Equivalence Relation on Fuzzy join hyperlattice

[1]D. Ameena Beave, [2]V. Senthamil Selvi
Thassim Beevi Abdul Kader College for Women, Kilakarai.

Abstract: **In this paper, we introduce the notion of fuzzy join hypercongruence and we derive the connection between a fuzzy join hypercongruence on a fuzzy join hyperlattice and a join hypercongruence on the associated join hyperlattice.**

Keywords: **Hyperlattice, fuzzy join hyperlattice, fuzzy join hypercongruence.**

INTRODUCTION:

Lattice theory [1] is the recent trend in mathematics. The fuzzy set has been combined with hyperlattice. And now we are going to introduce the equivalence relation on fuzzy join hyperlattice. And also we introduce the notion of fuzzy join hypercongruence. Hence we derive the connections between join hypercongruence on a fuzzy join hyperlattice.

I.PRELIMINARIES:

In this section we discuss about some of the basic definitions that we use throughout this paper.

Definition 1.1:

Let L be a non-empty set with two hyperoperations $\wedge$ and$\vee$. Then (L, $\vee$, $\wedge$) is called as hyperlattice [4], if the following identities holds for all a, b, c $\in$ L.

1) $a \in a \wedge a$ and $a \in a \vee a$
2) $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$
3) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ and $(a \vee b) \vee c = a \vee (b \vee c)$
4) $a \in a \wedge (a \vee b)$ and $a \in a \vee (a \wedge b)$.

Definition 1.2:

Let L be a non-empty set with two hyperoperation $\vee$ and$\wedge$. Then (L, $\vee$, $\wedge$) is called a fuzzy hyperlattice [2], if the following identities holds for all a, b, c $\in$ L.

1) $(a \vee a)(a) > 0$ and $(a \wedge a)(a) > 0$
2) $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$
3) $(a \vee b) \vee c = a \vee (b \vee c)$ and $(a \wedge b) \wedge c = a \wedge (b \wedge c)$
4) $(a \vee (a \wedge b))(a) > 0$ and $(a \wedge (a \vee b))(a) > 0$.

Definition 1.3:

Let L be a non-empty set, $\vee$: $L \times L \longrightarrow F^*(L)$,be a fuzzy hyperoperation and $\wedge : L \times L \longrightarrow L$ be a operation. Then, (L, $\vee$, $\wedge$) is a fuzzy join hyperlattice if for all x, y, z $\in$ L the following conditions holds:

1) $(x \vee x)(x) > 0$ and $(x \wedge x)(x) > o$
2) $x \vee y = y \vee x$ and $x \wedge y = y \wedge x$
3) $(x \vee y) \vee z = x \vee (y \vee z)$ and $(x \wedge y) \wedge z = x \wedge (y \wedge z)$
4) $(x \vee (x \wedge y))(x) > 0$ and $(x \wedge (x \vee y))(x) > 0$.

Definition 1.4:

Let (L, $\vee$, $\wedge$) be a fuzzy join hyperlattice and ρ be an equivalence relation on L. For any u, v $\in F^*(L)$, we say that u $\bar{\rho}$ v if the following condition holds:

(1) For all a $\in$ L, if $u(a) > 0$, then there exists b $\in$ L, such that $v(b) > 0$ and a ρ b;
(2) For all x $\in$ L, if $v(x) > 0$, then there exists y $\in$ L, such that $u(y) > 0$ and x ρ y.

II. EQUIVALENCE RELATION ON FUZZY JOIN HYPERLATTICE

In this section we introduce the notion of fuzzy join hypercongruence [3], and we derive the connections between a fuzzy join hypercongruence on fuzzy join hyperlattices and a join hypercongruence on the associated join hyperlattice.

Definition 2.1:

An Equivalence relation ρ on a fuzzy join hyperlattice (L, ∨, ∧) is said to be a fuzzy join hypercongruence on (L, ∨, ∧) if for all a, a', b, b' ∈ L, the following conditions holds:

1) a ρ a' implies that (a ∨ b) $\bar{\rho}$ (a' v b') and
2) b ρ b' implies that (a ∧ b) $\bar{\rho}$ (a' ∧ b').

The following theorem depicts that a homomorphism of fuzzy join hyperlattices can induce fuzzy join hypercongruence on a fuzzy join hyperlattice.

