On (i,j)*- Soft Preopen Sets In Soft Bitopological Spaces

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***Abstract*: The purpose of this paper is to introduce the concept of (i,j)*-soft preopen sets and (i,j)*-soft preoclosed sets, (i,j)*-soft preinterior, (i,j)*-soft preoclosure in soft bitopological space and some of their properties are also obtained. Also we state and prove the condition for collection of (i,j)*-soft preopen sets to be a soft bitopoloogy.**

***Keywords*: Soft Set, Soft Topology, (i,j)*-Soft Preopen sets, (i,j)*-Soft Preclosed sets, (i,j)*-Soft Preinterior, (i,j)*-Soft Preclosure sets.**

1. INTRODUCTION

The concept of soft set theory was initiated by Russian researcher D.Molodtsov [1] in the year 1999 as a recent mathematical tool and has been applied in several directions, such as smoothness of functions, game theory, operation research, engineering physics, economics, social science, Riemann integration, etc. Maji, Biswas, Roy [2], studied the theory of soft sets. They discussed the basic soft set definition with examples. The Muhammad shabir and munazza naz [3], introduced notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameter. Bitopological spaces was introduced by J. C. Kelly [8] in the year 1963 and specify a bitopological space, $(A, \tilde{\sigma}_i, \tilde{\sigma}_j)$ to be a set A for topologies $\tilde{\sigma}_i$ and $\tilde{\sigma}_j$ on A and introduced the systematic study of bitopological space. The concept of soft bitopological space was introduced by Basavaraj M. Ittanagi [5] in the year 2014. Andrijevic and Mashour [4] gave many results on preopen sets in general topology. Some properties of soft bitopological spaces was investigated by G.Senel et al [6]. Shabir Hussain et al [7] obtained some results on soft topological spaces. This paper aims at developing a soft bitopological via , (i,j)*-soft preopen sets.

PREMILINARIES

Definition 1.1 [1]: Let $V = (v_1, v_2, \ldots . v_n)$ be an initial universal set, T be the set of all parameters and $\rho(A)$ expressed the power set of V, A be a non-empty subset of T. A pair (P, A) expressed as P_A is called a soft set over A. Where P is a map from A to $\rho(A)$.

Definition 1.2 [3]: A soft set (P,T) in soft topological space is known as null soft set it is expressed as $P_{\emptyset}$ if for every $e \in T, P(e) = \tilde{\emptyset}$.

A soft set (P,T) in soft topological space is known as absolute soft set it is expressed as $P_{\tilde{A}}$ or $\tilde{A}$ if for every $e \in T, P(e) = \tilde{A}$.

Definition 1.3[3]: A topology on a set A is a collection $\tilde{\sigma}$ of subset of A having the following properties:

(i)Null soft sets and absolute soft sets belongs to $\tilde{\sigma}$.

(ii)Union of some member of soft set in $\tilde{\sigma} \in \tilde{\sigma}$. i.e., If $\{(P_j, T)\}_{j\in J} \in \tilde{\sigma}$, then $\cup_{j\in J}(P_j, T) \in \tilde{\sigma}$.

(iii)Intersection of some dual of soft set in $\tilde{\sigma} \in \tilde{\sigma}$. i.e., If $(P_1,T), (Q_1,T) \in \tilde{\sigma}$, then $(P_1,T) \cap (Q_1,T) \in \tilde{\sigma}$.

A set A for which a topology $\tilde{\sigma}$ has been specified is known as soft topological space**.** An any member of $\tilde{\sigma}$ are known as (i,j)*-soft open set in A and complement of them are called (i,j)*-soft closed set in A.

Definition 1.4 [5]: Let A be a non empty soft set over the universe V,$\tilde{\sigma}_i$ & $\tilde{\sigma}_j$ are dual diverse soft topologies over A. Then $(A, \tilde{\sigma}_i, \tilde{\sigma}_j)$ is called a soft bitopological space.

