# QUEUEING MODEL (M/M/C: ∞/FIFO) TO ERODE RAILWAY TICKET COUNTERS

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***Abstract*: In this paper the Queueing model (M/M/C: ∞/FIFO) is applied to the ticket counters of the Erode railway station. In this research the various characters of that queueing model have been analyzed and concluded.**

***Keywords*: Queue, Arrival rate, departure rate, traffic intensity.**

## 1. INTRODUCTION

A flow of customers from infinite or finite population towards the service facility forms a queue on account of lack of capability to serve them all at a time. Our main objectives of our research are

i. To apply the basic concepts of the selected queueing model to the Erode railway station, Tamil Nadu, India.

ii. To determine the various characteristics of the selected queueing model to the queueing system in the railway unreserved ticket centre at Erode Railway station.

## 2. DATA COLLECTION

We have visited the railway unreserved ticket counters at Erode railway Station on various days during 8.00a.m to 8p.m and conducted a survey about the arrivals of customers to the unreserved ticket counters and about the service rendered to the customers. The data pertaining to the arrival and Departures of customers to the unreserved ticket counters of Erode railway station are given in this section.

The customers arrive at the railway unreserved ticket counters under Poisson process and the service is done to the customers in the exponential rate. This queueing model involves three servers. The customers are given service under the FIFO (FIRST IN FIRST OUT) discipline. Thus the queueing model which will fit to our problem is (M/M/C: ∞ / FIFO) which is also known as the multi server queueing model.

The details regarding the arrivals and departures of customers obtained in the Railway Reservation counters at Erode railway station are given in the following table 1.

***Table 1: Arrival and Departure Passenger train details***

| Time intervals Per Hour | Arrival Time Interval (Counting for every ten minutes) | Departure Time Interval (Counting for every ten minutes) | No.of arrivals Per Hour | No.of Departure Per Hour |
|---|---|---|---|---|
| 8.00 – 9.00 | 10/7/10/8/6/12 | 0/8/6/6/9/13 | 53 | 42 |
| 9.00 – 10.00 | 3/7/2/5/6/7 | 4/5/7/6/7/4 | 30 | 33 |
| 10.00 – 11.00 | 4/7/4/6/7/8 | 6/5/7/4/6/7 | 36 | 35 |
| 11.00 – 12.00 | 10/7/10/8/6/9 | 10/8/7/8/4/11 | 50 | 48 |
| 12.00 – 13.00 | 10/8/5/6/10/5 | 7/8/9/10/5/5 | 44 | 47 |
| 13.00 – 14.00 | 3/3/3/2/2/3 | 6/8/7/5/4/4 | 16 | 33 |
| 14.00 – 15.00 | 0/1/3/3/6/5 | 0/3/3/5/5/6 | 18 | 22 |
| 15.00 – 16.00 | 4/4/1/3/2/1 | 7/3/3/4/6/3 | 15 | 24 |
| 16.00 – 17.00 | 2/4/5/2/3/1 | 3/4/3/4/4/3 | 17 | 21 |
| 17.00 – 18.00 | 5/5/4/2/6/1 | 4/5/4/7/3/3 | 23 | 26 |
| 18.00 – 19.00 | 7/4/5/3/3/4 | 7/8/4/9/7/5 | 26 | 35 |
| 19.00 – 20.00 | 3/4/4/4/4/0 | 4/7/4/6/5/4 | 19 | 23 |

## 3. EVALUATION OF THE CHARACTERISTICS

From the above table,

1. Average number of arrivals to the system per hour

$(\lambda) = \frac{347}{12}$ per hour = 28.92 per hour = 29 per hour (appr.)

2. Average number of departure from the system (μ) per hour $= \frac{393}{12}$ per hour = 32.75 per hour
μ = 33 ( appr.)
Since C = 4, μ = $\frac{33}{4}$ , i.e., μ = 8.2500 per queue.

3. To find the traffic intensity, $\rho = \frac{\lambda}{C\mu}$, Here $\lambda$ = 29, μ = 8.2500 and C = 4, $\rho = \frac{\lambda}{C\mu} = \frac{29}{33}$ = 0.88.

4. $P_n$= probability that there are n- customers in the system both Waiting and in service.

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 \; ; & 1 \leq n \leq C \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 \; ; & n \geq C \end{cases} \qquad \ldots\ldots\ldots\ldots(1)$$

Where $P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1}$ …………..(2)

To find $P_0 = \left[1 + \frac{1}{1!}\left(\frac{29}{8.2500}\right)^1 + \frac{1}{2!}\left(\frac{29}{8.2500}\right)^2 + \frac{1}{3!}\left(\frac{29}{8.2500}\right)^3 + \frac{1}{4!}\left(\frac{29}{8.2500}\right)^4 \frac{33}{33-29}\right]^{-1}$

$= [1 + 3.5151 + 6.1781 + 7.2390 + 52.4823]^{-1}$

$= [70.4145]^{-1}$

$P_0 = 0.0142$

5. The values of $P_n$ are calculated for n = 1,2,3........

Using the formulae (1) and (2) These values are given in the Table,

***Table 2: The values of Pn***

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_n$ | 0.01421 | 0.0499 | 0.0873 | 0.1023 | 0.0541 | 0.0324 | 0.0294 | 0.0231 | 0.0202 | 0.0176 | 0.0155 |

| N | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $P_n$ | 0.0136 | 0.0116 | 0.0104 | 0.0087 | 0.0081 | 0.0071 | 0.0062 | 0.0055 | 0.0046 | 0.0042 |

