Bianchi type-II Dark Energy Cosmological model in Saez-Ballester theory of Gravitation

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ABSTRACT: **Bianchi type-II dark energy cosmological model with variable equation of state (EoS) parameter in (Nordtvedt 1970) general scalar tensor theory of gravitation .To get a determinate solution of the field equation we take the help of special law of variation for Hubble parameter presented by Barman(1983) which yields a cosmological model with negative constant deceleration parameter. Some physical and kinematical properties of model are also discussed.**

Keywords**: Dark energy, cosmological model, Bianchi type-II**

1. INTRODUCTION

Einstein's theory of general relativity is one of the most beautiful structures of theoretical physics which describes the theory of gravitation in terms of geometry. It has provided a sophisticated theory of gravitation. It is based on the fundamental idea of relativity of all kinds of motion. The special theory of relativity formulated by Einstein (1905) makes a restricted use of this general idea since it merely assumes the relativity of uniform translator motion in a region of free space where gravitational effects can be neglected. As such it fails to study relative motion in accelerated frame of reference and is not applicable to all kinds of motion. Taking into account these limitations, Einstein (1916) generalized the special theory of relativity.

In recent years, there have been some interesting attempts to generalize the general theory of relativity by incorporating Mach's principle and other certain desired features which are lacking in the original theory. With this objective, various versions of scalar-tensor theories of gravitation have been suggested and widely discussed. Among these theories which are attracting more and more attention from physicists are the scalar-tensor theories proposed by Brans and Dicke (1961) [4], Nordtvedt (1970) [15], Saez and Ballester (1986) [30] etc.

Nordtvedt(1970) proposed a general class of scalar-tensor gravitational theories in which the parameter ω of the Brans and Dicke theory is allowed to be an arbitrary (positive definite) function of the scalar field $(\omega \to \omega(\phi))$.

The field equations of Saez - Ballester (1986) scalar tensor theory are

$$G_{ij} - \omega\phi^m\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij} \qquad (1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^m\phi^{i}_{,i} + m\phi^{m-1}\phi_{,k}\phi^{,k} = 0$$

where $G_{ij} = R_{ij} - \frac{1}{2}R g_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and m are constants, T_{ij} is the stress energy tensor of the matter.

The equation of motion is

$$T^{ij}_{,j} = 0 \qquad (2)$$

Recently, there has been considerable interest in cosmological models with dark energy in general relativity because of the fact that our universe is currently undergoing an accelerated expansion which has been confirmed by a host of observations, such as type Ia Supernovae (Reiss et al.1998 [29]; Perl Mutter et al.1999 [17]; and Tegmark et al.2004[35]). Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative presence making up around 70% of the present universe energy content to be responsible for this acceleration due to repulsive gravitation. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. Current studies to extract the properties of a dark energy component of the Universe from observational data focus on the determination of its equation of state $w(t)$, which is the ratio of the dark energy's pressure to its energy density

$w(t)=\frac{p}{\rho}$, which is not necessarily constant. The methods for restoration of the quantity $w(t)$ from expressional data have been developed (Sahni and Starobinsky 2006 [31]), and an analysis of the experimental data has been conducted to determine this parameter as a function of cosmological time (Sahni et al. 2008 [32]). Recently, the parameter $w(t)$ has been calculated with some reasoning which reduced to some simple parameterization of the dependences by some authors (Huterer and Turner 2001 [8]; Barker, B.M. (1978)[1]; Weller and Albrecht 2002 [38]; Linden and Virey 2008 [13]; Krauss et al. 2007 [10]; Usmani et al. 2008 [37]; Chen et al. 2009 [6]). The simplest dark energy candidate is the vacuum energy $(w=-1)$, which is mathematically equivalent to the cosmological constant (Λ) . The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence $(w>-1)$, phantom energy $(w<-1)$ and quintom (that can across from phantom region to quintessence region as evolved) and have time dependent EoS parameter. Due to lack observational evidence in making a distinction between constant and variable w, usually the equation of state parameter is considered as a constant (Kujat et al.2002 [11]; Bartelmann et al.2005 [2]) with phase wise value −1, 0,+1/3 and +1 for vacuum fluid, dust fluid, radiation and stiff dominated universe, respectively. But in general, w is a function of time or redshift (Jimenez 2003 [9]; Das et al. 2005 [7]).
For instance, quintessence models involving scalar fields give rise to time dependent EoS parameter w (Turner and white 1997 [36]; Caldwell et al.1998 [5]; Steinhardt et al. 1999 [34]). Ray et al. (2010) [28], Yadav and Yadav (2010) [39], Kumar (2010) [12] and Pradhan et al. (2011) [16] are some of the authors who have investigated dark energy models in general relativity with variable EoS parameter in different contexts.
Bianchi type space-times play a vital role in understanding and description of the early stages of evolution of the universe. In particular, the study of Bianchi type - II, VIII & IX universes are important because familiar solutions like FRW universe with positive curvature, the de Sitter universe, the Taub- Nut solutions etc correspond of Bianchi type-II, VIII & IX space- times. Rao et al (2008a, 2008b, 2008c) [23]-[25], have studied Bianchi type - II, VIII & IX various cosmological models in different theories of gravitation. Rao et al. (2012) have studied Bianchi type- I dark energy model in Saez – Ballester (1986) [30] scalar tensor theory of gravitation. Naidu et al. (2012) [14] have obtained LRS Bianchi type – II dark energy model in a scalar tensor theory of gravitation. Rao and Sreedevi Kumari (2012) [20] have discussed a cosmological model with negative constant deceleration parameter in a general scalar tensor theory of gravitation. Recently Rao et al. (2012) [18] have discussed LRS Bianchi type-I dark energy cosmological model in Brans-Dicke (1961) [4] theory of gravitation.

