# SPANNING TREE AND MINIMUM SPANNING TREE 

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#### Abstract

In this paper, we have given the mathematical properties of spanning tree and the Kirchhoff's matrix tree theorem to find the total number of possible spanning trees. And also the properties of minimum spanning tree, kruskal's method and prim's method to find the minimum spanning tree.

In this paper the total number of possible spanning trees are derived for cycle, wheel and complete graph with five and six vertices.


Keywords: Tree, Spanning tree, Minimum spanning tree, Weighted graph.

## INTRODUCTION:

Graph theory has direct application on many real world optimization problems. Many algorithms exists to calculate the possible spanning trees .Kruskal's algorithm and Prim's algorithm are the standard algorithm based on greedy technique. And the minimum spanning tree have direct applications in the design of network s, electrical grids, water supply networks and transportation networks. Here the application part is based on the transportation networks.

## TERMINOLOGY:

Simple graph: A Graph G consists of a set of objects $V=\{V 1, V 2, \ldots \ldots . . \mathrm{Vn}\}$ called vertices and another set $E=\{e 1, e 2, \ldots . . e n\}$ called edges. It is denoted by $G=(V, E)$.

Weighted graph: A weighted graph is a simple graph in which every edge is assigned a positive number called the weight of the edge.

Tree: A tree is a connected acyclic graph.
Spanning tree: A spanning tree of a connected graph contains every vertex of the graph.
Minimum spanning tree: Minimum spanning tree is a subset of the edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles and with the minimum possible total edge weight.

## MATHEMATICAL PROPERTIES OF SPANNING TREE:

1. Spanning tree has $n-1$ edges where $n$ is the number of nodes (vertices).
2. From a complete graph by removing maximum e-n+1 edges we can construct a spanning tree.
3. A complete graph can have maximum $n^{n-2}$ number of spanning trees. Thus we can conclude that spanning tree are a subset of connected graph G and disconnected graphs do not have spanning trees.

## KIRCHHOFF'S MATRIX TREE METHOD:

The total number of possible spanning trees from a graph is given by kirchhoff's matrix tree theorem. It is calculated as below.

1. Create adjacency matrix for the given graph.
2. Replace all the diagonal elements with the degree of nodes. For example, element at (1,1)position of adjacency matrix will be replaced by the degree of node 1 , element at $(2,2)$ position of adjacency matrix will be replaced by the degree of node 2 , and so on.
3. Replace all non-diagonal 1s with -1.
4. Calculate co-factor for any element.
5. The cofactor of any element in the matrix is the same and that is the total number of spanning trees for that graphs.

Find the total number of possible spanning trees from a graph having five vertex:

a) Adjacency matrix is given by, $\left[\begin{array}{lllll}0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0\end{array}\right]$
b) Replacing all the diagonal elements with degree of the corresponding vertex.
c) Non-diagonal elements that 1 with -1
d) Finding the co-factor of any element in the above matrix:

Co-factor of 3 is the determinant of the matrix $=24$.
Hence, the total number of possible spanning trees for the given graph is 24.
Find the total number of possible spanning trees from a graph having six vertex:


Adjacency Matrix of the given graph is, $\left[\begin{array}{cccccc}0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0\end{array}\right]$
a) Replacing all the diagonal elements with degree of the corresponding vertex
b) Non-diagonal elements that 1 with -1
c) Finding the co-factor of any element in the above matrix:

Co-factor of 3 is the determinant of the matrix $=36$.
Hence, the total number of possible spanning trees for the given graph is 36 .

Find the total number of possible spanning trees from a cycle having six vertex


Adjacency Matrix of the given graph is $\left[\begin{array}{llllll}0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0\end{array}\right]$
a) Replacing all the diagonal elements with degree of the corresponding vertex
b) Non-diagonal elements that 1 with -1
c) Finding the co-factor of any element in the above matrix:

Co-factor of 2 is the determinant of the matrix $=6$.
Hence, the total number of possible spanning trees for the given graph is 6 .
Find the total number of possible spanning trees from a wheel having six vertex.


Adjacency Matrix of the given graph is $\left[\begin{array}{llllll}0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0\end{array}\right]$
a) Replacing all the diagonal elements with degree of the corresponding vertex
b) Non-diagonal elements that 1 with -1
c) Finding the co-factor of any element in the above matrix:

Co-factor of 3 is the determinant of the matrix $=117$
Hence, the total number of possible spanning trees for the given graph is 117 .
Find the total number of possible spanning trees from a complete graph having six vertex.


Adjacency Matrix of the given graph is
$\left[\begin{array}{llllll}0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0\end{array}\right]$
a) Replacing all the diagonal elements with degree of the corresponding vertex
b) Non-diagonal elements that 1 with -1
c) Finding the co-factor of any element in the above matrix:

Co-factor of 5 is the determinant of the matrix $=1296$
Hence, the total number of possible spanning trees for the given graph is 1296.

## Kruskal's algorithm:

1. Sort all the edges in non-decreasing order of their weight.
2. Pick the smallest edge. check if it forms a cycle with the spanning tree formed so far. If cycle is not formed, include this edge. Else discard it.
3. Repeat step 2 until there are ( $\mathrm{v}-1$ )edges in the spanning tree.

## Prim's algorithm:

Prims Algorithm also use greedy approach to _nd the minimum spanning tree. In Prims Algorithm we grow the spanning tree from a starting position. Unlike an edge in kruskals, we add vertex to the growing spanning tree in prims.

1. Maintain two disjoint sets of vertices. One containing vertices that are in the growing spanning tree and other that are not in the growing spanning tree.
2. Select the cheapest vertex that is connected to the growing spanning tree and is not in the growing spanning tree and add it into the growing spanning tree. This can be done using priority queues. Insert the vertices, that are connected to the growing spanning tree into the priority queues.
3. Check for cycles, to do that mark the nodes which have been already selected and insert only those nodes in the priority queue that are not marked.

## APPLICATION OF MINIMUM SPANNING TREE:

Once upon a time there was a city that had no roads. Getting around the city was particularly difficult after rainstorms because the ground became very muddy cars got stuck in the mud and people got their boots dirty. The mayor of the city decided that some of the streets must be paved, but did not want to spend more money than necessary because the city also wanted to build a swimming pool. The mayor therefore specified two conditions:

1. Enough streets must be paved so that it is possible for everyone to travel from their house to anyone else house only along paved roads and
2. The paving should cost as little as possible. The number of paving stones between each house represents the cost of paving that route.


Using Kruskal's method and prim's method we found that the minimum spanning trees are 25 .
The number of paving stones between each house is 25 . This is the best route that connects all the houses. Hence the two conditions specified by the mayor is satisfied.

## CONCLUSION:

In this paper, we have discussed about the spanning trees and the minimum spanning trees and also their properties. We have found the total number of possible spanning trees using the kirchhoff's matrix tree theorem. Using the Kruskal's method and Prim's method we have found the minimum spanning trees. The total number of possible spanning trees for cycle graph, wheel graph, complete graph, etc having vertex more than six can be found for further research. Minimum spanning tree has numerous application in communication and transportation networks, it is very important to have efficient algorithms to find minimum spanning trees in weighted and connected graphs. Kruskal's method and Prim's method have made important ${ }^{\text {contributions }}$ in this area of graph theory.

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