Effect of backlash on bending stresses in spur gears

[1]Dyaneshwar Shinde, [2]Prof. K. S. Mangrulkar

[1]M. E. –Mech-,Design Engg. student,
[2]Associate Professor,
[1]Mechanical Dept., B. M. I.T., Solapur. ,
[2]Mechanical Engg. Dept., N. B. N. Sinhgad College of Engg., Solapur, India

***Abstract*— Backlash in gears is necessary to allow thermal expansion of gear teeth, lubrication and to avoid jamming of gears. Bending stresses in gears may lead to failure of gear teeth. In this research work, effect of backlash on bending stresses in spur gears is studied using Finite Element based software ANSYS. Here the backlash in spur gears is provided by reducing the tooth thickness. Models are prepared using the drafting software AutoCAD. IGES files are then imported to ANSYS where further processing is done. Maximum Von Mises stresses and Maximum deformation are noted during the analysis. For validation of the Finite Element model, use of analytical equation of bending stress is done. As the stresses obtained by finite element are near to the analytical stresses, further study is done using Finite Element method.**

IndexTerms*—Finite element analysis, backlash, bending stresses, deformation .(keywords)*

I. INTRODUCTION

Backlash is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles. In other words, when the sum of the thickness of two teeth out of mating gears on operating pitch circle would be less than circular pitch, it is said those two gears are engaged with backlash, meaning there is a gap between teeth of two mating gears. As actually indicated by measuring devices, backlash may be determined variously in the transverse, normal, or axial-planes, and either in the direction of the pitch circles or on the line of action. Such measurements should be corrected to corresponding values on transverse pitch circles for general comparisons.

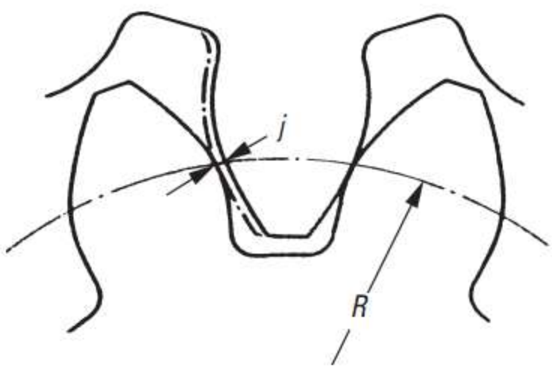

Figure 1 Backlash in gears

Being designed with standard features & being installed in standard center distance, the gears work ideally & there will be no backlash.

The basic purpose of designing backlash is to prevent locking the gears, as well as to prevent coming into contact in both side of one tooth simultaneously. A little of backlash is desirable to make necessary gap for lubrication & partially expansion of gears. Also, the amount of backlash should not be increased exceedingly, because the backlash causes high manufacturing costs.

In general, there are two ways for providing backlash in mating gears.

1. Reducing tooth thickness
2. Installing two mating gears in a distance which would be more than the standard center distance.

Both methods can happen by designing or unintentionally. It means that either errors in gear generation process & installing or designing conditions considered by designer in advance can be the reasons of making backlash.

If the designer wants to design backlash by reducing tooth thickness it is common to consider half the amount of backlash for each gear. So that, the sum of the decreased thickness amount in each gear would be equal to backlash backslash amount. While the gear generation process for making backlash, cuter tool should shave gear blank a little more than the standard cutting depth. Though, there are some exceptions to this rule.

II. PROBLEM DEFINITION

The stress field at the spur gear tooth root will be analyzed using a two-dimensional finite element solid model. The study will be for gears geometries used in the reduction gearing arrangement as shown in Figure 1.

