

Inverse Kinematics of Cable Driven Parallel Robot.

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Abstract— Now a days concept of Cable Driven Parallel Robot is the immerging area of research. Though it has great advantages like Reconfigurable, No special joints are required, Large workspace, High payload to weight ratio etc. In this paper basics of CDPR and several classification approach is discussed. For CDPR since cable use as links, it is important to find cable property so in that paper cable property was checked using standard methods and its result is shown. For any robotic application, calculation of forward and inverse kinematics solution is very much important. In this paper new method of solving inverse kinematics solution using Matlab Simulink tool is shown. In this paper inverse kinematics of CDPR is solved for making circular and helical trajectory and its time v/s coordinate and time v/s length of cables graph are shown.

Index Terms— Cable Driven Parallel Robot (CDPR), Inverse kinematics, Matlab Simulink, Classification of CDPR.

I. INTRODUCTION

Cable Driven Parallel Robot (CDPR) is a special class of parallel robot in which the rigid legs are replaced by cables. It has certain advantages in terms of intrusivity and workspace. But due to some special properties like unilateral property of cable it is necessary to work on proper tension of cable, work space analysis of CDPR, sagging and elasticity effect on cable etc.

Motion of the platform obtained either (1) by changing the length of the wire or (2) having fixed wire length and modifying the location of the attachment point A of the wires on the base. Number of kinematic equation will depends upon the cable configuration [1].

Kinematics of CDPR is classified as following two types [2]:

(1) CDPR categorized based on redundancy

(a) CRPM (Completely Restrained Parallel Manipulator): The pose of the robot is completely determined by the unilateral kinematic constraints defined by the tensed cables. For a CRPM at least $m = n + 1$ wires are needed.

(b) IRPM (Incompletely Restrained Parallel Manipulator): In addition to the unilateral constraints induced by the tensed wires at least one dynamical equation is required to describe the pose of the end effector.

(2) Based on the number of controlled degree of freedom:

- (a) 1T: linear motion of a point.
- (b) 2T: planar motion of a point.
- (c) 1R2T: planar motion of a body.
- (d) 3T: spatial motion of a point.
- (e) 2R3T: spatial motion of a beam.
- (f) 3R3T: spatial motion of a body.

Here T stands for translational and R stands for rotational degree of freedom [2].

Cable has unidirectional property i.e. it must be in proper tension if tension in cable is too much than the cable is broken and if tension is less than CDPR may not work properly. So in CDPR optimally safe tension distribution is very necessary.

For optimally safe tension distribution linear and quadratic programming formulation is done and introduce to new slack variable which enables rapid generation of feasible starting point from the solution of the previous servo loop. This algorithm is tested on NIMS-PL a four cable 2 degree of freedom robot and executed a circular trajectory and it satisfies the tension distribution and avoid near – slack operating condition and demonstrated continuous behavior [3].

Two different algorithms are proposed: one is for point wise trajectories and another is for continuous trajectories and algorithm is tested on a 3 degree of freedom planar CDPR to show the feasibility of the control strategy [4].

End effector's usable workspace is essential for trajectory planning, selection & design of robot configurations. Workspace of CDPR is classified as five types [5], static equilibrium work space, wrench closer work space, wrench feasible work space [6], dynamic work space, and collision free workspace. Algorithm is proposed which allows to determine exactly the location of the

end effector where interference between two wires will not occur [7]. It is applied to 6-6 cable suspended robot. The variations of workspace volume and global condition index of the robot vs. geometric configurations, size of moving platform and different orientations were determined [8].

While designing CDPR for large workspace sagging must be considered. Cable sag indeed large effect on both the inverse kinematics and the stiffness of a cable driven manipulator. The algorithm to solve forward kinematics for CDPR with sagging cables is developed and tested [9] Static analysis of Five hundred meters Aperture Spherical radio Telescope (FAST) is done and its mathematical modelling is proposed [10].

