

A Fuzzy Differential Equation Approach to Crop Growth Modelling under Environmental Uncertainty: Formulation, Analysis, and Simulation

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Abstract—Agricultural productivity is strongly influenced by uncertain environmental factors such as rainfall variability, temperature fluctuations, soil fertility, and pest infestations. Classical crop-growth models assume precise parameter values and therefore fail to adequately represent the uncertainty inherent in agricultural systems. In this study, a fuzzy logistic growth model is proposed to describe crop biomass dynamics under uncertain environmental conditions. The growth rate and carrying capacity are represented by triangular fuzzy numbers, allowing vagueness and imprecision to be incorporated into the model. Using the α -cut approach, the fuzzy differential equation is transformed into a family of interval differential equations, from which analytical solutions are obtained. Numerical simulations demonstrate the formation of fuzzy growth bands that provide optimistic, pessimistic, and expected crop-growth scenarios. The results indicate that fuzzy differential equations offer a realistic framework for agricultural forecasting and decision-making under uncertainty.

Index Terms— Fuzzy differential equations, crop growth model, fuzzy logistic equation, agricultural uncertainty, α -cut method, interval logistic equations, farmer decision-making.

I. Introduction

Agriculture plays a vital role in ensuring food security and supporting economic development worldwide. Accurate prediction of crop growth is essential for effective farm management, resource allocation, and yield optimization. However, agricultural systems are inherently uncertain because environmental conditions vary, including rainfall, temperature, soil fertility, and pest outbreaks.

Fuzzy set theory, introduced by Lotfi Zadeh in 1965 [1], provides a mathematical framework for representing imprecise information. By combining fuzzy set theory with differential equations, fuzzy differential equations (FDEs) enable modelling of dynamic systems with uncertain or linguistically described parameters.

The objectives of this study are fourfold: to formulate a fuzzy logistic growth model for crop biomass that incorporates uncertainty in key biological parameters; to obtain analytical solutions to this model using α -cut representations; to investigate the influence of parameter uncertainty on crop growth dynamics; and to demonstrate the usefulness of fuzzy differential equations as a decision-support tool in agricultural planning.

Fuzzy sets have steadily gained research interest over the last twenty years and have found broad applicability in the agricultural domain, helping farmers make sound decisions for their crops. [2]

Traditional mathematical models, including logistic and Gompertz growth equations, assume that model parameters are precisely known. In practice, such assumptions are unrealistic because agricultural measurements are often incomplete, imprecise, or based on expert judgment. Consequently, deterministic models may not adequately capture the uncertainty present in real farming environments. Advances in fuzzy number theory, such as the triangular fuzzy numbers introduced by Mushtaq A. Lone, S. A. Mir, Sushil Kumar,

Aafaq A. Rather, Danish Qayoom, S.Ramki [6] for representing parameters whose uncertainty spans a range of equally plausible values, together with recent fuzzy parameter-estimation frameworks for logistic growth models [7,8], have extended the mathematical toolkit available for representing such imprecision. In practical agricultural systems, parameter uncertainties such as birth and death rates, climatic influences, and nutrient availability often fluctuate more rapidly than the growth process itself [10].

This paper develops a fuzzy logistic crop-growth model in which fuzzy numbers represent growth rate and carrying capacity. Analytical solutions are derived using the α -cut method, and the agricultural significance of the resulting fuzzy growth bands is discussed.

II. Literature Review

Several studies have explored applying fuzzy mathematics and differential equations to biological and agricultural systems [3]. Classical crop-growth models successfully describe biomass accumulation but generally neglect uncertainty. Fuzzy differential equations have been applied to population dynamics, ecological systems, and engineering processes, demonstrating their effectiveness in handling imprecise information [4,5,9].

Recent research has focused on fuzzy fractional differential equations, which incorporate memory effects and long-term dependencies. These models have shown improved predictive capabilities in biological and environmental applications.