Definition 1.5 [8]: Let (Q, T) be a soft set in soft bitopological space $(A, \tilde{\sigma}, T)$. Then

(i) The (i,j)*-soft interior of (Q,T) is the soft set,

(i,j)*-int $(Q, T) = \cup \{(L, T): (L, T)$ is (i,j)*-soft open & $(Q, T) \tilde{\supseteq} (L, T)\}$

(ii) The (i,j)*-soft closure of (Q, T) is the soft set,

(i,j)*-cl $(Q, T) = \cap \{(L, T): (L, T)$ is (i,j)*-soft closed &$(Q, T) \tilde{\subseteq} (L, T)\}$.

The (i,j)*-soft closure of (Q,T) expressed as (i,j)*-cl(Q,T) is the intersection of each (i,j)*-soft closed of (Q,T). Obviously (Q,T) is the lowest (i,j)*-soft closed set contained (Q,T).

The (i,j)*-soft interior of (Q,T) expressed as (i,j)*-int(Q,T) is the union of each (i,j)*-soft open of (Q,T). Obviously (Q,T) is the greatest (i,j)*-soft closed set contained (Q,T).

2. Soft preopen sets

Definition 2.1: In a soft topological space $(A, \tilde{\sigma}_i, \tilde{\sigma}_j)$, a soft set,

(i) (Q,T) is known as (i,j)*-soft preopen set if $(Q,T) \subseteq$ -int($\tilde{\sigma}_{i,j}$-cl(Q,T)).

(ii) (P,T) is known as (i,j)*-soft preclosed set if $(P,T) \supseteq \tilde{\sigma}_{i,j}$-cl($\tilde{\sigma}_{i,j}$-int(P,T)).

Theorem 2.2: An arbitrary union of (i,j)*-soft preopen sets is a (i,j)*-soft preopen set.

Proof: We take the family of soft set $\{(Q,T)_\gamma | \gamma \in \chi\}$ be a collection of (i,j)*-soft preopen sets in $(A, \tilde{\sigma}, T)$. Then for each γ, $\tilde{\sigma}_{i,j}$-int($\tilde{\sigma}_{i,j}$-cl$(Q,T)_\gamma) \supseteq (Q,T)_\gamma$

Claim: $\cup\, \tilde{\sigma}_{i,j}$-int($\tilde{\sigma}_{i,j}$-cl$(Q,T)_\gamma) \supseteq \cup\, (Q,T)_\gamma$

From definition, $\cup\, \tilde{\sigma}_{i,j}$-int($\tilde{\sigma}_{i,j}$-cl$(Q,T)_\gamma) \supseteq \cup\, (Q,T)_\gamma$

$\tilde{\sigma}_{i,j}$-int($\cup\, \tilde{\sigma}_{i,j}$-cl$(Q,T)_\gamma) \supseteq \cup\, (Q,T)_\gamma$

$\tilde{\sigma}_{i,j}$-int($\tilde{\sigma}_{i,j}$-cl $(\cup\, (Q,T)_\gamma) \supseteq \cup\, (Q,T)_\gamma$

Hence $\tilde{\sigma}_{i,j}$-int($\tilde{\sigma}_{i,j}$-cl $(\cup\, (Q,T)_\gamma) \supseteq \cup\, (Q,T)_\gamma$.

Note: An arbitrary intersection of (i,j)*-soft preopen sets is a (i,j)*-soft preopen set.

Example 2.3:

Let $A = \{a_1, a_2, a_3, a_4\}, T = \{t_1, t_2, t_3\}$ and Let $P_1, P_2, P_3, P_4, P_5, P_6$ the maps from T to P(A) is defined by

$(P_1,T) = \{(t_1,\{a_1, a_2\}), (\{t_2, \{a_1, a_2\})\}$,

$(P_2,T) = \{(t_1,\{a_2\}), (\{t_2, \{a_1, a_3\})\}$,

$(P_3,T) = \{(t_1,\{a_1, a_3\}),(\{t_2, \{a_1\})\}$,

$(P_4,T) = \{(t_1,\{a_2\}),(\{t_2, \{a_1\})\}$,

$(P_5,T) = \{(t_1,\{a_1, a_2\}),(\{t_2, \{a_1, a_2, a_3\})\}$,

$(P_6,T) = \{(t_1,\{a_1, a_2, a_3\}), (\{t_2, \{a_1, a_2\})\}$ are soft sets in A.