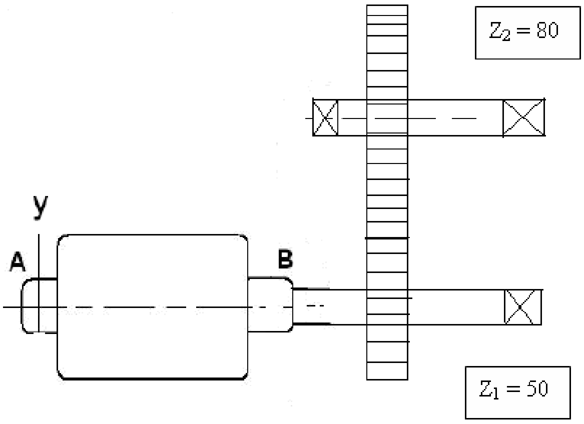

Figure 1 Gearing Arrangement of the problem considered

Table 1 Specification of Gears

Gear	**Speed**	*Rpm*	**Teeth**	*Nos.*	**P.C. D.**	*mm*
Pinion	n_1	750	Z_1	50	d_1	150
Gear	n_2	397	Z_2	80	d_2	240

As both the pinion & gear are made of same material, pinion is weaker. So the stress analysis of the pinion is carried out.

Parameters:

Motor Power (P) = 15 HP (8 Pole, Speed 750 Rpm)
Module (m) = 3 mm
Face Width (b) = 20 mm
Pressure angle = 20^o

Calculation of tangential force:

As the tangential force is responsible for bending stress, the same is calculated as below.
The forces between Gear No. 1 & 2 are calculated in the following way.
F_t = P/V where,
F_t is tangential force in Newton,
P is power in Watts, &
V is the pitch line velocity in m/s.

P = 15 H.P x 746 = 11190 Watts.

$$V = \frac{\pi . d_1 n_1}{60}$$

d_1 = 150 mm = 0.15 m (Given)

$$V = \frac{\pi \text{ x } 0.15 \times 750}{60}$$

V = 5.89 m/s
F_t = 11190/5.89
$\mathbf{F_t = 1899.83\ N \approx 1900\ N}$

III. ANALYTICAL APPROACH

Considering tangential load only, Bending stress is given by

$$\sigma = \frac{F_t}{b.m.J}$$

$$= \frac{1900}{20x3x0.45778}$$

σ = 69.17 Mpa

Determination of location of HPSTC & angle of loading for gearing system without backlash

Let HPSTC Radius = R_H

$$R_H = \sqrt{R_a^2 + P_b(1-m_p)[P_b(1-m_p) + 2\sqrt{R_a^2 - R_b^2}]}$$

R_a = Addendum Circle Radius
P_b = Base Pitch = cos ϕ x π m
m_p = Contact ratio = Length of Arc of Contact / π m
= (Length of path of contact /cos ϕ) /π m
Length of path of contact = $\sqrt{(R_a^2 - R^2 cos^2 \emptyset)}$ + $\sqrt{(r_a^2 - r^2 cos^2 \phi)}$ - (R+ r) sinϕ
R = Pitch Circle Radius

After calculations following results obtained.

HPSTC Radius = R_H = 75.267 mm

$\phi_1 = \phi$ - (tan ϕ - tan ϕ_H + $\frac{\pi}{2N}$)
But ϕ_H = $\cos^{-1}$ (R_b /R_H)
R_b = Base Circle Radius = Pitch Circle Radius x cos ϕ

ϕ_H is involute pressure angle at point H

ϕ_1 is operating pressure angle.

Angle at which the force acts = ϕ_1 = 18.826°

Tangential force = Ft = 1900 N
Hence,
Radial force = Fr = Ft X TAN ϕ_1 = 1900 tan 18.826
= 1900 x 0.341 = 647.9 N

Making backlash by reducing the tooth thickness:

Here $\phi_1 = \phi$ - (tan ϕ - tan ϕ_H + $\frac{\pi}{2N}$ - $\frac{B}{4R_p}$)
where B is final amount of backlash divided up into equal parts on each gear.