CDPR is almost used in all fields due to its advantages like intrusivity, large workspace, high payload to weight ratio. CDPR is used in Additive manufacturing, Rescue operation, Biomechanic and Rehabilitation, Cranes, Pic and place operation, Radio Telescope etc.[11].

For each application their inverse kinematics and its mathematical modeling is given like for contour crafting of large workspace C4 robot is designed and practically implemented and its simplest inverse and forward kinematics solution is developed and tested, its cost comparison is given [12].

For application like actuated sensing applications, a NIMS3D robot is developed which is used for rapid in-field deployments. Its kinematic and dynamic analysis of system have been provided and results from trajectory control experiments have been shown. Developed new method for generating energy efficient trajectories and proper tension distribution [13].

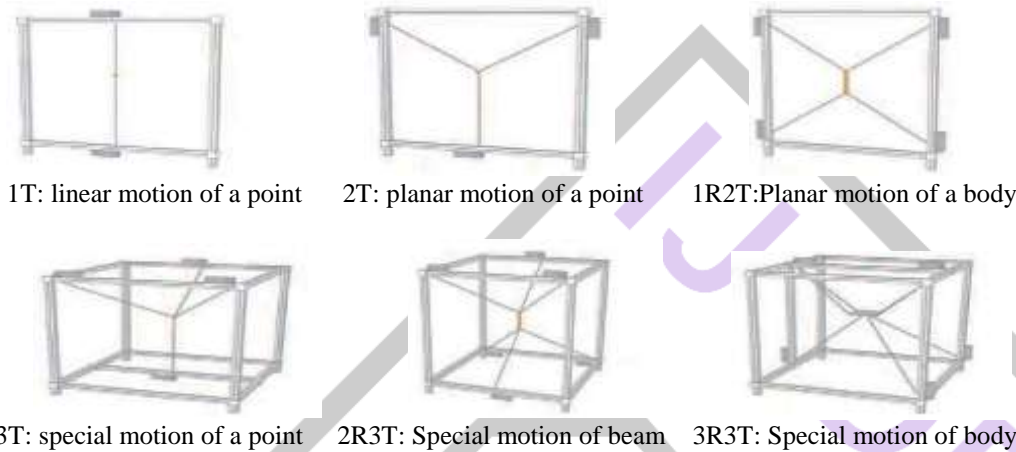


Figure 1. Classification of CDPR based on controlled D.O.F.[2]

Here T stands for translational and R for rotational d.o.f.. It is notable that this definition is complete and covers all wire robots. The classification given by Fang is similar to Verhoeven’s approach. Here, three classes are defined as [2]:

- IKRM (Incompletely Kinematic Restrained Manipulators), where $m < n$
- CKRM (Completely Kinematic Restrained Manipulators), where $m = n$
- RAMP (Redundantly Actuated Manipulators), where $m \geq n + 1$

II. PROPERTIES OF CABLE.

In CDPRs the rigid links of Parallel robot is replaced by cables. So in CDPRs cables act as main links therefore it is very much important to find the properties of cable. In following section properties of cables can be experimentally derived and calculated.

2.1 Experimental setup of checking properties of cable: At fixed rod tie one end of cable. The other end of cable is free. Measure length of cable from fixed end of cable to the free end of cable. Then gradually apply standard load 0.5kg to 4.5kg and convert weight from kg to N. After each two reading remove load and check for plastic deformation.

After taking reading of length and weight, find different properties
 Stress (N/mm^2) = Load / Area; $Area = \pi r^2 = \pi * (0.25)^2 = 0.20258024 (mm^2)$
 Strain = $\Delta L / L$ (572)

Young’s modulus of elasticity $Y (N/mm^2) = Stress / Strain$

Spring constant per unit length $K (N) = Y * Area$.

Spring constant $k (N/mm) = K / L$ (572)

After 10 readings takes average of it.