Theorem 2.2:

Let (L_1,$\vee_1$, $\wedge_1$) and (L_2, $\vee_2$, $\wedge_2$) be two fuzzy join hyperlattices. A map f : $L_1 \to L_2$ is a homomorphism of fuzzy join hyperlattices, then ρ = ker f = {(a, b) ∈ $L_1 \times L_1$| f(a) = f(b)} is a fuzzy join hypercongruence on (L,$\vee_1$, $\wedge_1$).

Proof:

We know that, ρ = ker f is an equivalence relation on L_1.

For all a, a', b, b' ∈ L_1, let a ρ a' and b ρ b', then,

f(a) = f(a') and f(b) = f(b').

then we get that f(a $\vee_1$, b) = f(a) $\vee_2$ f(b) = f(a') $\vee_2$ f(b').

Hence for all x ∈ L_1, let (a $\vee_1$ b) (x) > 0, then

f (a $\vee_1$ b) f(x) = sup {(a' $\vee_1$ b') (x')| f(x') = f(x), x' ∈ L_1} ≥ (a $\vee_1$ b) (x) > 0.

Therefore we have f(a' $\vee_1$ b') f(x) > 0, which implies that sup{(a' $\vee_1$ b') (x')| f(x') = f(x), x' ∈ L_1} > 0.

The above inequality implies that there exists x'∈ L_1 such that,

(a' $\vee_1$ b') (x') > 0 and

f(x') = f(x), implies (a' $\vee_1$ b') (x') > 0 and x ρ x'.

Conversely,

For all s ∈ L_1, let (a' $\vee_1$ b') (s) > 0, then there also exist t ∈ L_1, such that

(a $\vee_1$ b) (t) > 0 and s ρ t.

Hence (a ∨ b) $\bar{\rho}$ (a' ∨ b').

Similarly, we can show that (a ∧ b) $\bar{\rho}$ (a' ∧ b').

Therefore, ρ = ker f is a fuzzy join hypercongruence on (L,$\vee_1$, $\wedge_1$).

Now we introduce the fuzzy strong join hypercongruence on fuzzy join hyperlattices.

Definition 2.3:

Let (L, ∨, ∧) be a fuzzy join hyperlattice and ρ be an equivalence relation on L. For any u, v ∈ F*(L), we say that u $\bar{\bar{\rho}}$ v, if the following two identities hold:

(1) For all a ∈ L, if u(a) > 0, then there exists b ∈ L, such that v(b) > 0, u (a) = v (b) and a ρ b;
(2) For all x ∈ L, if v(x) > 0, then there exists y ∈ L, such that u(y) > 0, u (y) = v (x) and x ρ y.

Definition 2.4:

An equivalence relation ρ on a fuzzy join hyperlattice (L, ∨, ∧) is said to be a fuzzy strong join hypercongruence on (L, ∨, ∧), if for all a, a', b, b' ∈ L, then the following implications are satisfied:

1) a ρ a' implies (a ∨ b) $\bar{\bar{\rho}}$ (a' ∨ b') and
2) b ρ b' implies (a ∧ b) $\bar{\bar{\rho}}$ (a' ∧ b')

Theorem 2.5:

Let ρ be a fuzzy strong join hypercongruence on a fuzzy join hyperlattice (L, ∨, ∧). If we define the following fuzzy join hyperoperation on the quotient set L/ ρ : for all $\bar{x}$, $\bar{y}$, $\bar{z} \in$ L/ ρ,

$(\bar{x} \vee' \bar{y})\,(\bar{z})$ = sup {(x' ∨ y') (z') | x' $\in \bar{x}$, y'$\in \bar{y}$, z' $\in \bar{z}$} and

$(\bar{x} \wedge' \bar{y})\,(\bar{z})$ = sup {(x' ∧ y') (z') | x' $\in \bar{x}$, y'$\in \bar{y}$, z' $\in \bar{z}$}, then (L/ ρ, $\vee'$, $\wedge'$) is a fuzzy join hyperlattice.

Proof:

We shall show that $\vee'$and $\wedge'$ are well-defined.

For any x, x', y, y' ∈ L, let $\bar{x} = \overline{x'}$ and $\bar{y} = \overline{y'}$ then x ρ x' and y ρ y'.

Since ρ is a fuzzy strong join hypercongruence on (L, ∨, ∧), it follows that

(x ∨ y) $\bar{\bar{\rho}}$ (x' ∨ y') and (x ∧ y) $\bar{\bar{\rho}}$ (x' ∧ y').