Now, we consider $\tilde{\sigma}=\{\emptyset, A, (P_1,T), (P_2,T), (P_3,T), (P_4,T), (P_5,T), (P_6,T)\}$ a soft topology in A. Here, $(R,T) = \{(t_1,\{a_2\}),(\{t_2,\{a_2, a_3\})\}$ is (i, j)*-soft preopen set and also $(S,T) = \{(t_1,\{a_1, a_3\}),(\{t_2,\{a_1, a_3\})\}$ is (i, j)*-soft preopen set. But $(R,T) \cap (S,T) = \{(t_1,\emptyset),(\{t_2,\{a_3\})\}$ is (i, j)*-not soft preopen sets.

Theorem 2.4: Let (Q,T) be a (i, j)*-soft preopen set such that $(R,T) \subseteq (Q,T) \subseteq \tilde{\sigma}_{i,j}$-cl$(R,T)$. Then (Q,T) is a (i, j)*-soft preopen set.

Proof: Let $(A, \tilde{\sigma}, T)$ be soft bitopological space. The set (Q,T) is (i, j)*-soft preopen set if $(Q,T) \subseteq \tilde{\sigma}_{i,j}$-int cl$(Q,T)$. $\Rightarrow (R,T) \subseteq (Q,T) \subseteq \tilde{\sigma}_{i,j}$-cl(Q,T)$\subseteq \tilde{\sigma}_{i,j}$-cl$(R,T) \Rightarrow \tilde{\sigma}_{i,j}$-int$\tilde{\sigma}_{i,j}$-cl$(Q,T) \subseteq \tilde{\sigma}_{i,j}$- $int\ \tilde{\sigma}_{i,j}$-cl$(R,T) \Rightarrow (R,T) \subseteq (Q,T) \subseteq \tilde{\sigma}_{i,j}$-int $\tilde{\sigma}_{i,j}$- cl$(Q,T) \subseteq \tilde{\sigma}_{i,j}$-int $\tilde{\sigma}_{i,j}$-cl$(R,T) \Rightarrow (R,T) \subseteq (Q,T) \subseteq \tilde{\sigma}_{i,j}$-int $\tilde{\sigma}_{i,j}$-cl(R,T). Therefore (R,T) is (i,j)*-soft preopen set.

Note: Let (Q,T) be a (i,j)*-soft preclosed set such that $(R,T) \subseteq (Q,T) \subseteq \tilde{\sigma}_{i,j}$-int$(R,T)$. Then (Q,T) is a (i,j)*-soft preclosed set.

Theorem 2.5: If the (i,j)*-soft preclosure of (Q,T) is a soft set in soft bitopological space. Then,

(i) $(\tilde{\sigma}_{i,j}$-pcl$(Q,T))^c$ = $\tilde{\sigma}_{i,j}$-pint(Q^c,T).

(ii) $(\tilde{\sigma}_{i,j}$-pint$(Q,T))^c$ = $\tilde{\sigma}_{i,j}$-pcl(Q^c,T).

Proof: Let (Q,T) be a soft set over A.

(i) Let ($\tilde{\sigma}_{i,j}$-pcl$(Q,T))^c = (\cap\{(Q,T) \subseteq L,T)$ and $(L,T) \in$ PCSS$(A_T)\})^c$

= $\cup\{(L,T)^c$ & $(L,T)^c \subseteq (Q,T)^c$ & $(L,T)^c \in$ POSS$(A_T)\})$

= $\cup\{(L^c,T)$ & $(L^c,T) \subseteq (Q^c,T)$ & $(L^c,T) \in$ POSS$(A_T)\})$

= $\tilde{\sigma}_{i,j}$- pint(Q^c,T).