Table 1. Checking properties of cable

Sr.no.	F (N)	L(mm)	ΔL (mm)	Stress (N/mm^2)	Strain
1	0	572	0	0	0
2	4.905	577	5	24.21	0.0087
3	9.81	581.5	9.5	48.43	0.0166
4	14.715	587	15	72.64	0.0262

5	19.62	591.2	19.2	96.85	0.0335
6	24.525	596.7	24.7	121.06	0.0431
7	29.43	598.5	26.5	145.28	0.046
8	34.335	601.9	29.9	169.49	0.0522
9	39.24	605.26	33.26	193.70	0.0581
10	44.145	612	40	217.91	0.0699
	AVG=			108.96	0.0394

Young's modulus of elasticity Y	Per unit length spring constant K.	Spring constant k.
0.00	0	0
2769.92	561.13	3.55
2915.71	590.67	3.74
2769.92	561.13	3.55
2885.34	584.51	3.70
2803.57	567.95	3.60
3135.76	635.24	4.02
3242.39	656.84	4.16
3331.24	674.84	4.27
3116.17	631.27	4.00
2697.00	546.36	3.46

From above results, graph of Load F vs change in length of cable ΔL is plotted as shown in below fig.3. From above results it is seen that cable properties follows hook's law.

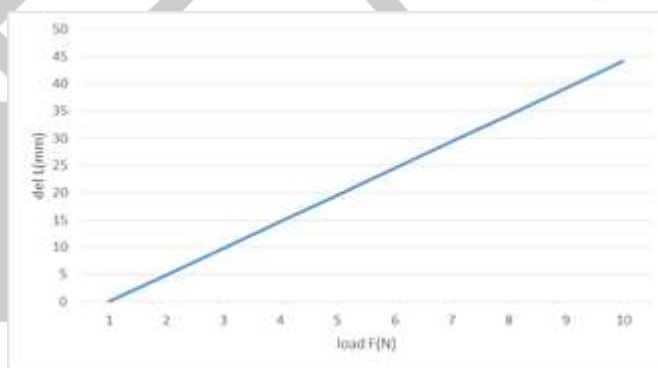


Figure 2. Load vs. ΔL

2.2 Calculation on actual setup:

P is the center point of the end effector platform.

$$P = (x, y, z);$$

$$P = (275, 290, 256.5);$$

A is the end effector point on which cable is connected.

$$A5 = (x + 23, y + 23, z + 10);$$

$$= (298, 313, 266.5);$$

$$A6 = (x + 23, y - 23, z + 10);$$

$$= (298, 267, 266.5);$$

$$A7 = (x - 23, y - 23, z + 10);$$

$$= (252, 267, 266.5);$$

$$A8 = (x - 23, y + 23, z + 10);$$

$$= (252, 313, 266.5);$$

B is the point on block in which hook is connected.

$$B5 = (0, 0, 585);$$

$$B6 = (0, 635, 585);$$

$$B7 = (640, 635, 585);$$

$$B8 = (640, 0, 585);$$

L_t shows theoretical length of cable.

$$L_{t5} = \sqrt{(A_{5x} - B_{5x})^2 + (A_{5y} - B_{5y})^2 + (A_{5z} - B_{5z})^2}$$

$$= \sqrt{(298 - 0)^2 + (313 - 0)^2 + (266.5 - 585)^2}$$

$$= 536.86$$

$$L_{t6} = \sqrt{(A_{6x} - B_{6x})^2 + (A_{6y} - B_{6y})^2 + (A_{6z} - B_{6z})^2}$$

$$= 570.6752$$

$$L_{t7} = \sqrt{(A_{7x} - B_{7x})^2 + (A_{7y} - B_{7y})^2 + (A_{7z} - B_{7z})^2}$$

$$= 622.4228$$

$$L_{t8} = \sqrt{(A_{8x} - B_{8x})^2 + (A_{8y} - B_{8y})^2 + (A_{8z} - B_{8z})^2}$$

$$= 591.5701$$

L shows measured length of cable and ΔL shows difference between measured length and theoretical length.

$$L_5 - L_{t5} = 514 - 536.86 = -22.86 = \Delta L_5$$

$$L_6 - L_{t6} = 554 - 570.68 = -16.68 = \Delta L_6$$

$$L_7 - L_{t7} = 535 - 622.42 = -87.42 = \Delta L_7$$

$$L_8 - L_{t8} = 540 - 591.57 = -51.57 = \Delta L_8$$

Table 3. Constant cartesian co-ordinates

constant cartesian co-ordinates			
	x	y	z
B5	0	0	585
B6	0	635	585
B7	640	635	585
B8	640	0	585

Following tables shows applied load and due to that change in length of cable and different position of endeffector center point.