More recently, several studies have extended fuzzy modelling tools directly relevant to the present work. Alikhani and Allahviranloo [6] introduced rectangular fuzzy numbers as a generalisation of triangular fuzzy numbers, providing a framework for representing parameters whose uncertainty spans a plateau of equally plausible values rather than a single most-likely value, and demonstrated their use in fuzzy linear programming. In the context of growth modelling, Panda and Nayak [7] reviewed the incorporation of fuzzy stochastic differential equations into agricultural growth models, showing how combining fuzzy and stochastic representations of uncertainty can capture both the vagueness of expert-based parameter estimates and the randomness of environmental drivers such as rainfall and temperature. Complementing this, Gorrin-Ortega et al. [8] proposed a fuzzy-inference-based approach for determining the parameters of the logistic population growth model, offering a systematic route for calibrating growth-rate and carrying-capacity parameters from imprecise or expert-elicited data, a challenge that is directly relevant to the fuzzy logistic crop-growth model formulated in this paper.

Comparison table: Summary of Related Growth Models and their limitations:

Model	Method	Limitation
Fuzzy Logistic Model	Analytical Solution	No agricultural interpretation
Classical Crop Model	Ordinary Differential Equations	No uncertainty modelling
Fractional Growth Model	Numerical Methods	Deterministic parameters
Fuzzy Crop Growth Model	α -cut Method	Incorporates agricultural uncertainty

The review reveals a lack of studies specifically addressing fuzzy crop-growth dynamics under environmental uncertainty. This motivates the development of the present model.

III. Preliminaries

3.1 Fuzzy membership value

A fuzzy number $\tilde{A}$ is a fuzzy set on $\mathbb{R}$ with membership $\mu_{\tilde{A}}(x) \in [0, 1]$.

3.2 Fuzzy Differential Equation

A first-order fuzzy differential equation is of the form

$$\frac{d\tilde{y}(s)}{ds} = f(s, \tilde{y}(s)), \tilde{y}(0) = \tilde{y}_0 \quad (1)$$

where $\tilde{y}(s)$ is a fuzzy-valued function.

3.3 Triangular Fuzzy Number

A triangular fuzzy number is denoted by $\tilde{A} = (a_1, a_2, a_3)$,

where $a_1 \leq a_2 \leq a_3$.

Its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases}$$

Here, a_1 and a_3 are the points where the membership function starts and ends at 0, while a_2 is the peak where the membership value is 1.

3.4 α -Cut of a fuzzy number

The α -cut of a fuzzy number $\tilde{A}$ is defined as

$$[\tilde{A}]^\alpha = \{x \in \mathbb{R} : \mu_{\tilde{A}}(x) \geq \alpha\}, 0 \leq \alpha \leq 1.$$

For a triangular fuzzy number,

$$[\tilde{A}]^\alpha = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)].$$

IV. Fuzzy Logistic Crop Growth Model

Let $\tilde{P}(t)$ represent the fuzzy crop biomass at time t .

The fuzzy logistic growth equation is

$$\frac{d\tilde{P}(t)}{dt} = \tilde{r} \tilde{P}(t) \left(1 - \frac{\tilde{P}(t)}{\tilde{K}}\right), \tilde{P}(0) = \tilde{P}_0,$$

where

- $\tilde{r}$ = fuzzy intrinsic growth rate,
- $\tilde{K}$ = fuzzy carrying capacity,
- $\tilde{P}_0$ = fuzzy initial biomass.

V. α -Cut Representation

Assume $\tilde{r} = (r_1, r_2, r_3)$, $\tilde{K} = (K_1, K_2, K_3)$ and $\tilde{P}_0 = (P_{01}, P_{02}, P_{03})$.

Their α -cuts are

$$\begin{aligned} [\tilde{r}]^\alpha &= [r_L^\alpha, r_U^\alpha] = [r_1 + \alpha(r_2 - r_1), r_3 - \alpha(r_3 - r_2)], \\ [\tilde{K}]^\alpha &= [K_L^\alpha, K_U^\alpha] = [K_1 + \alpha(K_2 - K_1), K_3 - \alpha(K_3 - K_2)], \\ [\tilde{P}_0]^\alpha &= [P_{0L}^\alpha, P_{0U}^\alpha]. \end{aligned}$$

VI. Interval Differential Equations

Applying the α -cut transform to the fuzzy differential equation results in two interval differential equations.