From the table 2 we can easily see that $\sum_{n=0}^{C-1} P_n + \sum_{n=C}^{\infty} P_n = 1$

## 4. THE RESULTS OF CHARACTERISTICS OF OUR SYSTEM $[(M/M/C):(\infty/FIFO)]$

(i) P (n≥ $C$) = Probability that an arrival has to wait

$$= \frac{C\mu\left(\frac{\lambda}{\mu}\right)^C}{C!(C\mu-\lambda)} P_0 = \frac{4(8.250)\left(\frac{29}{8.250}\right)^3 0.0142}{4!(33-29)} = 0.7452$$

(ii) Probability that an arrival enters the service without wait

$$= 1 - \mathrm{P}(\mathrm{n} \geq \mathrm{C}) = 1 - 0.7452 = 0.2548$$

(iii) Average queue length , Lq

$$\mathrm{Lq} = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_0}{(C-1)!(C\mu-\lambda)^2} = \frac{29(8.2500)\left(\frac{29}{8.2500}\right)^4 0.0142}{29}$$

Lq = 17.8862 …... (3)

(v) Average number of customers is the system $[E(n)]$

$\mathrm{Ls} = \mathrm{Lq} + \frac{\lambda}{\mu}$ = 17.8862+3.5151

Ls= 21.4013

(iv)Average waiting time of an arrival $[Wq]$

$\mathrm{Wq} = \frac{1}{\lambda}$ = 0.6167

$\mathrm{Lq} = \frac{17.8862}{29}$

$E(\omega)$ = 0.6167 or 37 minutes ……… (4)

(vi) Average waiting time an arrival spends in the system $[Ws]$

$\mathrm{Ws} = \mathrm{Wq} + \frac{1}{\mu}$

$= 0.6167 + \frac{1}{8.2500}$

$= 0.6167 + 0.1212$

Ws = 0.7379 or 44 minutes ……… (5)

### *4.1 Verification of the Little's formulae*

The relationships between the characteristics are given by means of the following formulae known as little formulae there are

(i) Ls = $\lambda$ Ws

(ii) Lq = $\lambda$ Wq

(iii) Ws = Wq + $\frac{1}{\mu}$

Our findings also satisfy the Little's formulae.

(i) To verify Ls = $\lambda$ Ws from equation (5)

In this section, we have obtained that Ls = 21.4013 from

Equation (5) we have found that $\lambda$ Ws = 29× 0.7379

= 21.3991

= Ls (appr.)

(ii) To prove Lq = $\lambda$ Wq

From the equation (3) we see that Lq = 17.8862

Also from equation, $\lambda$ Wq = 29 × 0.6167

= 17.8843

= Lq.

## 5. OBSERVATIONS, SUMMARY

### *5.1 Observations:*

In our study we have made the following observations the following results.

i. Traffic intensity, $\rho$ = 0.8787

ii. Probability that there are no customers in the system, $P_0$ = 0.0142

iii. Probability that an arrival has to wait, p(n≥c) = 0.7452

iv. Probability that an arrival enters the service without wait

1-p(n≥c) = 0.2548

v. Average queue length at any time Lq = 17.8862 customers

vi. Average number of customer in the system at any time

Ls = 21.4013 customers

vii. Average waiting time of an arrival in the queue, Wq = 0.6167 or

hour 37.002 minutes.

viii. Average waiting time of an arrival spends in the system Ws = 0.7379

or 44 minutes (appr)

ix. Average time of service = 8 minutes (appr)

x. Average number of arriving rate, $\lambda$ = 29 per hour.

xi. Average number of service rate, μ = 8.250 per hour.

### *5.2 Summary:*

After conducting the survey and the analysis of data obtained from the reservation centre of Erode railway station the following predictions are made regarding the characteristics of the queueing made 1 (M/M/C: ∞/FIFO):

i. An average of 29 customers are arriving to the unreserved ticket counters in an hour.

ii. Averages of 33 customers are being served in the counters in an hour.

iii. The traffic intensity $\rho = \frac{\lambda}{c\mu}$ =0.8787 < 1 and therefore the queueing system followed in the Erode railway unreserved ticket centre attains stability.

iv. The probability that all the servers bring idle is nearly 12 percent.

v. The probability that an arrival has to wait before getting served is 88 percent.

vi. The probability that an arrival enters the service straight away without having to wait is 11 percent.

vii. At any time an average of 17 customers are found waiting in the queue.

viii. At any time an average of 21 customers are found waiting in the system.

ix. On arrival, a customer has to wait in the queue for 37 minutes before getting served.

x. On arrival, a customer has to wait in the system for 44 minutes before getting served.

xi. A clerk takes an average of 8 minutes of reserve a ticket.

## 6. CONCLUSION

In our study we have found that it is optional that there are four clerks employed in the Erode railway ticket counter. It means that if we add one more counter to the existing set up, the chance for a clerk to be idle would increase enormously except the holidays and festival days. But we can suggest them to open the non working counter to work on the holidays and festival days. Also, if one counter is removed from the existing set up, the no of customers waiting in the queue would increase rapidly, thus it would end up in chaos.

Also we have analyzed that the traffic intensity of the queueing system prevailing in Erode railway station always remains less then unity and therefore the system attains marginally stability.

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