(ii) Let ($\tilde{\sigma}_{i,j}$-pint$(Q,T))^c$ = ($\cup$\{(L,T) $\subseteq$ (Q,T) & (L,T) $\in$ POSS$(A_T)\})^c$

= $\cap\{(L,T)^c$ & $(Q,T)^c \subseteq$ $(L,T)^c$ & $L,T)^c \in$ PCSS$(A_T)\}$

= $\cap\{(L^c,T)$ & $(Q^c,T) \subseteq (L^c,T)$ & $(L^c,T) \in$ PCSS$(A_T)\}$

=$\tilde{\sigma}_{i,j}$-pcl(Q^c,T).

Theorem 2.6: Let $(A, \tilde{\sigma}_i, \tilde{\sigma}_j)$ be a $soft$ bitopological space and (Q,T) be a soft set in A.

(i) $\tilde{\sigma}_{i,j}$-pcl$\big((Q,T) \cup (M,T)\big) =$ $\tilde{\sigma}_{i,j}$-pcl$(Q,T) \cup \tilde{\sigma}_{i,j}$-pcl$(M,T)$

(ii) $\tilde{\sigma}_{i,j}$-pint$(Q,T) \cap (M,T) =$ $\tilde{\sigma}_{i,j}$-pint(Q,T) $\cap$ $\tilde{\sigma}_{i,j}$-pint(M,T)

Proof: We take, $(Q,T) \cup (M,T) \supset (Q,T)$ & $(Q,T) \cup (M,T) \supset (M,T)$

We have, $(Q,T) \subseteq (M,T), \Rightarrow \tilde{\sigma}_{i,j}$-pcl$\big((Q,T) \cup (M,T)\big) \supset \tilde{\sigma}_{i,j}$-pcl$(Q,T)$ & $\tilde{\sigma}_{i,j}$-pcl$((Q,T) \cup (M,T)) \supset \tilde{\sigma}_{i,j}$-pcl$(M,T) \Rightarrow \tilde{\sigma}_{i,j}$-pcl$((Q,T) \cup (M,T)) \supset \tilde{\sigma}_{i,j}$-pcl(Q,T) $\supset \tilde{\sigma}_{i,j}$-pcl(M,T). (1)

Since, (i,j)*-soft preclosure(Q,T), (i,j)*-soft preclosure(M,T) ϵ soft preclosed set(A_T) and (i,j)*-soft pre-closure $(Q,T) \cup$ (i,j)*-soft preclosure(M,T) ϵ (i,j)*-soft preclosed set(A_T). Now, $(Q,T) \subset \tilde{\sigma}_{i,j}$-pcl$(Q,T)$ and $(P,T) \subset \tilde{\sigma}_{i,j}$-pcl$(P,T)$. Implies, $\tilde{\sigma}_{i,j}$-pcl$(Q,T) \cup \tilde{\sigma}_{i,j}$-pcl$(M,T) \supset ((Q,T) \cup (M,T))$. That is, $\tilde{\sigma}_{i,j}$-pcl$((Q,T) \cup (M,T))$ is the lowest (i,j)*-soft preclosed set containing $((Q,T)\,) \cup (M,T))$ and $\tilde{\sigma}_{i,j}$-pcl $(Q,T) \cup \tilde{\sigma}_{i,j}$-pcl$(M,T)$ is a (i,j)*-soft preclosed set containing $((Q,T) \cup (M,T))$. Therefore, $\tilde{\sigma}_{i,j}$-pcl$\big((Q,T) \cup (M,T)\big) \subset$ $\tilde{\sigma}_{i,j}$-pcl$(Q,T) \cup \tilde{\sigma}_{i,j}$-pcl$(Q,T)$ (2)

From (1) & (2), $\tilde{\sigma}_{i,j}$-pcl$\big((Q,T) \cup (M,T)\big) \subset \tilde{\sigma}_{i,j}$-pcl$(Q,T) \cup \tilde{\sigma}_{i,j}$-pcl$(M,T)$.

(ii) Similar to (i).

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