Table 4. Applied load of 0.2kgf different position of points and change in length.

F = 0.2 kgf = applied load						
	x	y	z	L_t	L	ΔL
P	2	2	2			
	75	90	56.5			
A	2	3	2	536.86	5	-22.86
5	98	13	66.5		14	
A	2	2	2	570.67	5	-16.68
6	98	67	66.5	53	54	
A	2	2	2	622.42	5	-87.42
7	52	67	66.5	29	35	
A	2	3	2	591.57	5	-51.57
8	52	13	66.5	02	40	

Table 5. Applied load of 0.3kgf different position of points and change in length.

F = 0.3 kgf = applied load						
	x	y	z	L_t	L	ΔL
P	2	2	25			
	75	90	1.58			
A	2	3	26	539.7	5	-20.79
5	98	13	1.58	9	19	
A	2	2	26	573.4	5	-14.44
6	98	67	1.58	357	59	
A	2	2	26	624.9	5	-84.95
7	52	67	1.58	548	40	
A	2	3	26	594.2	5	-49.23
8	52	13	1.58	335	45	

Table 6. Applied load of 0.7kgf different position of points and change in length.

F = 0.7 kgf = applied load						
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	x	y	z	L_t	L	ΔL
P	2 75	2 90	25 1.58			
5	A 98	2 13	3 1.58	26 9	539.7 19	5 -20.79
6	A 98	2 67	2 1.58	26 357	573.4 59	5 -14.44
7	A 52	2 67	2 1.58	26 548	624.9 40	5 -84.95
8	A 52	2 13	3 1.58	26 335	594.2 45	5 -49.23

Table 7. Applied load of 0.8kgf different position of points and change in length.

F = 0.8 kgf = applied load						
	x	y	z	L_t	L	ΔL
P	2 75	2 90	24 9.02			
5	A 98	2 13	3 9.02	25 3	541.3 21	5 -20.33
6	A 98	2 67	2 9.02	25 834	574.8 61	5 -13.88
7	A 52	2 67	2 9.02	25 835	626.2 42	5 -84.28
8	A 52	2 13	3 9.02	25 307	595.6 47	5 -48.63

Table 8. Applied load of 1.2kgf different position of points and change in length.

F = 1.2 kgf = applied load						
	x	y	z	L_t	L	ΔL
P	2 75	2 90	24 4.62			
5	A 98	2 13	3 4.62	25 9	543.9 24	5 -19.99
6	A 98	2 67	2 4.62	25 898	577.3 64	5 -13.39
7	A 52	2 67	2 4.62	25 849	628.5 45	5 -83.58
8	A 52	2 13	3 4.62	25 501	598.0 50	5 -48.05

Table 9. Applied load of 1.3kgf different position of points and change in length.

F = 1.3 kgf = applied load						
	x	y	z	L_t	L	ΔL
P	2 75	2 90	24 2.06			
5	A 98	2 13	3 2.06	25 5	545.5 27	5 -18.55
6	A 98	2 67	2 2.06	25 584	578.8 67	5 -11.86
7	A 52	2 67	2 2.06	25 342	629.9 48	5 -81.93
8	A 52	2 13	3 2.06	25 681	599.4 53	5 -46.47