Table 2 Maximum Von Mises Stress & Maximum Deformation for different values of backlash

Sr. No.	Amount of Backlash (B) in mm	ϕ_1 (in degrees)	F_t (N)	$F_r = F_t$ X TAN ϕ_1 (N)
1	0.15	18.88326	1900	649.895
2	0.20	18.90235		650.602
3	0.25	18.92145		651.310
4	0.30	18.94054		652.017
5	0.35	18.95963		652.725
6	0.40	18.97872		653.433

IV. FINITE ELEMENT ANALYSIS

a) Finite element analysis of pinion without backlash

a) Pre-processing: During the pre-processing step, model of the pinion is prepared using the facilities in AutoCAD. Then files are exported from AutoCAD in IGES form. These IGES files are then imported in ANSYS. Figure 2 shows model with load and boundary conditions. Meshed model is having 240 nodes and 67 elements.

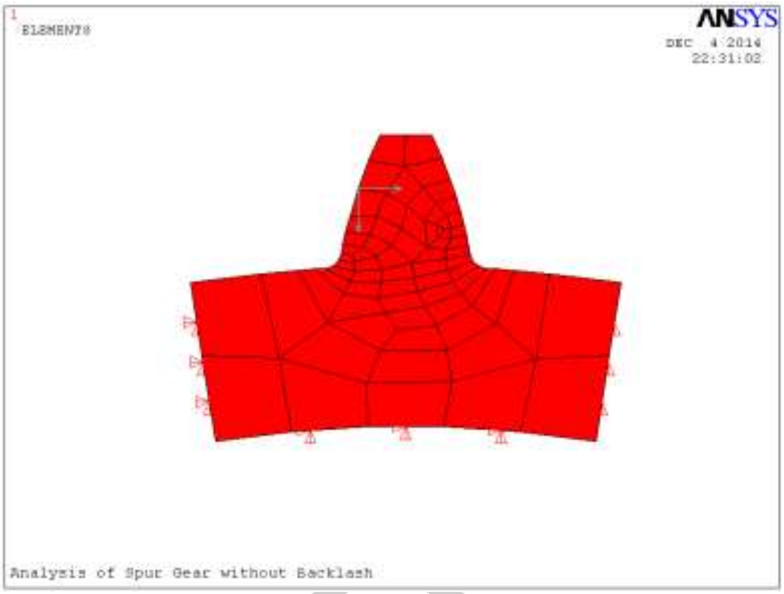

Figure 2 Load conditions along with boundary conditions

Material properties: As the material of the pinion is steel, the following material properties are used for analysis.

Table 3 Input properties

Young's Modulus	2.1 X 10^5 Mpa
Poisson's ratio	0.3

Boundary condition: Rotary & linear movements of all points lying on the radial lines & inner circumferential line are restricted. i.e. All degrees of freedom have been locked for these lines. Figure 2 shows model with the boundary conditions applied. The small triangles on the surfaces show that the displacement of these surfaces is restricted.

Load condition: The tangential load acting is equal to 1900 N for 20 mm thick gear. i.e. A load of 1900/20 = 76.15 N/mm is applied to the model at the tip of the tooth. Figure 2 shows the model with both boundary conditions & load conditions.

b) Processing: In this step, discritized model with load and boundary condition is submitted to the solver.

c) Post-processing: In this step, necessary steps are taken in order to observe the Von Mises stresses and deformation of the model.

__**Validation of finite element model**__

Bending stresses in pinion without backlash

Figure 3 shows the bending stresses. Here as the load is applied at the HPSTC point of the tooth, the maximum stresses are at this tip as this load is point load. So neglecting the stresses at the point of application of load, stresses at the root of the teeth are observed.

