Multi-Objective Stochastic Programming with a Joint Probability Constraint approach for Production Planning Problems: A Model for SMEs

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Abstract- **In India, 95% of enterprises are small and medium enterprises (SMEs), and they play a major role in generating employment and contribute significantly to the export of the country. In recent years, SMEs are finding themselves in an intensely competitive environment and as a result the growth of these enterprises has slowed down considerably. Large and unpredicted variations in supply and demand fluctuations are the critical factors for the SME which leading to increase in manufacturing cost. In this context, literature has suggested that implementation of appropriate production planning approaches would be beneficial to the SMEs as this would help them to overcome the constraints of infrastructural resources. In this paper an attempt is made to develop a methodology to find optimal solutions under supply and demand variations environment. A statistical model has been developed using chance constrained programming technique and a deterministic model based on the stochastic model has been made. To solve the multi-objective deterministic problem obtained, we use only one objective at a time and ignoring the others.**

IndexTerms— **Small and medium Manufacturing Enterprises, Production Planning. Multi-Objective Stochastic Programming**

I. Introduction

Small and medium enterprises (SMEs) occupy a place of strategic importance in the economic growth of India. In addition to generating employment and fostering entrepreneurship, their contribution in the growth of exports is also significant According to the third all India census of small industries, there are around 11.85 million small and medium enterprises (SMEs) in India out of which,1.63 million units are registered and rest are unregistered. It is estimated that nearly, 95% of industrial units are in the SME sector. With a total size of US $140 billion, this sector forms the backbone of industrial development and is one of the driving forces for the long term growth of India with a substantial share in industrial production, export and employment.

Recognizing the market potential, many of the global players are shifting their production bases to the developing countries like India. In the last few years, nearly 508 new TNCs have been established in various sectors and this number is expected to grow further (UNCTAD, 2008). In order to achieve cost reductions, more and more the large enterprises (LEs) are transferring some of their production and process activities to SMEs through industrial subcontracting. As most of the SMEs are components manufacturers they largely depend on the original equipment manufacturers (OEMs) for their growth and survival. Entry of the global giants has created intensified competition amongst the suppliers. High quality standards with increased component complexities have become primary requirements. In addition to high variety with low volumes, delivery of components on time becomes a difficult task for the SMEs with the existing infrastructural resources. Due to high investment and lack of government subsidies, many of the small firms could not enhance their technologies to enable them for higher scale of operations. As a result, the growth of these enterprises has slowed down considerably and their survival is at stake (Okada, 2004 and Saranga, 2009). In this context, it is important to study and identify the determinants that would ensure the sustenance and growth of these enterprises.

Production Planning is an important strategy for effective operations management for any manufacturing industry. Particularly for SMEs effective production planning approaches plays a crucial role since they are contained in terms of limited resources. Main objective of production planning is meant to arrive at the frame work of the manufacturing operations during the period planned. Customer requirements, availability of resources are the important considerations for effective planning for the given time frame. Customer demands for products are sometimes may uncertain and non-stationary. Accordingly raw material and other direct materials have its own non-homogeneous and random characteristics. Thus, the quantities of products that can be produced by each process varies randomly. So, the production plans have to be arrived in such a way that they must respond to the in the face of external and internal uncertainties, namely uncertain product demands, uncertain raw material supplies etc., so,SMEs should be in a position to address a multi-period, multi-product (MPMP) production planning problem in a manufacturing environment, where alternative processes produce simultaneously multiple products from several varieties of raw materials. In this paper, a solution methodology is developed for SMEs when both supply and demands are uncertain with a joint constraint. Related literature on production planning is presented in the next section.

II. RELATED LITERATURE

Owing to the importance of minimization of cost of manufacturing,Concentration on production planning problems gaining importance.many researchers (Dantzig 1955; Kall and Wallace1994; Mulvey et al.,1995; Birge and Louveaux 1997; and Kall and Mayer 2005) have presented various approaches and robust optimization methods for problems in production. A detailed review of the published literature on production planning applications is also provided in Mula et al. (2006). Chern and Hsieh have proposed heuristic approaches algorithms to provide solution for production planning problems of various manufacturing applications.