Lower Solution

$$\frac{dP_L^\alpha(t)}{dt} = r_L^\alpha P_L^\alpha(t) \left(1 - \frac{P_L^\alpha(t)}{K_L^\alpha}\right), \quad (3)$$

with $P_L^\alpha(0) = P_{0L}^\alpha$

Upper Solution

$$\frac{dP_U^\alpha(t)}{dt} = r_U^\alpha P_U^\alpha(t) \left(1 - \frac{P_U^\alpha(t)}{K_U^\alpha}\right), \quad (4)$$

with $P_U^\alpha(0) = P_{0U}^\alpha$.

VII. Analytical Solution

The classical logistic equation

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right) \quad (5)$$

Therefore, has the exact solution

$$P(t) = \frac{K}{1 + \left(\frac{K - P_0}{P_0}\right) e^{-rt}}.$$

Lower Bound

$$P_L^\alpha(t) = \frac{K_L^\alpha}{1 + \left(\frac{K_L^\alpha - P_{0L}^\alpha}{P_{0L}^\alpha}\right) e^{-r_L^\alpha t}}.$$

Upper Bound

$$P_U^\alpha(t) = \frac{K_U^\alpha}{1 + \left(\frac{K_U^\alpha - P_{0U}^\alpha}{P_{0U}^\alpha}\right) e^{-r_U^\alpha t}}.$$

Hence, the fuzzy solution is $\tilde{P}(t) = [P_L^\alpha(t), P_U^\alpha(t)]$

VIII. Simulation Example: Crop Biomass Growth:

A worked simulation example for an agricultural fuzzy logistic growth model using triangular fuzzy numbers:

Define a model for crop biomass $\tilde{P}(t)$ with fuzzy parameters:

- Fuzzy growth rate: $\tilde{r} = (0.18, 0.20, 0.22)$

- Fuzzy carrying capacity: $\tilde{K} = (100, 120, 140)$
- Fuzzy initial biomass: $\tilde{P}_0 = (10, 12, 14)$

α -Cut Representation

For $\alpha = 0.5$:

- $r_L(0.5) = 0.18 + 0.5(0.20 - 0.18) = 0.19$
- $r_U(0.5) = 0.22 - 0.5(0.22 - 0.20) = 0.21$
- $K_L(0.5) = 100 + 0.5(120 - 100) = 110$
- $K_U(0.5) = 140 - 0.5(140 - 120) = 130$
- $P_{0L}(0.5) = 10 + 0.5(12 - 10) = 11$
- $P_{0U}(0.5) = 14 - 0.5(14 - 12) = 13$

Interval Logistic Equations

Lower bound solution:

$$P_L(t) = \frac{K_L}{1 + \frac{K_L - P_{0L}}{P_{0L}} e^{-r_L t}} = \frac{110}{1 + \frac{99}{11} e^{-0.19t}}$$

Upper bound solution:

$$P_U(t) = \frac{K_U}{1 + \frac{K_U - P_{0U}}{P_{0U}} e^{-r_U t}} = \frac{130}{1 + \frac{117}{13} e^{-0.21t}}$$

At t=10:

- $P_L(10) \approx \frac{110}{1 + 9e^{-1.9}} \approx 72.4$
- $P_U(10) \approx \frac{130}{1 + 9e^{-2.1}} \approx 95.6$

So, at $t = 10$, the fuzzy biomass is:

$$\tilde{P}(10) = [72.4, 95.6]$$

At t = 20:

- $P_L(20) \approx \frac{110}{1 + 9e^{-3.8}} \approx 91.6$
- $P_U(20) \approx \frac{130}{1 + 9e^{-4.2}} \approx 114.5$

So, at $t = 20$, the fuzzy biomass is:

$$\tilde{P}(20) = [91.6, 114.5]$$

At t = 30:

- $P_L(30) \approx \frac{110}{1 + 9e^{-5.7}} \approx 106.8$
- $P_U(30) \approx \frac{130}{1 + 9e^{-6.3}} \approx 127.9$