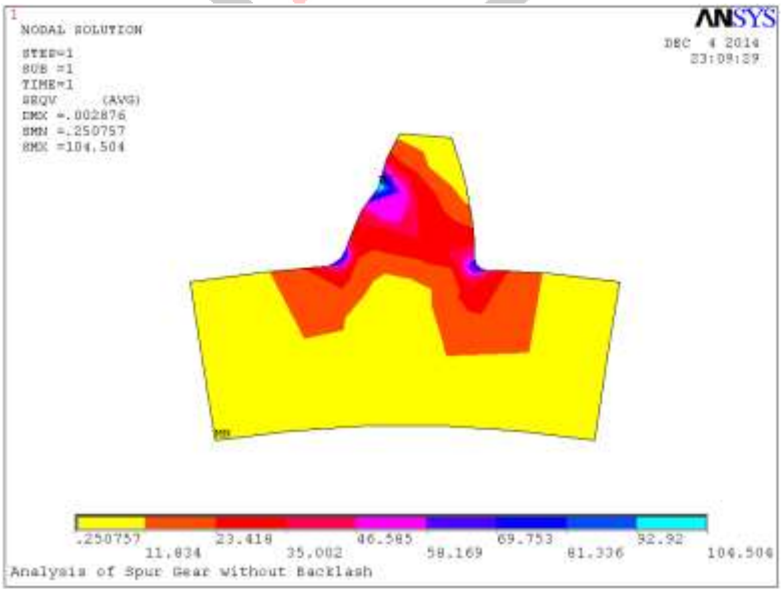

Figure 3 Bending Stresses (Von Mises)

These stresses are very much near to the analytical value of bending stress.

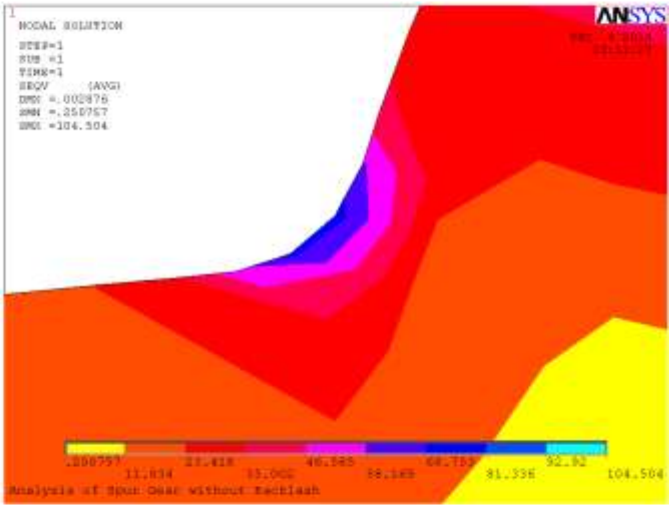

Figure 4 Exaggerated view of Bending Stresses (Von Mises)

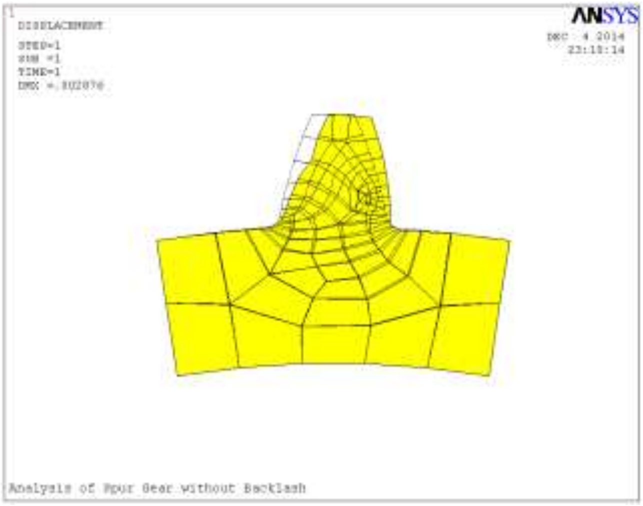

Figure 5 Deformed shape of pinion without backlash

Hence the model is validated & further analysis is done using the same mesh density and type of element.

V. RESULTS AND DISCUSSIONS

Finite element analysis of pinion with backlash by reducing tooth thickness

Analysis of pinion with back lash is carried out and results are summarized in the table 4. Here maximum Von Mises stresses are taken as a measure of bending stresses. Maximum deformation is also noted in each case.