To tackle uncertainties in the resources few researchers developed stochastic programming based mathematical models. So,we can now be concentrate on the stochastic programmes and model applications are extended to multi-stage multi period and multi product models. Huang (2005) emphasized on the multistage stochastic programming models for production and capacity planning with uncertainty.a multi-stage stochastic programming approach for multi item capacitated lot-sizing with uncertain demand proposed by Brandimarte (2006). a two-stage stochastic model production planning with uncertain yield for multi- period multi product based planning proposed by Kazemi Zanjani et al. (2007) also he proposed robust optimization models with different recourse cost variability measures to address production planning with uncertain yield.

The current approaches in the literature for solving uncertainties are two categories one is the single parameter based i.e. either for demand or supply (Escudero et al. 1993, Bakir and Byrne 1998, Brandimarte 2006, 4702 M. Kazemi Zanjani et al. Kazemi Zanjani et al. 2007, 2008a, 2008b) and the other is set of parameters based for simultaneous handling of both supply and demands (Leung and Wu2004, Huang 2005, Wu, 2006, Leung et al. 2007, Khor et al. 2008). To optimize the production costs, researchers concentrated on multi-objective perspective to generate solutions for production planning problems. Researchers developed (Leclercq, 1982; Goicoechea et al., 1982; Charles and Dutta, 2006 and Abdelaziz et al., 2007) multi-objective stochastic programming problems for production planning problems.

From the literature it can be revealed that most of the researchers have attempted to solve production problems pertain to large scale manufacturers and to the knowledge of the authors; no much work has been done aiming on SMEs or small scale manufacturers. Until now many of the methodologies presented in the literature are for problems of uncertainties in supply and demands separately and combination of both. In addition to that

In this paper, in addition to the combined uncertainties in supply and demands, a cumulative uncertainty effect on both supply demand is also considered.

III. PRODUCTION PLANNING PROBLEM OF AUTOMOBILE MANUFACTURING UNIT

A small scale automotive component manufacturing industry produces and supply parts to an automotive vehicle manufacturer based on lators work orders. Initially, it is supplied a tentative quarterly work order and later on orders are confirmed before commencement of production for every month. Intermittently, vehicle manufacturer also revises work orders and component manufacturer supplies accordingly. By the end of the month there will be variations in work orders given by the manufacturer and supplies made by the component manufacturers also. Due to variations in demands as well as supplies, manufacturing cost for SMEs would increase.To optimize the Cost a methodology has been proposed in this paper.

IV. A CASE STUDY

Application of proposed methodology to CPL
Consider a three month period in which a particular product is manufactured by an automobile manufacturing unit. Let us also assume we can predict the average demand and supply along with certain variance exhibited by the product in a three month period. The manufacture of the number of units of a product depends on the supply but the option of selling at a later date is reserved with the enterprise. Also the number of units of the product sold in a month depends on the demand of the product.

Let, Cost of manufacturing = Rs M per unit.

Cost of storage of already produced product = Rs K per month per unit.

Cost of over production to meet demand in a month = Rs P per month per unit.

Mean supplies & variances predicted in given period = S_1, S_2, S_3 & sv_1, sv_2, sv_3

Mean demands & variances predicted in given period = D_1, D_2, D_3 & dv_1, dv_2, dv_3

	Sold Per Month				Supply	
Manufactured Per Month		1	2	3	Mean	Variance
	1	x_1	x_2	x_3	S_1	sv_1
	2	x_4	x_5	x_6	S_2	sv_2
	3	x_7	x_8	x_9	S_3	sv_3
Demand	Mean	D_1	D_2	D_3		
	Variance	dv_1	dv_2	dv_3		

Where $x_2 = M+K$, $x_3 = M+2K$, $x_4 = M+K$, $x_6 = M+K$, $x_7 = M+2K$, $x_8 = M+K$.