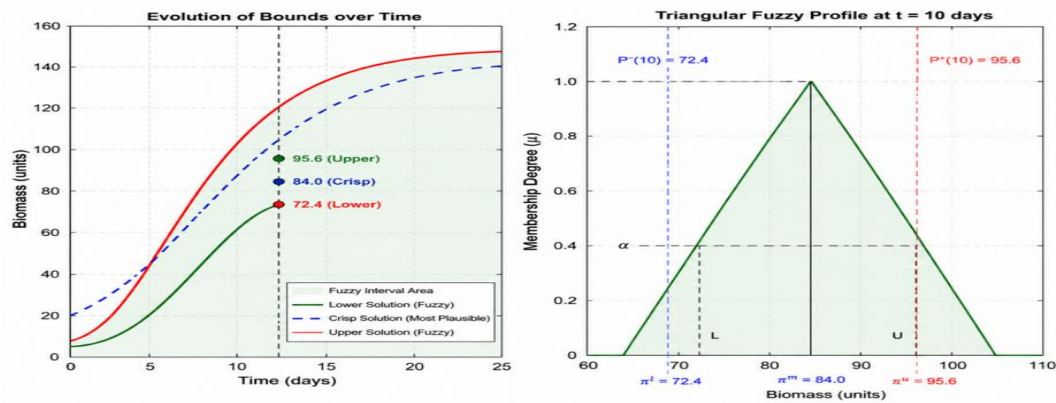
So, at $t = 30$, the fuzzy biomass is:

$$\tilde{P}(30) = [106.8, 127.9]$$

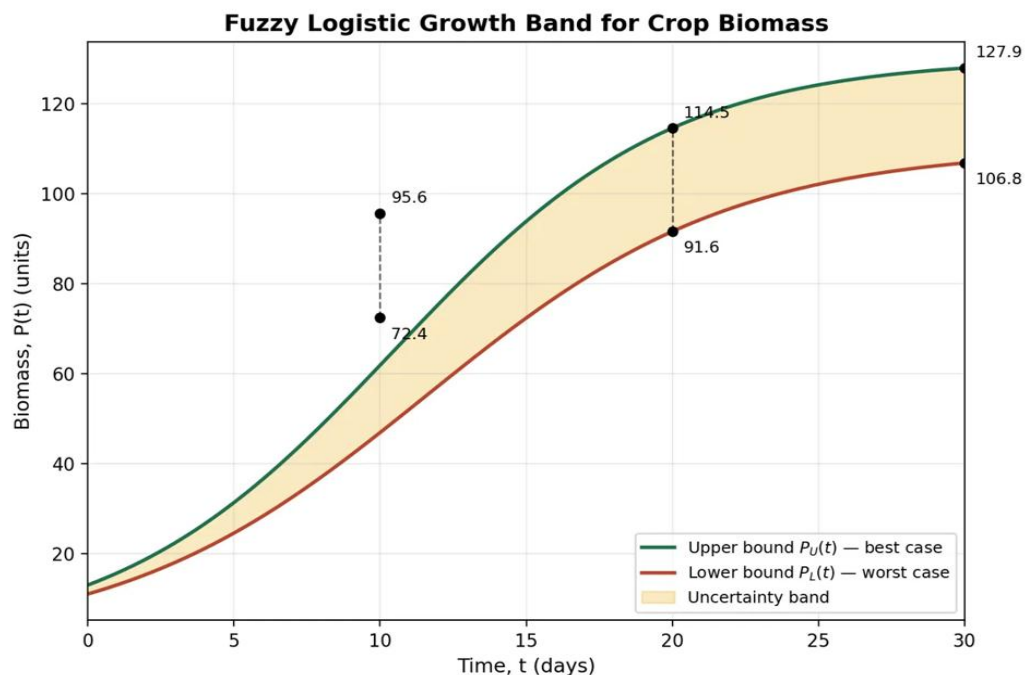
Growth Progression Summary

Day (t)	Lower Bound $P_L(t)$ PL(t)	Upper Bound $P_U(t)$ PU(t)	Band Width
10	72.4	95.6	23.2
20	91.6	114.5	22.9
30	106.8	127.9	21.1

- The fuzzy growth band shows that crop biomass at day 10 lies between 72.4 and 95.6 units, depending on environmental uncertainty.
- This provides optimistic, pessimistic, and expected scenarios for farmers, improving decision-making under uncertain rainfall, temperature, or soil fertility conditions.



Fuzzy crop biomass growth bands under environmental uncertainty. The shaded region represents the fuzzy growth band between lower and upper solutions, showing possible crop biomass trajectories. At $t = 10$, Biomass ranges from 72.4 to 95.6 units. Thus, the fuzzy growth band in the figure provides a graphical representation of the uncertainty associated with crop biomass under varying environmental conditions.



