarrier OFDM system using N_p subcarriers as pilots, the pilot pattern according to the CDS (N, N_p) is optimal if the CDS (N, N_p) exists. For most practical OFDM systems with N being a power of two, e.g., $N = 64, 256, 512$, or 1024 , the CDS does not exist. Even if the condition of N for the existence of the CDS is satisfied, various choices of N_p might be needed in practical systems. Moreover, we usually set the length of the cyclic prefix of OFDM to be $N/4$, which is much larger than the CIR length L , i.e., $N/4 \geq L$. Therefore, it is important to explore practical schemes to design near optimal pilot pattern for any setting of (N, N_p) and any value of L .

IV. NEAR OPTIMAL PILOT DESIGN SCHEMES

We propose three low-complexity practical schemes to obtain near-optimal pilot patterns for any given pair of (N, N_p) and for any value of L . The first two schemes are based on the stochastic search, which searches for the near-optimal pilot pattern with two loops of iterations. The third scheme, which is called IGS, forms the resulting pilot pattern in a reversed fashion by sequentially removing subcarriers from the subset of all available OFDM subcarriers until the number of the remaining OFDM subcarriers reaches the desired size of the pilot pattern.

A. Stochastic Search Schemes

The two stochastic search schemes consist of two levels of loops. In the outer loop, we randomly generate pilot patterns as the initializations of the inner loop. In the inner loop, we iteratively update the resulting pilot pattern in a greedy manner. For the pilot update in the inner loop, we propose two alternatives, i.e., the sequential search and the parallel search.

1) SSS: Given the maximum numbers of iterations for the outer and inner loops, i.e., M1 and M2, respectively, the SSS scheme is described as follows.

In each iteration of the outer loop, we randomly generate a pilot pattern $p \subset N = \{1, \dots, N\}$ as the initialization of the inner loop. In each iteration of the inner loop, we perform a sequential update of each entry of p according to the following step.

For $k = 1, \dots, N_p$, given the latest p from the last iteration, we update the k^{th} entry of p with the best one selected from $N \setminus \{p(i) \mid i = 1, \dots, N_p, i \neq k\}$, which results in the minimum MIP.

$$\hat{p}_{p,k} = \arg \min_{\substack{p^*(i)=p(i), i=1,2,\dots,N_p, i \neq k \\ p^*(k) \in N \setminus \{p(i), i=1,2,\dots,N_p, i \neq k\}}} g(p^*). \quad (4)$$

For each initial pilot pattern in the outer loop, we obtain a corresponding optimized pilot pattern. With M outer loop iterations, we then obtain M1 optimized results, from which we select the one with the minimum MIP as the final output.

2) SPS: To be more conservative, we present a parallel search scheme for the inner-loop update. Therefore, the only difference between this scheme and the SSS scheme is in the inner-loop iteration. In each iteration of the inner loop, we perform a parallel update of p according to the following steps.

For $k = 1, \dots, N_p$, given the latest p from the previous inner-loop iteration, we obtain $\hat{p}_{p,k}$ according to (4) for the k^{th} entry. Note that unlike SSS, we do not immediately update p with $\hat{p}_{p,k}$.

We then obtain $p_{p1}, p_{p2}, \dots, p_{pN_p}$, each one corresponding to an entry update of p . From them, we select the best update with the minimum MIP as the final update for this inner-loop iteration.

B. Iterative Group Shrinkage

A generalized tree-based scheme called IGS is proposed in order to further improve the performance. With this scheme, in each iteration, we jointly select and remove N_r rows from the latest updated sub matrix so that the sub matrix with the remaining rows has the smallest coherence. For that, we first extend the definition of MIP function $g(p)$ in (3) and define

$$gw(q) = \max_{0 \leq m < n \leq L-1} | \langle Wq(m), Wq(n) \rangle | \quad (5)$$

Where q can be a subset with any size smaller than N , Wq is a sub matrix formed by the selected rows of W with row indices given by q , and $Wq(n)$ denotes the n^{th} column of Wq . Given the predetermined N_r and N_t , the IGS scheme is described as follows.

With the initial N , we exhaustively search a subset $\Phi \subset N$ with $|\Phi|_0 = N_r$, and for each obtained Φ , we calculate the objective

$gw(q)$ according to (5) where $q = N \setminus \Phi$. From all of $[N \setminus N_r]$ obtained subsets, we choose N_t best subsets, i.e., $\Phi_1, \Phi_2, \dots, \Phi_{N_t}$, with N_t smallest objective.

Define $N^* = N - N_r$ and $N_i^* = N \setminus \Phi_i$, $i = 1, \dots, N_t$. In other words, we construct a tree whose top level includes N_t parent nodes representing N_t best subsets $\Phi_1, \Phi_2, \dots, \Phi_{N_t}$. Then iteratively select N_r rows from the remaining N^* rows and update N_i^* , $i = 1, \dots, N_t$. From the perspective of the tree structure, we understand it as iteratively generating N_{2t} leaf nodes from the N_t parent nodes, and then selecting N_t best leaf nodes as parents nodes for the next iteration. Eventually, we select a best branch stringed by the surviving nodes and then remove the branch from N and output the remaining row indices as the designed pilot pattern. The iterative steps can be described as below.