Table 4 Effect of Backlash on maximum Von Mises stresses and Deformation

Sr. No.	Amount of Backlash (B) in mm	Maximum Von Mises Stress (MPa)	Maximum Deformation (mm)
1	0.15	90.996	0.00296
2	0.20	91.422	0.002996
3	0.25	103.667	0.003047
4	0.30	107.612	0.003096
5	0.35	106.375	0.003153
6	0.40	107.404	0.003198

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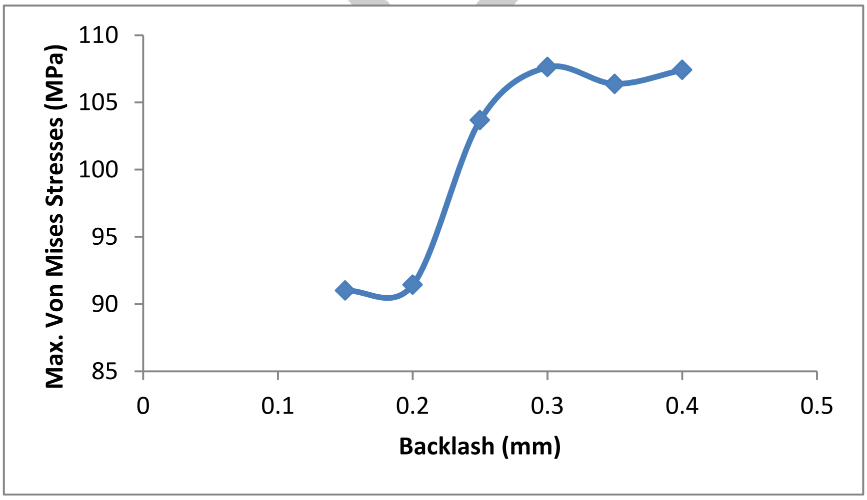

Figure 6 Effect of backlash on maximum Von Mises stresses

From the figure 6, it is seen that as backlash increases maximum Von Mises stresses increase slightly initially. But afterwards there is much increase in these stresses with increase in backlash. At the end, it nearly remains constant.

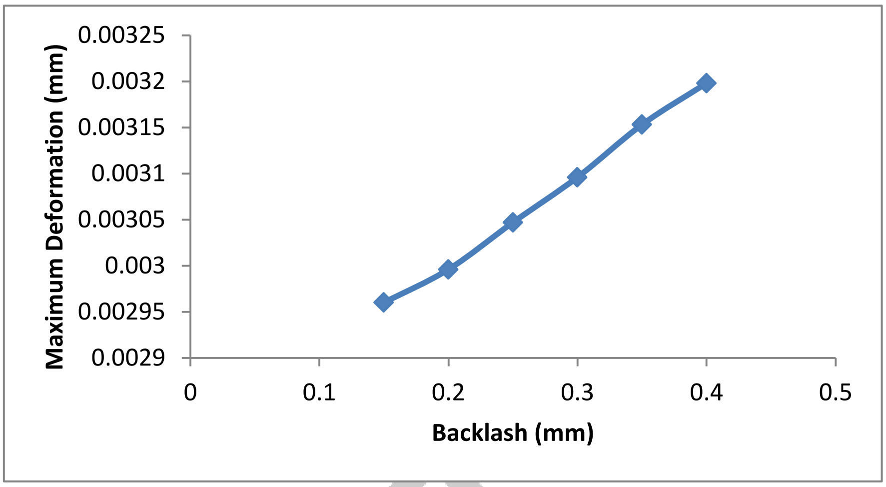

Figure 7 Effect of backlash on maximum deformation

From figure 7, it is seen that as backlash increases, maximum deformation increases. Maximum deformation is almost directly proportional to backlash.

V. CONCLUSIONS

During this research work, effect of backlash on bending stresses is studied. Backlash in this case is produced by reducing the tooth thickness. Following conclusions can be drawn.

- As backlash increases, maximum Von Mises stresses increase slightly initially. But afterwards there is much increase in these stresses with increase in backlash. At the end, it nearly remains constant.
- As backlash increases, maximum deformation increases. Maximum deformation is almost directly proportional to backlash.
- Finite Element method is a good tool to analyze these types of problems.

VI. ACKNOWLEDGMENT

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