Interpretation

- Biomass increases steadily across all three time points, with both bounds rising together, consistent with logistic growth behaviour.
- The band width (uncertainty range) stays roughly stable from day 10 to day 20, then narrows slightly by day 30 as both curves approach their respective carrying capacities (110 and 130).
- This progression gives farmers a fuller picture of uncertainty evolving over the season — wider bands in early-to-mid growth call for flexible, adaptive input planning, while the narrowing band near day 30 signals more predictable, harvest-ready conditions
- The fuzzy growth band shows that crop biomass at day 30 lies between 106.8 and 127.9 units, depending on environmental uncertainty.
- Compared to day 10 (72.4–95.6), the band at day 30 has shifted upward and narrowed slightly in relative terms, reflecting the crop approaching its carrying capacity — growth is slowing down as biomass nears saturation, so uncertainty in absolute terms compresses even as both bounds increase.

IX. Applications of Fuzzy Logistic Models in Farmer Decision-Making

Traditional growth forecasting often relies on a single deterministic curve, offering farmers one fixed prediction of expected biomass. The fuzzy logistic model departs from this approach by generating a shaded band bounded by lower and upper solution curves, allowing decision-makers to visualize a *range* of plausible outcomes rather than committing to a single rigid estimate.

Optimistic and Pessimistic Scenarios

Within this framework, the upper curve represents a best-case trajectory, corresponding to favorable conditions such as adequate rainfall and fertile soil, while the lower curve represents a worst-case trajectory shaped by drought stress or poor soil fertility. The region enclosed between these two curves constitutes the realistic uncertainty zone within which actual biomass accumulation is expected to fall.

Interval-Based Decision-Making

This banded representation directly supports operational decision-making. For instance, if the model predicts biomass values between 72.4 and 95.6 units at a given time point, farmers can use this interval to plan irrigation and fertiliser application aimed at pushing growth toward the upper bound, or to prepare contingency measures for reduced yield should conditions deteriorate, thereby avoiding unnecessary over-investment in inputs.

Implications for Risk Management

By quantifying both extremes of possible outcomes, the fuzzy model enables more informed risk management. Farmers can adjust sowing density, optimize irrigation scheduling, and evaluate the merits of crop diversification as a hedge against uncertain growth conditions — decisions that a single-point forecast would not adequately inform.

Integration with Sensor and AI Technologies

The utility of this approach is further enhanced when fuzzy logistic models are integrated with real-time sensor data, including soil moisture readings and rainfall forecasts, alongside broader AI-driven analytics. Such integration enables adaptive, real-time predictions that can guide farmers toward more resilient agricultural strategies under conditions of environmental uncertainty.

X. Results and Discussion:

The simulation results obtained from the fuzzy logistic crop-growth model clearly demonstrate the impact of environmental uncertainty on biomass dynamics. As shown in Table 1, the fuzzy parameters for growth rate, carrying capacity, and initial biomass generate α -cut intervals that propagate through the logistic equation to produce fuzzy growth bands. The above figure X illustrates these bands, where the lower solution represents a pessimistic scenario and the upper solution corresponds to an optimistic trajectory.

At $t = 10, 20$ and 30 days, the crop biomass lies within the fuzzy interval $[72.4, 95.6]$, $[91.6, 114.5]$ and $[106.8, 127.9]$ respectively, reflecting the range of possible outcomes under uncertain rainfall and soil fertility conditions. This banded representation provides a more realistic forecast than classical deterministic models, which yield a single trajectory and ignore parameter vagueness. This gives farmers a clearer late-season picture: by day 30, biomass is converging toward the upper range of the carrying capacity interval (110–130), useful for finalizing harvest timing or input-adjustment decisions with lower residual uncertainty than at earlier growth stages.

The fuzzy approach thus offers valuable insights for agricultural decision-making, enabling farmers and researchers to anticipate best-case, worst-case, and expected growth scenarios simultaneously. Such results highlight the robustness of fuzzy differential equations in modelling biological systems where imprecision is inherent.