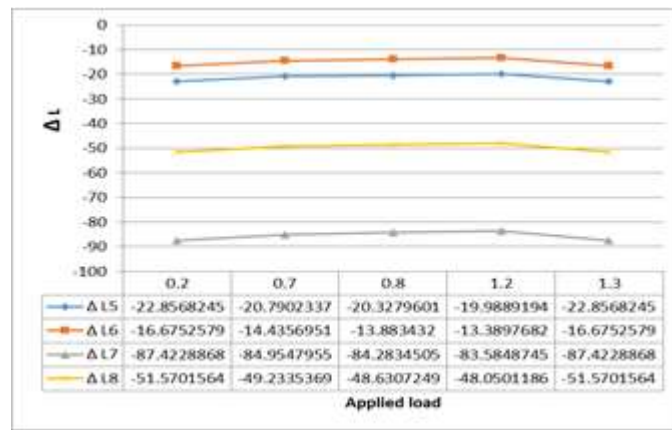


Figure 3. Different load vs. ΔL

From above data, graph of different load applied v/s ΔL is plotted. It is seen that for various load there is difference between actual and theoretical length of cable. This error is modified by compensate it by adding or subtracting value of coordinate point. So perfect position of end effector is obtained.

III. INVERSE KINEMATICS SOLUTION FOR DIFFERENT TRAJECTORIES.

Using Matlab Simulink block inverse kinematics of Cable Driven Parallel Robot (CDPR) is solved. In following example, for Circular trajectory inverse kinematics of CDPR is solved using Matlab Simulink block. It gives theoretical length of cables (we can take four cable driven parallel robot so in output we get different lengths of all four cables) for making any given geometry like Circle, Helical, and Ellipse etc. This data of different cable length is used to generate a program for elastic and sagging compensation of CDPR.

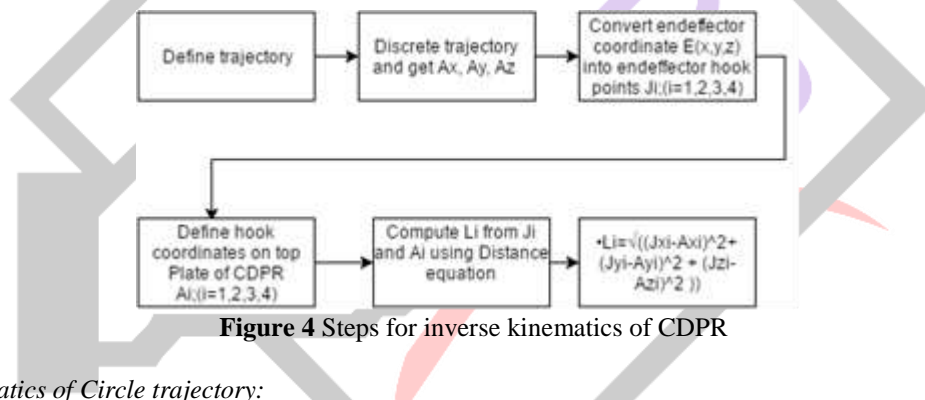


Figure 4 Steps for inverse kinematics of CDPR

3. Inverse kinematics of Circle trajectory:

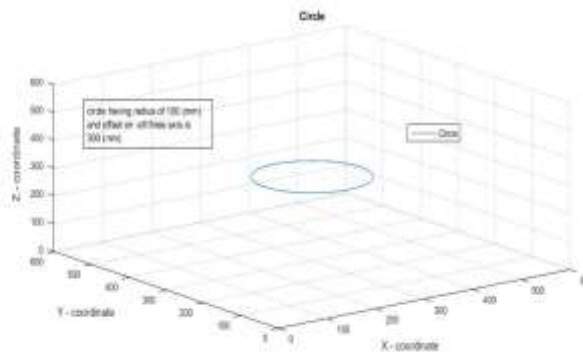


Figure 5. Circle trajectory

Using parametric equation of circle we can plot circular trajectory as shown in above fig.7

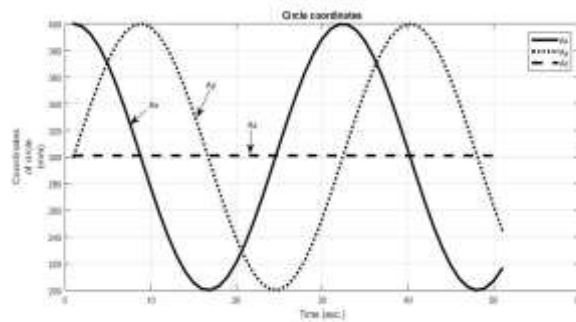


Figure 6. Circle coordinates

Above fig.8 shows graph of Time (sec) v/s coordinates of circle (Ax, Ay, Az).