2. METRIC AND FIELD EQUATIONS:

We consider a spatially homogeneous Bianchi type-II metric of the form

$$ds^2=dt^2-R^2\left[d\theta^2+f^2(\theta)\,d\phi^2\right]-S^2\left[d\varphi+h(\theta)\,d\phi\right]^2 \qquad (3)$$

where (θ,ϕ,φ) are the Eulerian angles, $f(\theta)=1$ and $h(\theta)=\theta$

R and S are functions of t only.
The energy momentum tensor of the fluid can be written in diagonal form as

$$T_j^i=diag\,[T_0^0,T_1^1,T_2^2,T_3^3] \qquad (4)$$

We can parameterize the components of the Energy Momentum tensor as follows:

$$\begin{aligned} T_j^i &= diag[\rho,-p_x,-p_y,-p_z] \\ &= diag[1,-w_x,-w_y,-w_z]\rho \\ &= diag[1,-w,-(w+\gamma),-(w+\gamma)]\rho \end{aligned} \qquad (5)$$

where ρ is the energy density of the fluid and p_x, p_y and p_z are the pressures and w_x, w_y and w_z are the directional EoS parameters along the x,y and z axes respectively. $w(t)=\frac{p}{\rho}$ is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $w_z=w$ and then introducing skewness parameter γ which is the deviation from w along both x and y axes.
The non vanishing components of Einstein tensor are given by

$$G_1^1 = -\left(\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4}\right) \tag{6}$$

$$G_2^2 = -\left(\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4}\right) \tag{7}$$

$$G_3^3 = -\left(2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3S^2}{4R^4}\right) \tag{8}$$

$$G_4^4 = -\left(2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} - \frac{S^2}{4R^4}\right) \tag{9}$$

3. BIANCHI TYPE -II DARK ENERGY COSMOLOGICAL MODEL

Now with the help of (10) & (11), the field equations (1) for the metric (3) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = -8\pi(w+\gamma)\rho \tag{12}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3S^2}{4R^4} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = -8\pi w\rho \tag{13}$$

$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} - \frac{S^2}{4R^4} + \frac{\omega}{2}\phi^m\dot{\phi}^2 = 8\pi\rho \tag{14}$$

$$\ddot{\phi} + \dot{\phi}\left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) + \frac{m}{2\phi}\dot{\phi}^2 = 0 \tag{15}$$

$$\dot{\rho} + 2\frac{\dot{R}}{R}(w+\gamma+1)\rho + \frac{\dot{S}}{S}(w+1)\rho = 0 \tag{16}$$

Here the over head dot denotes differentiation with respect to ‘t’.

The field equations (12) to (16) are only four independent equations with six unknowns $R, S, \rho, w, \gamma \;\&\; \phi$ which are functions of ‘t’. Two additional constraints are required to obtain explicit solutions of these field equations. We solve the above set of highly non-linear equations with the help of special law of variations of Hubble’s parameter proposed by Bermann (1983)[3] which yields constant deceleration parameter of the models of the universe. We consider the constant deceleration parameter of the model defined by

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \text{constant} \tag{17}$$

where $a = (R^2 S\, f(\theta))^{\frac{1}{3}}$ is the overall scale factor. Here the constant is taken as negative so that it represents an accelerating model of the universe.
From (17), we get

$$a=(R^2 S\, f(\theta))^{\frac{1}{3}}=(c_1 t+c_2)^{\frac{1}{(1+q)}} \tag{18}$$

where $c_1 \neq 0$ and c_2 are constants of integration. This equation implies that the condition of expansion is $1+q>0$. To get the deterministic solution, it has been assumed that the expansion θ in the model is proportional to the shear scalar σ. This condition leads to

$$S=R^n \tag{19}$$

where n is an arbitrary constant.
From (18) & (19), we get

$$R=\left(\frac{1}{f(\theta)}\right)^{\frac{1}{n+2}}(c_1 t+c_2)^{\frac{3}{(1+q)(n+2)}} \tag{20}$$

$$S=\left(\frac{1}{f(\theta)}\right)^{\frac{n}{n+2}}(c_1 t+c_2)^{\frac{3n}{(1+q)(n+2)}} \tag{21}$$