This shows that for all a ∈ L, if (x ∨ y) (a) > 0, then there exists b ∈ L, such that (x' ∨ y') (b) > 0,

(x ∨ y) (a) = (x' ∨ y') (b) and a ρ b.

Conversely, for all s ∈ L, if (x' ∨ y') (s) > 0, then there exists some t ∈ L, such that (x ∨ y) (t) > 0,

(x ∨ y) (t) = (x' ∨ y') (s) and s ρ t,

Hence we can get that for all $\bar{z} \in$ L/ ρ, then

$(\bar{x} \vee' \bar{y})\,(\bar{z})$ = sup {(x' ∨ y') (z') | x' $\in \bar{x}$, y'$\in \bar{y}$, z' $\in \bar{z}$}

= sup {(x" ∨ y") (z") | x" $\in \bar{x}$, y"$\in \bar{y}$, z" $\in \bar{z}$}

= sup {(x" ∨ y") (z") | x" $\in \overline{x'}$, y"$\in \overline{y'}$, z" $\in \overline{z'}$}

= $(\overline{x'} \vee' \overline{y'})\,(\bar{z})$, whence $(\bar{x} \vee' \bar{y}) = (\overline{x'} \vee' \overline{y'})$.

Similarly, we show that $(\bar{x} \wedge' \bar{y}) = (\overline{x'} \wedge' \overline{y'})$.

Therefore, $\vee'$and $\wedge'$ are well-defined.

Theorem 2.6:

An equivalence relation ρ is a fuzzy join hypercongruence on a fuzzy join hyperlattice (L, ∨, ∧), if and only if ρ is a join hypercongruence on corresponding join hyperlatice (L, ⊗, ⊕).

Proof:

Set a ρ a', b ρ b', where a, a', b, b' ∈ L.

We have, (a ∨ b) $\bar{\rho}$ (a' ∨ b') and

(a ∧ b) $\bar{\rho}$ (a' ∧ b') if and only if the following conditions are satisfied:

1) for all x ∈ L, if (a ∨ b) (x) > 0, then there exists y ∈ L, such that (a' ∨ b') (y) > 0 and x ρ y;
2) for all s ∈ L, if (a' ∨ b') (s) > 0, then there exists t ∈ L, such that (a ∨ b) (t) > 0 and s ρ t;
3) for all x ∈ L, if (a ∧ b) (x) > 0, then there exists y ∈ L, such that (a' ∧ b') (y) > 0 and x ρ y;
4) for all s ∈ L, if (a'∧ b') (s) > 0, then there exists t ∈ L, such that (a ∧ b) (t) > 0 and s ρ t.

these conditions are equivalent to the following ones,

1) for all x ∈ L, if x ∈ a ⊗ b, then there exists y ∈ a' ⊗ b', such that x ρ y;

2) for all $x \in L$, if $s \in a' \otimes b'$, then there exists $t \in a \otimes b$, such that $s \ \rho \ t$;
3) for all $x \in L$, if $x \in a \oplus b$, then there exists $y \in a' \oplus b'$, such that $x \ \rho \ y$;
4) for all $x \in L$, if $s \in a' \oplus b'$, then there exists $t \in a \oplus b$, such that $s \ \rho \ t$.

which shows that $(a \otimes b) \ \bar{\rho} \ (a' \otimes b')$ and $(a \oplus b) \ \bar{\rho} \ (a' \oplus b')$.

Therefore, ρ is a fuzzy join hypercongruence on $(L, \vee, \wedge)$, if and only if ρ is a join hypercongruence on $(L, \otimes, \oplus)$.

III.CONCLUSION

Hence, in this paper we introduced the notion of fuzzy join hypercongruence and we derive the equivalence relation on fuzzy join hyperlattice. We also depicted the connection between a fuzzy join hypercongruence on a fuzzy join hyperlattice and a join hypercongruence on the associated join hyperlattice.

REFERENCES

[1] https://en.wikipedia.org/wiki/Lattice_(order)
[2] https://www.sciencedirect.com/science/article/pii/S0898122111009357
[3] https://www.researchgate.net/publication/303348737_On_complete_congruence_lattices_of_complete_lattices
[4] https://www.insa.nic.in/writereaddata/UpLoadedFiles/IJPAM/Vol46_2015_5_ART04.pdf