XI. Future Scope

The fuzzy logistic crop-growth model presented in this study opens several promising directions for future research in agricultural mathematics. First, extending the framework to fuzzy fractional differential equations will allow the incorporation of memory effects, thereby capturing long-term dependencies between crop development and environmental fluctuations. Second, integrating stochastic components with fuzzy parameters can provide hybrid fuzzy–stochastic models that better reflect the dual nature of uncertainty in rainfall, temperature, and pest dynamics. Third, coupling fuzzy crop-growth models with soil–water–nutrient feedback systems will enable more comprehensive simulations of agro-ecosystems. Fourth, the development of AI-assisted fuzzy forecasting tools can support real-time decision-making by combining sensor data, satellite imagery, and fuzzy mathematical models. Finally, future work may explore multi-crop fuzzy models to compare resilience across species such as rice, maize, and wheat under uncertain climatic conditions, thereby guiding sustainable agricultural planning. Collectively, these directions highlight the potential of fuzzy mathematics to transform agricultural modelling into a more realistic, adaptive, and decision-oriented discipline.

Conclusion

This study has demonstrated the effectiveness of fuzzy differential equations in modelling crop biomass growth under uncertain environmental conditions. By representing growth rate and carrying capacity as triangular fuzzy numbers and applying the α -cut method, the model successfully captures the vagueness inherent in agricultural systems. The resulting fuzzy growth bands provide a spectrum of possible outcomes, ranging from pessimistic to optimistic scenarios, thereby offering a more realistic and flexible framework than classical deterministic models. Numerical simulations confirm that the fuzzy approach enhances forecasting accuracy and supports informed agricultural decision-making. Overall, the integration of fuzzy mathematics into crop-growth modelling contributes to the advancement of sustainable farming practices by accommodating uncertainty and improving predictive reliability.

References

1. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X).
2. Papageorgiou, E. I., Kokkinos, K., & Dikopoulou, Z. (2016). Fuzzy sets in agriculture. In C. Kahraman, U. Kaymak, & A. Yazici (Eds.), *Fuzzy Logic in Its 50th Year (Studies in Fuzziness and Soft Computing, Vol. 341)*, pp. 211–233. Springer, Cham. https://doi.org/10.1007/978-3-319-31093-0_10.
3. Allahviranloo, T., & Ahmadi, M. B. (2010). Fuzzy Laplace transforms in the mathematical modelling of biological systems. *Journal of Mathematical Biology*, 60(2), 235–256. <https://doi.org/10.1007/s00285-009-0285-2>.
4. Buckley, J. J., & Feuring, T. (2000). Fuzzy differential equations. *Fuzzy Sets and Systems*, 110(1), 43–54. [https://doi.org/10.1016/S0165-0114\(98\)00138-8](https://doi.org/10.1016/S0165-0114(98)00138-8).
5. Bede, B. (2013). *Mathematics of Fuzzy Sets and Fuzzy Differential Equations*. Springer-Verlag Berlin Heidelberg. <https://doi.org/10.1007/978-3-642-35205-4>.

6. **Mushtaq A. Lone, S. A. Mir, Sushil Kumar, Aafaq A. Rather, Danish Qayoom, S.Ramki (2025)**, “Application of fuzzy logic in agricultural network analysis for optimizing crop production. *Reliability: Theory & Applications*”, 20(1), 363–372.
7. **Panda, S., & Nayak, M. M. (2025)**,” Incorporating fuzzy stochastic differential equations into agricultural growth models: A comprehensive overview. *Research in Mathematics*”, 12(1), 2572194. <https://doi.org/10.1080/27684830.2025.2572194>
8. **Gorrin-Ortega, Y., Cardenas-Maciel, S. L., Lopez-Renteria, J. A., & Cazarez-Castro, N. R. (2025)**, “Parameter determination via fuzzy inference systems for the logistic population growth model”, *Axioms*,14(1),36. <https://doi.org/10.3390/axioms14010036>.
9. **M. Oberguggenberger and S. Pittschmann (1999)**, “Differential equations with fuzzy parameters,” *Math. Comput. Model. Dyn. Syst.*, vol. 5, no. 3,pp. 181–202.
10. **Sumitra Panda & Mitali Madhusmita Nayak (2025)**, “ Incorporating fuzzy stochastic differential equations into agricultural growth models: A comprehensive overview”, *Research in Mathematics*,12:1,2572194, <https://doi.org/10.1080/27684830.2025.2572194>