For each N_i^* , $i = 1, \dots, N_t$, from the last iteration, we exhaustively search over all subsets $\Phi \subset N_i^*$ to find the N_t best results. Considering that if N_r does not divide $(N - N_p)$, in the last iteration, we remove $N^* - N_p$ rows instead of removing N_r rows. For each obtained Φ , we calculate the objective $gW(q)$ according to (5) where $q = N_i^* \setminus \Phi$. From all of $(N^* - 1)$ obtained subsets, we find N_t best subsets, i.e., $\Phi_{i1}, \Phi_{i2}, \dots, \Phi_{iN_t}$, with N_t smallest objective. Then we update $N^* = N^* - 1$. If it is the last iteration, i.e., $N^* \leq N_p$, we find the best result Φ_{i^*,k^*} from $\{\Phi_{ik}, i = 1, \dots, N_t, k = 1, \dots, N_t\}$ obtaining the final result $p^* = N_i^* \setminus \Phi_{i^*,k^*}$. Then we terminate the procedure and output p^* . If $N > N_p$, from $\{\Phi_{ik}, i=1, \dots, N_t, k=1, \dots, N_t\}$ with N_t^2 entries, we select N_t best entries $\Phi_{is,ks}$, $s = 1, \dots, N_t$. We obtain $N_s^* = N_{is}^* \setminus \Phi_{is,ks}$, $s = 1, \dots, N_t$. Then we update $N_s^* = N_s^* - 1$, $s = 1, \dots, N_t$. The proposed stochastic search schemes, including SSS and SPS, can be readily extended to MIMO systems with N_u transmit antennas and N_v receive antennas.

V. SIMULATION RESULTS

TABLE I: Performance of the IGS pilot design scheme with various N_r settings for $N = 73$, $N_p = 9$, $N_t = 2$, and $l = 37$

N_r	Q	Running time
1	4.815 0	1.76
2	4.377 6	22.52
3	4.125 2	274.98
4	3.974 2	2824.45

Fig.3 shows the updating histories of $g(p)$ as a function of running time for the proposed SSS and SPS, as well as some results of the IGS scheme from Table I and marked as points corresponding to the resulting Q and the running time needed. The stopping time for all these schemes is set as 1893 s, which is the time required by the IGS scheme with $N_t = 2$ and $N_r = 2$. It is shown in Fig.3 that all the proposed methods, including the SSS, SPS, and IGS schemes perform much better than the cross-entropy optimization and the exhaustive search in terms of both Q and the convergence rate. It is also observed that the SSS

and SPS schemes outperform the IGS. The MSE performance in Fig 4 shows that SSS and IGS offer better results.

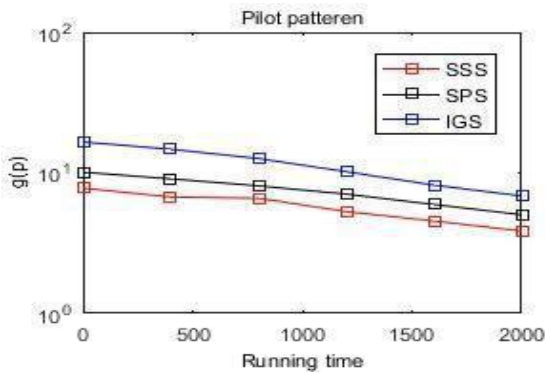


Fig.3 Comparison of proposed schemes to design the pilot pattern.

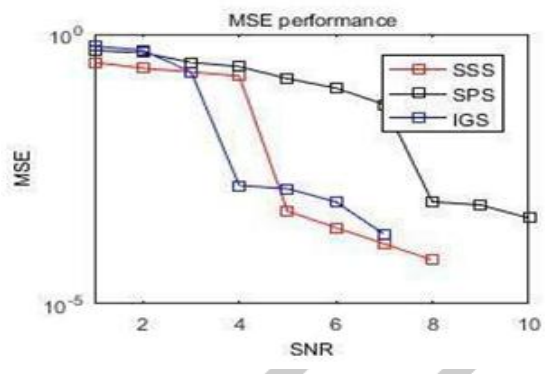


Fig.4 MSE performance of proposed algorithms

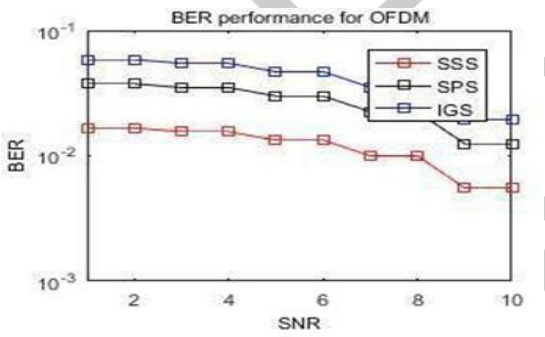


Fig.5 BER performance comparisons of channel estimation for proposed pilot design schemes.

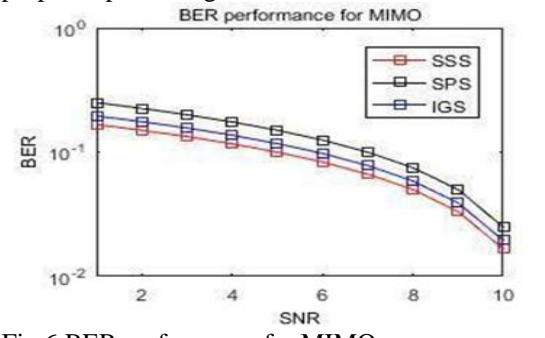


Fig.6 BER performance for MIMO

While comparing the 3 proposed schemes in OFDM, the best performance is offered by SSS and moderate performance is offered by SPS which is shown in Fig 5. In Fig 6 it is observed that the pilot obtained from SSS performs slightly better than IGS.

VI. CONCLUSION

MIP based pilot design schemes for sparse channel estimation in OFDM systems is considered in this paper. Three pilot design schemes, including SSS, SPS, and IGS to obtain a near-optimal pilot pattern are proposed and also extended to MIMO systems to design multiple orthogonal pilot patterns. Simulation results have demonstrated that the proposed SSS, SPS and IGS schemes are efficient out of other design schemes and SPS schemes outperform the IGS scheme in terms of channel estimation performance.

VII. REFERENCES

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