Multi Objective Stochastic Programming problem:

Minimization of overall Production cost $(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9)$

Minimization of overall storage cost $(x_2+2x_3+x_6)$

Minimization of overall over production cost $(x_4+2x_7+x_8)$

Subject to:

$$Pr\,[x_1+x_2+x_3 >= b_1;$$
$$x_4+x_5+x_6 >= b_2;$$
$$x_7+x_8+x_9 >= b_3;$$
$$x_1+x_4+x_7 >= b_4;$$
$$x_2+x_5+x_8 >= b_5;$$
$$x_3+x_6+x_9 >= b_6;\ ] >= 1-\alpha$$

The mean and standard deviations of b_1, b_2, b_3, b_4, b_5, b_6 are:

$E(b_1)= S_1$ $(b_1)= sv_1$

$E(b_2)=S_2$ $(b_2)=sv_2$

$E(b_3)=S_3$ $(b_3)=sv_3$

$E(b_4)=D_1$ $(b_4)=dv_1$

$E(b_5)=D_2$ $(b_5)=dv_2$

$E(b_6)=D_3$ $(b_6)=dv_3$

Using chance constrained programming technique, the equivalent deterministic programming problem is:

$$\text{Min: } M^*(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9)$$
$$\text{Min: } k^*(x_2+2x_3+x_6)$$
$$\text{Min: } p^*(x_4+2x_7+x_8)$$

Subject to: $x_1+x_2+x_3- sv_1*B_1=S_1$;
$x_4+x_5+x_6- sv_2*B_2=S_2$;
$x_7+x_8+x_9- sv_3*B_3=S_3$;
$x_1+x_4+x_7- dv_1*B_1=D_1$;
$x_2+x_5+x_8- dv_2*B_2=D_2$;
$x_3+x_6+x_9- dv_3*B_3=D_3$;

$$1.2533141(1+2y_1)(3-B_1^2) <= 3B_1 exp(-B_1^2/2)$$

$$1.2533141(1+2y_2)(3-B_2^2) <= 3B_1 exp(-B_2^2/2)$$

$$1.2533141(1+2y_3)(3-B_3^2) <= 3B_1 exp(-B_3^2/2)$$

$$1.2533141(1+2y_4)(3-B_4^2) <= 3B_1 exp(-B_4^2/2)$$

$$1.2533141(1+2y_5)(3-B_5^2) <= 3B_1 exp(-B_5^2/2)$$

$$1.2533141(1+2y_6)(3-B_6^2) <= 3B_1 exp(-B_6^2/2)$$

$$\prod (y_1 y_2 y_3 y_4 y_5 y_6) >= 1- \alpha$$

$y_1, y_2,\dots y_6$ <=1 $x_1, x_2,\dots x_9$ >=0
$B_1, B_2,\dots B_6$ are unrestricted in sign.

DATA OF AUTOMOBILE MANUFACTURING UNIT:

An example of the application of the given methodology to real parameters is given below. M= 50; K= 5; P= 10;

		Sold Per Month			Supply	
		1	2	3	Mean	Variance
Manufactured per Month	1	50	55	60	85	5
	2	60	50	55	110	10
	3	70	60	50	130	10
Demand	Mean	105	70	150		
	Variance	5	10	10		

1- α =0.84

RESULT:

Production Plan for the given data using the suggested methodology is obtained by Matlab software:

	Sold/ Month				Supply	
Manufactured/ Month		1	2	3	Mean	Variance
	1	70	10	-	85	5
	2	25	-	60	110	10
	3	-	50	90	130	10
Demand	Mean	105	70	150		
	Variance	5	10	10		

V. Solution Methodology

Aim is to maximize profits of a SME by minimizing its manufacturing cost & optimize production schedules to match predicted demand with supply considering variations based on past data. The methodology is presented below

- Step 1: Formulate a stochastic programming problem from the given real life situation taking various constraints.
- Step 2: Using chance constrained programming technique, convert the problem to an equivalent deterministic programming problem.
- Step 3: Solve the multi-objective deterministic problem obtained from Step 2, using only one objective at a time and ignoring the others. Repeat the process K times for the K different objective functions. Let X(1), X(2)…, X(K) be the respective ideal solutions of the K objective functions.
- Step 4: Using the solutions obtained in Step 3, find corresponding values of all the objective functions at each of the solutions.
- Step 6: From step 4, obtain the upper and lower bounds (Uk and Lk , k=1,…,K) for each of the objective functions.
- Step 7: Using a linear membership function, formulate a crisp model. By introducing an augmented variable formulate single objective non-linear programming problem.
- Step 8: Provide production plans based on crisp model obtained in Step 7.

VI. CONCLUSION

This paper proposed the mathematical model which will provide the solution for production planning with variable demand and variable supply with uncertainty in small and medium scale enterprises which will be helpful for its growth.

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