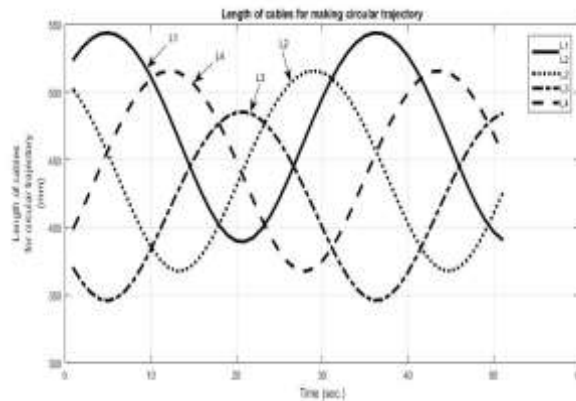


Figure 7. Length of cables for circular trajectory

For making circular trajectory above fig. shows Time (sec) v/s length of cables (L1, L2, L3, L4)

Table 10. Coordinates of circle and length of cables for paarticular time interval.

Time (sec.)	Ax	Ay	Az
0	400	300	301
0.2	398.007	319.867	301
0.4	392.106	338.942	301
0.6	382.534	356.464	301
0.8	369.671	371.736	301
1	354.03	384.147	301
1.2	336.236	393.204	301
1.4	316.997	398.545	301
1.6	297.08	399.957	301
1.8	277.28	397.385	301

Time (sec.)	L1 (mm)	L2 (mm)	L3 (mm)	L4 (mm)
0	523.995	501.569	369.555	399.463
0.2	532.622	492.35	359.504	412.941
0.4	538.893	481.227	351.911	427.397
0.6	542.642	468.522	347.249	442.187
0.8	543.772	454.633	345.824	456.718
1	542.256	440.03	347.733	470.461
1.2	538.131	425.25	352.847	482.958
1.4	531.505	410.896	360.831	493.822
1.6	522.552	397.617	371.195	502.739
1.8	511.517	386.078	383.352	509.463

Above data shows at particular time, discretize geometry coordinates and according to that the different cables length. Matlab automatically takes time interval of 0.2 seconds. And according to that it will automatically discretize geometry in 10 small parts. Here only 10 results shown in table

3.2 Inverse kinematics of helical trajectory:

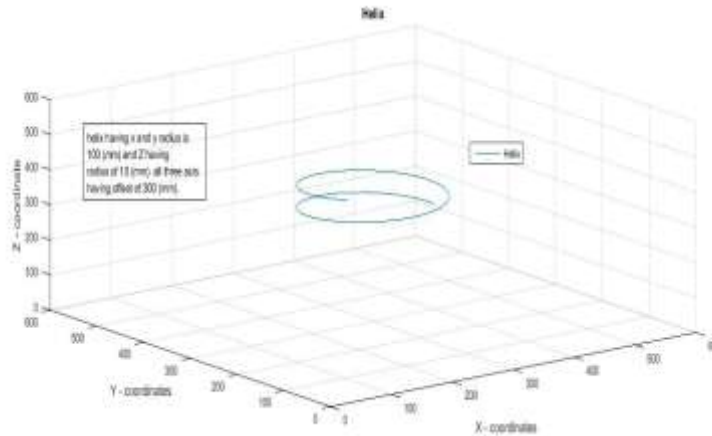


Figure 8. Helical trajectory

Using parametric equation of Helix, we can plot helical trajectory as shown in above fig.10