From equations (15), (20) & (21), we get

$$\phi^{\frac{m+2}{2}}=\frac{m+2}{2}\left(\frac{c_3}{c_1}\frac{(q+1)}{(q-2)}f(\theta)(c_1 t+c_2)^{\frac{q-2}{q+1}}+c_4\right),\quad q\neq 2 \tag{22}$$

where c_3 and c_4 are constants of integration.
From equations (16) & (20)-(22), we get the energy density ρ as

$$8\pi\rho=\left(\frac{9(2n+1)}{(1+q)^2(n+2)^2}\right)\frac{c_1^{\,2}}{(c_1 t+c_2)^2}-\frac{1}{4(c_1 t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}}+\frac{\omega\, c_3^{\,2}}{2(c_1 t+c_2)^{\frac{6}{(1+q)}}},\quad (n+2)\neq 0 \tag{23}$$

From equations (13) & (20)-(22), we get the EoS parameter w as

$$w=\frac{1}{8\pi\rho}\left[\left(\frac{6(1+q)(n+2)-27}{(1+q)^2(n+2)^2}\right)\frac{c_1^{\,2}}{(c_1 t+c_2)^2}+\frac{3}{4(c_1 t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}}+\frac{\omega\, c_3^{\,2}}{2(c_1 t+c_2)^{\frac{6}{(1+q)}}}\right] \tag{24}$$

From equations (12) & (20)-(22), we get the skewness parameter γ as

$$\gamma=\frac{1}{8\pi\rho}\left[\left(\frac{3(q-2)(n-1)}{(1+q)^2(n+2)}\right)\frac{c_1^{\,2}}{(c_1 t+c_2)^2}-\frac{1}{(c_1 t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}}\right] \tag{25}$$

The metric (3), in this case can be written as

$$ds^2 = dt^2 - (c_1 t + c_2)^{\frac{6}{(1+q)(n+2)}}\left(d\theta^2 + d\phi^2\right) - (c_1 t + c_2)^{\frac{6n}{(1+q)(n+2)}}\left(d\varphi + \theta\, d\phi\right)^2. \qquad (26)$$

Thus the equation (26) together with (23), (24) & (25) constitutes a Bianchi type-II dark energy cosmological model with variable EoS parameter in a scalar tensor theory of gravitation proposed by Saez and Ballester (1986)[30].

4. Physical and geometrical properties:

The spatial volume for the models is

$$V = (-g)^{\frac{1}{2}} = (c_1 t + c_2)^{\frac{3}{(1+q)}} \qquad (26)$$

since $a = (R^2 S f(\theta))^{1/3}$, which is the average scale factor.

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{,i} = \frac{3c_1}{(1+q)(c_1 t + c_2)} \qquad (28)$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7c_1{}^2}{2(1+q)^2(c_1 t + c_2)^2} \qquad (29)$$

The components of Hubble Parameter H_1 & H_2 are given by $H_1 = \frac{\dot{R}}{R} = \frac{3c_1}{(1+q)(n+2)(c_1 t + c_2)}$

$$H_2 = \frac{\dot{S}}{S} = \frac{3nc_1}{(1+q)(n+2)(c_1 t + c_2)}$$

Therefore the generalized mean Hubble parameter (H) is

$$H = \frac{1}{3}\left(2H_1 + H_2\right) = \frac{c_1}{(1+q)(c_1 t + c_2)}. \qquad (30)$$

The average anisotropy parameter are defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2(n-1)^2}{(n+2)^2} \qquad (31)$$

where $\Delta H_i = H_i - H,\ (i = 1,2,3)$.

5. Conclusions:

In this chapter we have presented spatially homogeneous and anisotropic Bianchi type – II dark energy cosmological model with variable EoS parameter in a scalar tensor theory of gravitation proposed by Saez and Ballester (1986)[30]. The power law solution represents a non-singular model where the spatial scale factors and volume vanish at $t = \frac{-c_2}{c_1}$. We observe that the model (3.2.15) has no initial singularity at $t = \frac{-c_2}{c_1}$ and the spatial volume is increasing as time t increases, i.e. the present model (3.2.15) is expanding. The Hubble parameter is zero as t approaches to infinity. The scalar expansion θ and the shear scalar σ^2 tend to infinity at $t = 0$ while they become zero as $t \to \infty$. Since the mean anisotropy parameter $A_m \neq 0$ the model is anisotropic for $n \neq 1$. If $n = 1$, $A_m = 0$ and hence the model will become isotropic. Also, since $1 + q > 0$, the model represents an accelerating universe. Therefore, it follows that our dark energy model in Saez – Ballester (1986)[30] theory is consistent with the recent observations of Type – Ia Supernovae[29] ,Perl mutter et al. (1999)[17]; Riess, et al. (1998). Finally we can conclude that our model is accelerating, more general and represent not only the early stages of evolution but also the present stage of the universe.

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