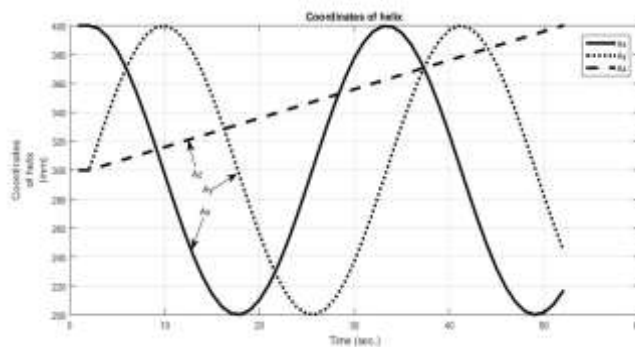


Figure 9. Helix coordinates

Above fig.11 shows graph of Time (sec) v/s coordinates of helix (Ax, Ay, Az).

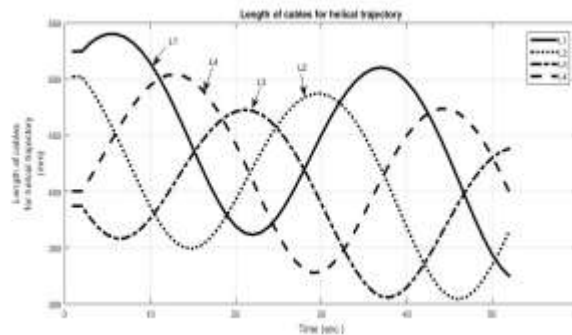
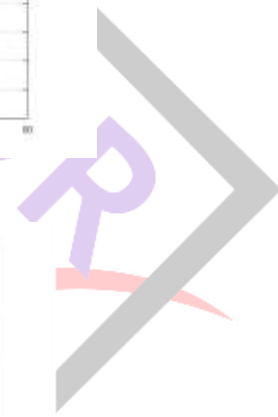


Figure 10. Length of cables for helical trajectory

For making circular trajectory above fig. shows Time (sec) v/s length of cables (L1, L2, L3, L4)

Table 11. Coordinates of helix and length of cables for paarticular time interval.

Time (sec.)	Ax	Ay	Az
0	400	300	300
0.2	398.007	319.867	302
0.4	392.106	338.942	304
0.6	382.534	356.464	306
0.8	369.671	371.736	308
1	354.03	384.147	310
1.2	336.236	393.204	312
1.4	316.997	398.545	314
1.6	297.08	399.957	316
1.8	277.28	397.385	318



Time (sec.)	L1 (mm)	L2 (mm)	L3 (mm)	L4 (mm)
0	524.547	502.145	387.112	400.187
0.2	532.08	491.764	376.094	412.242
0.4	537.29	479.431	367.321	425.374
0.6	539.995	465.454	361.249	438.935
0.8	540.084	450.216	358.192	452.321
1	537.513	434.172	358.281	464.987
1.2	532.305	417.852	361.442	476.457
1.4	524.55	401.859	367.406	486.328
1.6	514.408	386.852	375.746	494.269
1.8	502.108	373.522	385.925	500.016

IV. CONCLUSION

In this paper classification of CDPR was shown. A new and easy method of solving inverse kinematics of CDPR is developed using Matlab Simulink tool and its step by step procedure is shown in block format. By applying this method for getting graph of Time v/s coordinates and Time v/s Lengths of cables for Circular and Helical trajectories were shown. Here fishing line is used as cable, its mechanical property was obtained by using standard method.

Abbreviations and Acronyms

CDPR – Cable Driven Parallel Robot.
 CRPM - Completely Restrained Parallel Manipulator.
 RAMP - Redundantly Actuated Manipulators.
 IRPM - Incompletely Restrained Parallel Manipulator.
 RRPM – Redundantly Restrained Parallel Manipulator.
 IKRM - Incompletely Kinematic Restrained Manipulators.
 CKRM - Completely Kinematic Restrained Manipulators.

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